DECLARATION

I declare that this research is my own work. It is being submitted for the degree of Masters of Philosophy (MPhil) in Mathematics at the University of Mines and Technology (UMaT), Tarkwa. It has not been submitted for any degree or examination in any other University.

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(Signature of Candidate)

………………day of………………(year)

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ABSTRACT

This study uses the Geometric Fractional Brownian Motion (GFBM) model to simulate stock price path and test whether the simulated stock prices mimic the actual stock returns. The model incorporates Hurst index which delimit the constant volatility assumptions and was estimated using the moment generating function. The sample for this study was based on the large Ghanaian companies listed on the Ghana Stock Exchange (GSE). Daily stock price data was obtained from the GSE database over the period January 2018 to December 2018. The results find increasing evidence that, the GFBM model consistently predict the stock price over all time horizon. There was a little above 80% chance that a stock price simulated using GFBM move in the same direction as the actual stock price. Finally, the average percentage error of the GFBM model was 16.68% or an accuracy of 83.32% while the GBM model generated an average percentage error of 20.90% or an accuracy of 79.10%. This indicates that, the GFBM model yielded better predicting accuracy than that of the GBM on almost all the selected stocks in the long-run and partly in the short-run.

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DEDICATION

This work is dedicated firstly to the Almighty God for his kindness and mercies. Again, to my lovely mother, Emilia Ama Dapaah and to my caring Siblings: Florence Okyere and Sophia Okyere. I dedicate this thesis to my former HoD Dr Sampson Takyi Appiah and many other relations whose names I cannot mention all, for their support, advice and words of encouragement. I also dedicate this work to my father, Mr Kwabena Kyere for his support financially and spiritually towards this achievement.

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CHAPTER 1

INTRODUCTION

1.1 Background of the Research

Stock Market can be described as a legal framework surrounding the trading of shares of companies by which prices of publicly listed assets are determined by buy and sell instructions, which arrive at random intervals and in random quantities (Osinubi and Arnaghionyeodiwe, 2003).

According to Estember and Marana (2016), the stock market is one of the few financial institutions that offer higher returns on investment. This is achieved, if future price of stocks traded on the market can be predicted accurately. In view of this, the stock market is seen by many investors, researchers and stock brokers as the platform for lucrative investment (Fafcharnps, 2001).

However, this assertion cannot be made by many potential investors with interest in the Ghana Stock Exchange (GSE), because of inability to make optimal investment decisions due to future price uncertainty (Kyereboah-Colernan and Agyire-Tettey, 2008). As a result, investors face challenges in predicting the behaviour and dynamics of GSE stock price (Chandra, 2008). Although stock trading is noted for its likelihood of yielding high returns, earnings of stock brokers or market players depends on the degree of stock price fluctuations and other market interactions (Mettle *et al.*, 2014).

Stock price prediction focuses on developing a successful model for predicting index values or stock prices. The ultimate purpose is to earn high return by means of well-defined investment which does not conflict with the laws of the market regulators.

However, predicting stock prices is very difficult because of its volatile nature. This makes earnings highly uncertain, which in return is associated with high risks and sometimes significant losses (Segal *et al.*, 2015).

In financial Mathematics, early works such as (Ait-Sahalia and Lo, 1998; Scott, 1991) to solve the problem uses the Geometric Brownian Motion (GBM). The assumption of constant risk in the GBM model was problematic as it lead to some market crashes such as Black-Monday in 1987, the Asian Crises in 1989 and the housing bubble, and this resulted in studies like Bartram *et al.*, (2007); Bakshi and Tso (2002) to research into the area. Later works from (Heston, 1993; Comte and Renault, 1998; Chopin *et al.*, 2013) rejected the constant volatility hypothesis and proposed that in dealing with volatility, a more robust approach may be paramount.

Brownian motion dates back to the nineteenth century when it was discovered by biologist Robert Brown examining particles floating in water under microscope (Reddy and Clinton, 2016). Brown observed that, the pollen particles exhibit a jittery motion, and concluded that the particles were 'alive'. This hypothesis was later confirmed by Albert Einstein in 1905 who observed that, under the right conditions, molecules of water move at random. A common assumption for stock market is that, they follow a Brownian motion, where asset prices are randomly changing over time. This concept has led to the development of a number of models based on radically different theories (Reddy and Clinton, 2016).

Two common approaches for predicting stock prices are those based on the theory of technical analysis and those based on theory of fundamental analysis (Kapoor and Sachan, 2015). Technical theorist assume that history repeats itself, that is, past patterns of price behaviour tend to, recur in the future. The fundamental analysis approach assumes that, at any point in time an individual security has an intrinsic value that depends on the earning potential of the security, meaning some stocks are over-priced or under-priced (Kapoor and Sachan, 2015).

Many also believe in the theory that; stock prices exhibit random walk. The random walk theory is the idea that stocks take a random and unpredictable path, making it difficult to outperform the market without assuming additional risk. The GBM model incorporates this idea of random walk in stock prices through its uncertain component, along with the idea that stocks maintain price trend over time as a certain component. The uncertain component of the GBM model is describe as the product of the stock's volatility and a stochastic process called the Brownian process, which incorporates random volatility and a time interval (Brewer *et al.*, 2013).

For effectiveness and efficiency, Sengupa (2004) claim that, the GBM model must be subjected to the following assumptions.

i. The company is a growing entity and its stock price are continuous in time and value.

ii. Stocks follow Brownian process, meaning only the current stock price is relevant for predicting future prices.

iii. The proportional return of a stock is log-normally distributed.

iv. The continuously compounded return for a stock is normally distributed.

Marathe and Ryan (2005); Bhattacharyya and Timilsina (2010); Higgins *et al.* (2011); Hadavandi *et al.* (2010) argued that, the validation of the GBM model on stocks on the developed stock market gives better prediction whilst those in the developing stock market do not predict better. The reason backing these findings from those studies were that, stock on developing economies usually show less volatility. Therefore, to validate the Geometric Brownian Motion (GBM) model on the GSE, the Hurst exponent is introduction to change the model to Geometric Fractional Brownian Motion (GFBM).

1.2 Statement of the Problem

Predicting stock prices in Ghana has been one of the challenging tasks for investors (Antwi, 2017). Ganai (2019), reiterate that the rise (fall) of an investment is based on the rise (fall) of the stock price and accurate prediction of stock price result in higher returns. The stock market and its associated challenges continue to be the headache of many investors (Gilpin, 2018). Literature proposed by Antwi (2017), suggested that, mathematical model to estimate and predict the behaviour and dynamics of stock prices on the GSE have been made but still need improvement. He stipulated that, most existing literature and models are mainly based on deterministic evaluation of market variables using past data. This means most investors trading on the GSE find it difficult in predicting the behaviour of stocks and returns on their investment.

Even with the few literatures on stochastic modelling of the Ghana Stock Exchange, most of the authors apply Geometric Brownian Motion (GMB) model (Antwi, 2017; Damptey, 2017; Quayesam, 2016). But the deficiencies of the GBM according to Zili, (2006) is that, it cannot account for long-run effect, stochastic volatility and also continuity of price path. The long-run effect tells the volatility nature of the stock and for a high volatile stock, the long-run effect is very difficult to predict using the GBM model as in the case of many developed stock exchange whereas less volatile stocks predict better using the GFBM model as in the case of many emerging stock exchange.

Other techniques such as artificial neural networks, fuzzy logic and support vector machines all have been used to remedy the problem but the problem still persist in their models (Khan *et al.*, 2011; Tiwari *et al.* 2015; Estember and Marana 2016; Hadavandi *et al.* 2010). Hence, this research uses the Geometric Fractional Brownian Motion which takes into account the Hurst index to model stock price in Ghana.

1.3 Objectives of the Research

The objectives of the study are to:

i. develop a GFBM model to predict stock price in Ghana.

ii. estimate parameter of the developed model.

iii. predict stock price using the developed model and compare with existing model.

1.4 Methods Used for this Research

The methods used include:

i. Geometric Brownian Motion process; the benchmark model. It is given by

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t) \tag{1.1}$$

ii. Modified Geometric Fractional Brownian process. It will be used to model the stock price. This can be done by finding a solution to the process:

$$dX(t) = \mu X(t)dt + \sigma X(t)dB_H(t) \tag{1.2}$$

iii. Moment Generating Function (MGF). This will be used to estimate the drift, speed constant and the predictor parameters. The formula for the MGF is given by;

$$M_x(\theta) = E\left(e^{\theta x}\right) = \sum_x e^{\theta x} P(X = x) \tag{1.3}$$

$$M_x(\theta) = E\left(e^{\theta x}\right) = \int_{-\infty}^{\infty} e^{\theta x} f_x(x)\,dx \tag{1.4}$$

equations (1.2) and (1.3) are for discrete and continuous cases of x respectively.

iv. Resealed range analysis (RS), Ito process and Wick calculus.

v. Mean Absolute Percentage Error (MAPE) will be used to check the predicting accuracy of the model. It is calculated as;

$$MAPE = \left(\frac{1}{n}\sum\left(\frac{|Y_t - F_t|}{|Y_t|}\right)\right)100 \tag{1.5}$$

1.5 Organisation of the Research

This thesis is divided into five main chapters. Chapter 1 talks about the background of the study, statement of the problem, objectives of the research, among others. Chapter 2 focuses on literature review. Here various studies on the study area were reviewed in terms of the methods, findings and the limitations. Moreover, the Chapter 3, methodology encompasses the solutions to the various stochastic processes that were applied in the study in order to achieve the desired objectives. Furthermore, Chapter 4 is in two sections; the first section deals with parameter estimation of the developed model and the second section focuses on simulation of price path for the various stocks considered. Finally, Chapter 5 focuses on the conclusion, contributions and recommendations of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter outlines the history and development of the Ghana Stock Exchange and how price of stock can be modelled through the theoretical frame work which will include proxies on the exchange such as volumes of stock traded, liquidity of the capital market, efficient market hypothesis, volatility, opening price and other economic indicators as well as review of studies that relates to Geometric Fractional Brownian Motion.

2.2 History and Development of the Ghana Stock Exchange

As part of the Financial Sector Adjustment Programme (FINSAP), the Ghana Stock Exchange was established in July, 1989, as a private company limited by guarantee under the country's Companies' Code of 1963 (Act 179). Plans of the establishment dates back to the 1960s, when a government study concluded that the establishment of a stock market was essential for the economic development of the country. This led to the promulgation of the Stock Market Act of 1971. The act laid the foundation for the establishment of the Accra Stock Market Ltd. (ASML) in 1971 (Apio, 2014).

However, the idea of establishing a stock market failed because of political tensions, unfavourable economic environment and the lack of government support. In spite of these unsuccessful attempts, two stock brokerage firms, namely National Trust Holding Company Ltd. (NTHC) and National Stockbrokers Ltd., now Merban Stockbrokers Ltd. did Over-The-Counter (OTC) trading in shares of some foreignowned companies prior to the establishment of the Ghana Stock Exchange (Amo-Yartey, 2006).

The GSE started operations in November 1990. The set-up was made up of 3 stockbrokers, 11 equity listings and 1 Commemorative bond. The processes were mostly manual. Trading was based on a call over system that was conducted three times each week to fix the price of each listed security. Clearing and settlement was manual. The three days a week was later expanded to daily price fixing. After that the call-over was replaced by continuous trading, still manual with traders posting their orders on whiteboards. The present operations of the GSE are in stark contrast to how the GSE started. Today the GSE can boast of the following: 42 Listed equities, 21 Licensed Dealing Members, 97 Government bonds, fully automated trading system, Central securities depository, Alternative Market (GAX) for SMEs, Ghana Fixed Income Market and GHS 2.1 billion in equity finance (Manu, 2017).

Despite these improvements the exchange has faced several difficult challenges and disappointments. One of these challenges is low liquidity. According to Drehmann and Nikolaou (2013) market liquidity refers to the ability of buyers and sellers of securities to transact efficiently and is measured by the speed with which large purchases and sales can be executed and the transaction costs incurred in doing so. The GSE liquidity as measured by the market turnover ratio is well below key African markets such as Botswana, Nigeria, Kenya and South Africa (Piesse and Hearn, 2005).

Low liquidity increases volatility thus creating additional risk for investors. If there are many potential buyers and sellers and they can transact quickly, easily, and cheaply, then price movements tend to be smoother as news events are factored into prices quickly based on the market consensus about their significance. The high volatility of the market is reflected by the fact that, in the five-year period 2011-2015, returns on the GSE Composite Index ranged between 78.81% in 2011 and 76.12% in 2015 with a fall of -2.69%. Such volatility drives away investors (Shiller,1990).

2.3 Performance of the Ghana Stock Exchange

The Ghana Stock Exchange, although relatively young has performed remarkably well in terms of returns on investment. For example, in 1994, it was ranked the 6th best performing stock market index among all the emerging markets, gaining 124.3 percent; an assertion made by Birinyi Associates, a Research Group based in the USA. It was also adjudged the best performer among all stock markets in Africa and the third best in emerging markets in 1998 in terms of capital appreciation by the Standard Chartered Bank London Limited. The GSE was again adjudged the world's best-performing market at the end of 2003 with a yearly return of about 154.7% (or 144 % in US dollar terms) compared with 30 percent return by Morgan Stanley Capital International Global Index (Adjasi and Yartey, 2007).

The performance of the composite index has, however, not always been remarkable. For instance, in 1999, 2005, 2009 and 2011, the composite index experienced negative returns with -15.22%, -30%, -47% and -3.1% respectively. The growth rate of almost -47% in 2009 was the worst since trading commenced on the exchange. Financial analysts attribute it to the spill over effects of the global financial crisis of 2007 and 2008. The 2005 performance was attributed to rising oil prices, inflation and interest rates (Nkwede *et al.*, 2016).

The composite index rose sharply from 69.77 points in 1991 to 334.02 points in 1994 representing a gain of 124.34% before fluctuating in 1995 to 6.33%. The fall is attributed to high levels of inflation and interest rate at the period. Thereafter, the trend of the composite index showed both increasing and decreasing trends but the most significant of the period is in 2003, 2004, 2008 and 2013 where it gained 154.67%, 91.33%, 58.06% and 78.81% respectively. The composite index performance in 2017 is also worth mentioning. It reached 1,698.20 points representing a gain of 52.73% in the composite index.

The GSE market index summary from 1990 to 2018 is shown in Figure 2.1. The good performance of the stock market in 1994, 1998, 2003, 2008, 2013 and 2017 according to Bhuyan and Dash (2018) is attributed partly to favourable macroeconomic indicators (inflation, interest rate) and mainly to the listing of the Ashanti Goldfields Company Limited in 1994. In 1998 in particular, there was high demand for equity shares on the market that led to a remarkable increase in share prices on the market.

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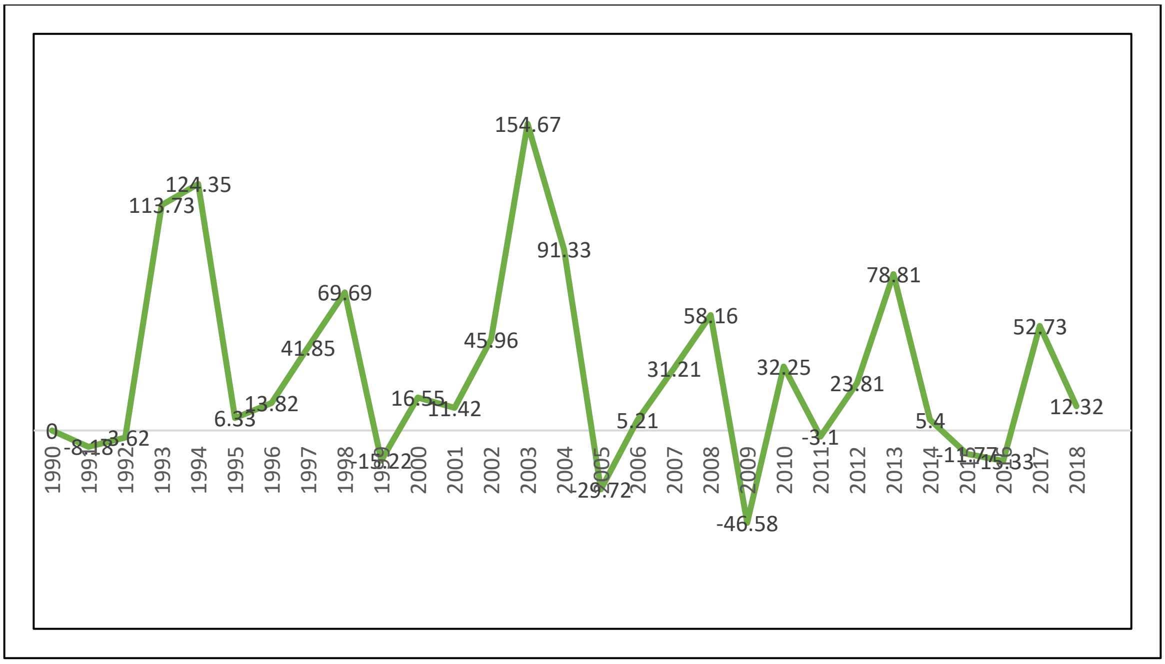

Figure 2. 1 Trend of GSE Market Returns

2.4 Liquidity of the Ghana Stock Exchange

Liquidity generally refers to the ability to buy and sell securities easily (Demirgüç-Kunt and Levine, 2018; White and Mala, 2006). Two main traditional stock market performance indicators used to gauge market liquidity are total value traded ratio and turnover ratio. The volume and value of shares traded have improved considerably over the years. The value of shares traded in particular rose substantially from GHS 7.31 million in 1994 to GHS 65.59 million in 2004 primarily due to the listing of the Ashanti Goldfields Limited and AngloGold Ashanti through a merger as noted earlier and other companies (Awiagah and Choi, 2018).

The trend of the liquidity of the GSE relative to the size of the economy shows a rising and falling trend through the various years. Its highest point was in 1994 with a market liquidity of 1.40%. The second highest peak is in 2008 with a market liquidity of 1.21%. Its lowest was in 2012 with a liquidity of 0.14%. The liquidity of the GSE relative to the economy of Ghana as a whole has been relatively small, especially, with periods prior to 1994 before jumping to 1.4% in 1994 (Korkpoe and Howard, 2019).

The average liquidity of 0.45% of the Ghana Stock Exchange is far below the 1990s world average of 31% which is indicative of the fact that the Ghana Stock Exchange is relatively small relative to the size of the economy as already indicated in Amo-Yartey (2006). The other indicator of stock market liquidity is the turnover ratio. The turnover ratio is the total value of shares traded during the period divided by the market capitalization for the period. It measures the activity of the stock market relative to its size. The turnover ratio is often used to capture the efficiency of the domestic stock market. High turnover ratio is used as an indicator of low transaction costs which can be attributed to the efficient market hypothesis (Cici *et al.*, 2018).

2.5 Efficient Market Hypothesis (EMH)

The efficient market hypothesis, alternatively known as the efficient market theory, is a hypothesis that states that share prices reflect all information and consistent alpha generation is impossible. According to the EMH, stocks always trade at their fair value on exchanges, making it impossible for investors to purchase undervalued stocks or sell stocks for inflated prices. Therefore, it should be impossible to outperform the overall market through expert stock selection or market timing, and the only way an investor can obtain higher returns is by purchasing riskier investments.

2.5.1 Empirical Evidence from Developed Financial Market

Earlier studies mostly probed into the behaviour of developed financial markets, mostly of European and US financial markets. Traditionally markets of developed economies are more efficient as compared to emergent markets (Gupta and Basu, 2006). Rossi and Gunardi (2018) investigated British industrial and US commodity share price indices. The study supported random walk on zero correlation rationale. Similar rationale was provided by Dimson and Mussavian (2000) with small sample. Cootner (1962) picked 45 stocks from New York stock exchange and found similar results at low levels of correlation. Lo and MacKinlay (1988) conducted a vital study on US security prices for the period 1962-1985, by first introducing variance ratio test. The study rejected random walk based on positive serial correlation of weekly and monthly returns. Fama and French (1988) conducted a study on New York Stock Exchange (NYSE) stocks for the 1926-85 period and found large negative autocorrelations for longer periods. Poterba and Summers (1986;1988) applied variance ratio test on Standard and Poor composite stock index for the period 1928-1984, for US stocks market as whole for the period 1871-1986, and for sixteen other countries for 1957-1985. They rejected the random walk and found the evidence of positive serial correlation over short periods and negative autocorrelation for longer periods. Contradictory to this Lee and Rui (2002) found existence of random walk for the stock markets of US and ten other industrialised nations namely United Kingdom, France, West Germany, Australia, Belgium, Netherlands, Switzerland, Italy, Japan and Canada, for 1967–1988. Similarly, Choudhry (1994) examined stock indices of United States, United Kingdom, Canada, France, Japan, Italy and Germany for the period 1953–1989, by applying unit root test and Johansen method of cointegration using monthly return series and also found unit root and presence of random walk in all stocks.

Poon (1996) tested UK stock markets for random walk, serial correlation, and persistence of volatility and found presence of random walk. Al-Loughani and

Chappel (1997) found heteroscedasticity in FTSE 30 index of London Stock Exchange for the period 1983-1989, by employing Lagrange Multiplier (LM) serial correlation test, unit root test and Generalized Autoregressive Conditional Heteroscedasticity-Mean (GARCHM) model. Chan *et al.*, (1997) conducted a study on 18 international stock markets (Australia, Belgium, Canada, Denmark, Finland, France, Germany, India, Italy, Japan, Netherlands, Norway, Pakistan, Spain, Sweden, Switzerland, the United Kingdom, and the United States) with 16 amongst them belong to developed world and the rest two; Pakistan and India among the emergent markets for the period 1962-1992. The study was aimed at testing weak-form efficiency. The result from unit root testing revealed the weak-form efficiency in developed market. However, cointegration test revealed significant cointegration in the return series. Groenewold (1997) vetted the markets of Australia (Statex Actuaries 'Index) and New Zealand (NZSE-40 Index) for the period 1975-1992. The study tested weak-form and semi strong form efficiency in those markets and used stationarity and autocorrelation tests and found result consistent with weak-form efficiency. However, the granger causality rejected the semi-strong form and at the same time revealed cointegration between the two stock markets.

Lee *et al.*, (2000) tested French futures and options markets using unit root and variance ratio tests. The study found presence of random walk in the markets. Worthington and Higgs (2004) examined sixteen European equity markets for random walk including, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom and four emerging markets of Czech Republic, Hungary, Poland and Russia. Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and multiple variance ratio (MVR) tests were applied. It was found that only Hungary amongst the emergent markets and Germany, Ireland, Portugal, Sweden and the United Kingdom amongst the developed markets follow random walk criterion. Gan *et al.* (2005) looked into the stock markets of New Zealand, Australia, US and Japan for the period 1990-2003, and reaffirmed the findings of Groenewold (1997) except for the granger causality between New Zealand and Australian stock markets. The study used conventional methods (ADF and PP unit root test) for finding efficiency levels. Nakamura and Small (2007) used small-shuffle surrogate method to investigate random walk on Standard and Poor's 500 in US market and Nikkei225 in Japanese market, exchange rate and commodity markets and found existence of random walk in markets whose first differences are independently distributed random variables. Torun and Kurt (2008) conducted a study on European Monetary Union Countries taking panel data of stock price index, consumer price index and purchasing power of euro for the period 2000-2007 to investigate weak-form and semi-strong efficiency. The study used panel unit root test, panel cointegration and causality test and found result consistent with weak-form efficiency. Borges (2010) investigated the stock markets indices of France, Germany, UK, Greece, Portugal and Spain, from January 1993 to December 2007 for the presence of random walk by taking monthly and daily stock returns. He used both parametric and non parametric tests including serial correlation test, runs test, multiple variance ratio test proposed by Lo and MacKinlay (1988), and ADF test. Evidence of random walk was found in all six countries for monthly returns. However, for the daily returns hypothesis of random walk was rejected for Greece and Portugal. Shaker (2013) tested the weak-form efficiency of Finnish and Swedish stock markets by using ADF, variance ratio test proposed by Lo and MacKinlay (1988). This particular study rejected the hypothesis of random walk in these markets. The above empirical literature revealed the evidence weak-form efficiency and random walk in most of the developed financial markets.

2.5.2 Empirical Evidence from Emerging Financial Market

An emerging economy is a transitional phase between a developed and developing economy. Compared to developed markets, emerging markets are relatively isolated from capital markets of other countries and have relatively low correlation with developed markets. But during last two decades huge amount of capital inflow from developed economies as a result of globalization and liberalization of financial markets have attracted the researchers to investigate the implications of these changing trends on market efficiency of emerging markets. Therefore, particular attention is being paid by researchers to find trends in emerging markets. However, contribution of equity markets in the process of development in developing countries is less and the resultant is weak markets with restrictions and controls Gupta and Yang (2011). In emerging stock markets, stock price manipulation by intermediaries (brokers) is a common issue. Greater returns by inside traders (brokers) than outside traders in emerging markets accounts for weak market reforms and limited capital increase (Khwaja and Mian, 2005). China's worst stock market crime came out as a result of collusion of brokers in the market (Green 2004). In 2005, the Securities and Exchange Board of India barred 11 brokers for engaging in price manipulation. An intermediary (broker) can manipulate outcomes in equilibrium without losing credibility in the market (Khwaja and Mian, 2005; Siddiqi, 2007). Therefore, efficiency levels in emerging economies are sensitive to the manipulation capacities in the markets (Magnusson and Wydick, 2002).

Areal and Armada (2002) studied Portuguese stock market to check for weak-form efficiency. Parametric and non-parametric test were used and found mixed evidence mostly sensitive to methodology used. The study did not reject weak-form efficiency. Siourounis (2002) investigated Athens stock exchange (ASE) for weak-form efficiency and heteroscedasticity from 1988-1998. The study employed GARCH model and concluded that current volatility is positively related to past realizations. It was also concluded that negative shocks have an asymmetric impact on returns. Chow test, Granger causality test and Newbold test were used for non linearity. Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests with first difference log values of return series were employed to check unit root in the daily return series. The two tests failed to reject random walk at 5% significance level. However, for first difference there is no evidence of unit root. Later another study examined Athens stock market (Samitas and Kenourgios 2004) by using Johansson's Maximum likelihood procedure and unit root test which showed consistency with the former study. While a recent study by Dicle and Levendis (2011) revealed appearance of inefficiency with DOW effects after performing runs test and Granger causality test on Athens stock market. Smith and Ryoo (2003) analysed five European emerging markets namely Greece, Hungary, Poland, Portugal and Turkey, by employing multiple variance ratio test. The hypothesis of random walk was rejected in all markets except for Istanbul stock exchange due to higher turnover than other markets. Guidi *et al.*, (2011) investigated Central and Eastern Europe (CEE) equity markets for the period 1999-2009. Study used autocorrelation analysis, runs test, and variance ratio test for test the hypothesis of random walk. It was concluded that most of the CEE markets don not follow random walk and abnormal profits can be accrued by a well-informed investor.

Another study on Istanbul stock exchange (ISE) National 100 index was conducted by Kapusuzoglu (2013) for the period 1996-2012 found contradictory evidence as compared to the findings of Smith and Ryoo (2003). The study aimed at detecting the presence of random walk in the returns by using unit root test on daily stock returns and rejected the hypothesis of random walk.

A very recent study by Dragota and Tilica (2014) examined Post Communist East European Countries. The study aimed at tracing any improvement in efficiency based on the past record and used 20 countries namely Bosnia-Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Georgia, Hungary, Kazakhstan, Latvia, Lithuania, Former Yugoslav Republic of Macedonia, Moldova, Montenegro, Poland, Romania, Russia, Serbia, Slovakia, Slovenia, and Ukraine for the period 2008-2010, a period of financial crises. Unit root tests, runs test, filter rules test and variance ration tests were used.

However, the results were not consistent in all markets. Moreover, the heterogeneity of results was revealed suggesting variable portfolio management techniques for different levels of market efficiency. Mixed results were observed in case of Eastern European financial markets with traces of weak form efficiency in stock exchanges of Athens and Turkey. Urrutia (1995) scrutinised Latin American emerging markets namely Argentina, Brazil, Chile and Mexico stock markets for random walk hypothesis for the period 1975-1991. By applying variance ratio test it was found that serial correlation is present in all markets. However, runs test indicated weak-form efficiency in studied Latin American markets. However, when parametric test along with non-parametric test were applied by Worthington and Higgs (2003) to investigate weak-form market efficiency in Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela; it rejected the random walk hypothesis. In another study Mexico and Brazil stocks markets are re-examined for random walk by Grieb and Reyes (1999) by employing variance ratio tests on individual firms and on indices. The result revealed greater tendency of Brazil stock index for random walk than for Mexican stock markets.

Smith *et al.* (2002) conducted a study on five medium-sized African stock markets namely, Egypt, Kenya, Morocco, Nigeria and Zimbabwe and two small and comparatively newer markets of Botswana and Mauritius for testing the hypothesis of random walk. The stock market of South Africa was also put under question of random walk. The study used multiple variance ratio tests of Chow and Denning (1993). The hypothesis of random walk was rejected in all seven markets, except for South African market which was found to follow random walk. Another study in the same era by Magnusson and Wydick (2002) found presence of weak form efficiency on monthly returns of six out of eight African markets namely Botswana, Cote d 'Ivoire, Kenya, Mauritius, South Africa, Ghana, Nigeria and Zimbabwe. The results were then compared with US stock market, Latin American and Asian emerging markets and it was concluded that efficiency levels are sensitive to the efficiency hurdles in developed and emerging economies and market manipulation capacities. Shamsir and Mustafa (2014) tested weak-form efficiency of 11 African stock markets comprising of Nigeria, Egypt, Kenya, Zimbabwe, Mauritius, Morocco, Botswana, Ghana, Ivory Coast, Swaziland and South Africa. Conditional volatility (heteroscedasticity) was captured by using exponential GARCH-M model. Their result showed evidence of weak-form efficiency in Egypt, Kenya, and Zimbabwe. The result also revealed traces of efficiency in Mauritius and Moroccan stock exchanges.

Rejection of weak-form efficiency in Botswana stock market was also found very recently when parametric and non parametric (autocorrelation test, Kolmogorov Smirnov Test, Runs Test, ADF and Phillips-Parron (PP) unit root test) were applied (Chiwira and Muyambiri, 2012). Jefferis and Smith (2005) studied changing patterns of market efficiency of African stock markets of South Africa, Egypt, Morocco, Nigeria, Zimbabwe, Mauritius and Kenya over time. The study period started in early 1990's and ended in June 2001. GARCH approach with time varying parameters and test of evolving efficiency (TEE) were used to detect efficiency over the period of time. The study found Johannesburg stock market (JSE) weak-form efficient throughout the study period, while Egypt, Morocco and Nigeria became efficient at the end of the period. However contradictory results were revealed with respect to Shamsir and Mustafa (2014) in case of Kenya and Zimbabwe stock markets which show no tendency towards weak form efficiency over time. Mauritius stock market exhibits slow tendency to eliminate inefficiency, which is consistent with (Shamsir and Mustafa 2014).

Gupta and Basu (2007) investigated market efficiency on emerging African stock markets of Egypt, Kenya, Zimbabwe, Morocco, Mauritius, Tunisia, Ghana, Namibia, Botswana and the West African regional stock exchanges. Non parametric tests (Kolmogorov-Simirnov correlation test and runs tests) were applied. Random walk was rejected except for Namibia, Kenya and Zimbabwe. The results are consistent with Shamsir and Mustafa (2014) in case of Kenya and Zimbabwe.

Mlambo and Biekpe (2007) examined ten African stock markets including, Botswana, Egypt, Ghana, Johannesburg, Kenya, Mauritius, Morocco, Namibia, Tunisia and Zimbabwe and West African Regional Stock Exchange (Bourse Regionale des Valeurs

Mobilieres (BRVM). In order to cater thin- trading in almost all the markets returns were calculated on trade-to-trade basis. In Namibia random walk hypothesis was not rejected due to its correlation with Johannesburg stock exchange. Similarly, Kenya and Zimbabwe were not rejected as weak-form efficient. On the other hand, Mauritius, Egypt, Botswana and BRVM deviated from random walk hypothesis. The study suggested the need for non linear serial correlation testing in these markets for testing efficiency level, since markets with weak microstructures where return generating process is expected to be non-linear. Therefore, a test on linear correlation could lead to wrong inferences.

2.5.3 Empirical Evidence from Ghana

According to the systematic study of Afego (2015), it was observed that most studies have empirically proven that the returns on the Ghana Stock Exchange (GSE) are predictable. Most of these studies, which were of the weak-form type of the Efficient Market Hypothesis (EMH), concluded that the GSE was weak-form inefficient. This means that the market is predictable and an investor can make abnormal returns by simply studying past prices on the GSE. For example, Osei (1998), studied the returns on the Ghana stock market between 1993 and 1995 using serial correlations. It was concluded that the market was not weak-form efficient. Also, Magnusson and Wydick (2002) studied countries including South Africa, Nigeria, Ghana and Zimbabwe using PACF and White test. The conclusion from the study was a weak-form inefficiency for Ghanaian market.

Simons and Laryea (2005) analysed the South African, Ghanaian, Mauritian and Egyptian markets using runs test, Multiple Variance Ratio (MVR) test and ARIMA and concluded that the GSE is weak-form inefficient. Also, Smith (2008) studied the South African, Zimbabwean, Ghanaian and Nigerian markets using MVR tests. Again, none of the markets was found to be weak-form efficient. The inconsistency of the EMH on the GSE by the literatures cited above makes the stock market vulnerable and with the right mathematical model in its prediction an investor can make abnormal returns.

2.6 Geometric Fractional Brownian Motion (GFBM)

In financial mathematics the Black-Scholes option pricing model consists of a risky asset, stock $S(t)$ and a risk-free asset, a bond. The risky asset is a stochastic process $S(t)$ which follows a geometric Brownian motion and is defined by the stochastic differential equation;

$$dS(t) = \mu S(t)\,dt + \sigma S(t)\,dB_H(t) \tag{2.1}$$

In the geometric Brownian motion model, the returns are independent of each other, that is, today's price change has no correlation with previous price changes. Some studies (Mandelbrot, 1967) have shown long-range dependency does exist between the returns in some markets. It is proposed to replace Brownian motion in modelling derivatives with fractional Brownian motion $B^H(t)$.

2.6.1 Empirical Evidence from the world

Long-range dependency has been investigated by many literatures. It has been shown that many of the emerging markets do exhibit a Hurst exponent that is larger than 0.5, thus implying that the returns have long-term memory. Cheung and Lai (1995) investigated long memory in 18 countries and only 5 showed persistent behavior. Cajueiro and Tabak (2003) investigated the Brazilian equity market and found persistency more importantly their results suggest that the Hurst parameter is time varying even after adjusting for short-range dependency. Cajueiro and Tabak (2004) investigated 11 emerging markets and the U.S. and Japan, their results concluded significant long range-dependency in Asian countries, less in the Latin American countries, except Chile, the U.S. and Japan were the most efficient. Sadique and Silvapulle (2001) found persistency in Korea, New Zealand, Malaysia, Singapore, while no or little evidence of persistency was found in Japan, the U.S. and Australia. The returns of the Standard and Poor's 500 and the Dow Jones Industrial Average returns did not display trend reinforcing behaviour. Lo (1989) found little evidence of long-term memory in U.S. stock market returns.

Cheung (1993) investigated long memory in foreign exchange rates and found evidence of long-memory. Wei and Leuthold (2000) investigated the agricultural market and found long memory in the sugar market. Jamdee and Los (2007) show evidence of long memory on European options through a time-dependent volatility. Qian and Rasheed (2004) suggests that if a stock time series has a high Hurst exponent, then the stock will be less risky and there will be less noise in the data set. Motivated by these results the application of fractional Brownian motion is proposed. Replacing Brownian motion with the fractional Brownian motion is suggested to reduce model risk. Fractional Brownian motion is self-similar and captures long-range dependency. The fractional option pricing models depend on an extra parameter, the Hurst parameter H.

2.6.2 Hurst Parameter

The Hurst parameter $0 \le H \le 1$ classifies a time series into three different groups. If $H = 0.5$ then events follow a random walk. The returns are uncorrelated and random. If $0 \le H \le 0.5$ then the time series is said to have anti-persistent behaviour, that is mean reverting and if $0.5 \le H \le 1$ then the time series is said to have persistent behaviour, that is trend reinforcing. If the stock prices have a $H > 0.5$ this shows that long-range dependence exists in the stock prices. Long-range dependency is the same as a long-memory process where past events have a decaying effect on the future. Mandelbrot (1982) pointed out two characteristics of the stock market price behaviour and called them the Noah and Joseph effect. The Noah-effect refers to the observed instances of large discontinuous jumps in the stock prices, or outliers. The Joseph-effect refers to the tendency of the stock prices to have long term trends with non-periodic cycles.

Fractional Brownian motion is a continuous Gaussian process that depends on the Hurst parameter H and is defined by its covariance function. When $H = 0.5$, fractional Brownian motion becomes the ordinary Brownian motion.

Mandelbrot and Van Ness (1968) defined a stochastic integral representation of fractional Brownian motion. When $H \ne 0.5, B^H(t)$ is not a semi martingale, and therefore the application of classical Itô calculus is not possible. Incorporating fractional Brownian motion to price options using partwise integration theory is not possible as it allows for arbitrage possibilities. Under partwise integration fractional Brownian motion does not have zero expectation, which already implies the possibility of a riskless gain. Duncan *et al.* (1991) introduce another integration theory based on the Wick product and a so-called Wick Itô Skorohod integral for fractional Brownian motion. The Wick Itô stochastic integral has a zero expectation.

2.7 Volatility

Volatility is a measure of the spread of positive and negative outcomes, unlike risk which is a measure of uncertainty of the negative outcome of some event/process like the stock market returns. A good forecast of asset price volatility over the investment period is a good process towards the assessment of investment risk. There are two general classes of volatility models, namely:

Volatility models that formulate the conditional variance directly as a function of observables (including historical and implied volatility) and others like the ARCH and GARCH models that are not functions of purely observable parameters like the stochastic volatility models. The stochastic volatility model is very popular in option pricing where semi-closed form solution exists.

Ruotolo *et al.* (2008) assert that stochastic volatility models are less common as time series model when compared with GARCH models, since the estimation of stochastic volatility model using time series data is a non-trivial task. This is because maximum likelihood function cannot be written straightforwardly when the volatility itself is stochastic. Stochastic models are usually approximated through Markov Chain Monte Carlo methods. These stochastic volatility models are usually simulated, and they are difficult to estimate.

A good volatility model should be able to forecast volatility, which is the central requirement in almost all financial applications. In modelling volatility of a financial system, one should take into cognizance the stylized facts of volatility which include:

pronounced persistence and mean reversion, asymmetry such that the sign of an innovation also affects volatility, and the possibility of exogenous or pre-determined variables affecting volatility, Patton (2001). Essentially, all the financial uses of volatility models entail forecasting aspects of future returns and a typical volatility model used to forecast the absolute magnitude of returns can also be used to predict quartiles or the entire density.

The forecasts of volatility for absolute magnitude of returns are therefore applied by the stakeholders in financial industry in risk management, derivatives pricing and hedging, market making, market timing, portfolio selection, and a host of other financial activities. Volatility is the most important variable in the pricing of derivative securities, the volume of which in the world trade has increased tremendously in recent years. To price an option, one needs to know the volatility of the underlying asset from the time of entering into the contract to expiration date of the contract.

Poon and Granger (2003) assert that nowadays it is possible to buy derivative written on volatility itself, in which case the definition and measurement of volatility will be clearly specified in the derivative contracts. In such case, volatility forecast and a second prediction on the volatility over the defined period is needed to price such derivative contracts.

A risk manager should know as at today the likelihood that his portfolio will rise or decline in future just like a stakeholder in option contract would wish to know the expected volatility over the entire life span of his contract. A farmer on his own side may wish to write a forward contract to sell his agricultural product, to hedge against fall in price of his produce at the time of harvesting and so on. Dynamic risk management uses the correct estimate of historical volatility and short-term forecast in risk management process. Volatility (historical) is, therefore, from Poon and Granger (2003) given by:

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(r_t - \mu\right)^2} \tag{2.2}$$

where $r_t = \log\left(\frac{X_t}{X_{t-1}}\right)$, μ is the expected return in a quantified measure of market risk.

The main characteristic of any financial asset is its return which is considered as a random variable. The spread of this random variable is known as asset volatility which plays pivotal role in numerous financial applications. The primary role is to estimate the market risk and serve as a key parameter for pricing financial derivatives like the option pricing as seen earlier. It is also used for risk assessment and management and to a larger extent in portfolio management.

2.7.1 Market Risk

Market risk is one of the main sources of uncertainties for any financial establishment that has a stake in given risky asset(s). This market risk refers to the possibility that an asset value will decrease owing to changes in interest rates, currency rates, and the price of securities. The method of estimating a financial institution's exposure to market risk is the value-at-risk methodology. The value at risk methodology adopts a system of dynamic risk management whereby the market risk is monitored on daily basis. GARCH models, as stated above, are also referred to as volatility models and are usually formulated in terms of the conditional moments. GARCH (p, q) lags denoted by GARCH (p, q) has a volatility equation written as:

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$$\sigma_t^2 = \vartheta_0 + \vartheta_1\varepsilon_{t-1}^2 + ... + \vartheta_p\varepsilon_{t-p}^2 + \lambda_1\sigma_{t-1}^2 + ... + \lambda_p\sigma_{t-p}^2 \quad (2.3)$$

when the coefficient of the term σ_{t-1}^2 is insignificant in GARCH (1, 1) model, the implication is that ARCH (1) model is likely to be good enough for the volatility data estimation.

As stated earlier in the stylized facts, financial asset returns (stock returns) exhibit volatility clustering, leptokurtosis and asymmetry. These characteristics of asset return indicate increase in financial risk which can affect investors adversely. Volatility clustering refers to the situation when large stock price changes are followed by large price change, of either sign, and similarly small changes are followed by periods of small changes. Leptokurtosis refers to the market condition where the distribution of stock return is not normal but rather exhibits fat tails. In other words, leptokurtosis means that there are higher propensities for extreme values to occur more regularly than the normal law predicts in a series.

Asymmetry, otherwise known as leverage effect, means that a fall in asset return is followed by an increase in volatility greater than the volatility induced by increase in returns. These three characteristics mentioned above make investors to pay higher risk premium to insure against the increased uncertainty in the portfolio of investments. Volatility clustering for instance makes investors to be more averse to holding stocks due to high stock price uncertainty. Emenike (2010) advocates for the use of GARCH (1,1) model to capture the nature of volatility, the Generalised Error Distribution (GED) to capture fat tails, Alberg *et al.* (2008), GDR-GARCH (1, 1) model which is a modification of GARCH (1, 1) to capture the leverage (asymmetry) effects of stock return.

2.7.2 Implied Volatility (IV)

The market's assessment of the underlying assets volatility as reflected in an option is known as implied volatility (IV) of the option. This is obtained through an observation of the market price of the option, and through an inversion of option pricing formula, we can determine the volatility implied by the market, Mayhew (1995). In other words, given the Geometric Brownian motion, with some other assumptions, Black and Scholes (1973) obtained exact formula for pricing European call and put options.

Usually, options are traded on volatility with implied volatility serving as an efficient and effective price of the option and therefore implied volatility is important in financial assets risk management. To this end, investors can adjust their portfolios in order to reduce their exposure to those instruments whose volatilities are predicted to be on the increase, thereby managing effectively their exposure to risk in investment.

Traditionally, due to their robustness, implied volatility (IV) has been calculated using either the BS formula or the Cross-Ross-Rubinstein binomial model for option pricing, and from the underlying stock price assumption of the BS model, IV could be interpreted as the option market's estimate of the constant volatility parameter.

The BS assumption of constant variance does not hold exactly in the markets due to jumps in the underlying asset prices, movement of volatility over time, transaction cost on the assets, and non-synchronous trading which will therefore cause the observed implied volatility to differ across options.

If the underlying asset volatility, as opposed to the assumptions of the BS mode, is allowed to vary deterministically over time, IV is interpreted as the market's assessment of the average volatility over the remaining life of the option. However, when the options pricing formula cannot be inverted analytically as is usually the case, IV is calculated through numerical approximations.

Many options with varying strike price and time to expiration could be written on the same underlying asset and by the BS model (with constant variance) these options should be priced so that they all have exactly the same IV which of course is not true. This systemic deviation from the predictions of the BS constant variance model is referred to as "volatility smile". Volatility smile refers to the use of different values of implied volatility by practitioners in the derivatives contract for different strike prices. As IV are not necessarily the same across the life span of the option, some literature suggested calculating implied volatilities for each option and then using a weighted average of these implied volatilities as a point estimate of future volatilities.

2.8 Speculation

As noted, the major motivation for entering into a forward or futures and in fact any derivatives contract is to speculate and or hedge an existing market exposure so as to reduce cash flow uncertainties resulting from the market exposure. While the forward or futures contract is mainly for hedging, an option contract provides a form of financial insurance to their holders. Thus, holding a call/put option provides the investor with the protection (insurance) against an increase/decrease in the price above/below the contract's price. The writer of the call/put option who takes the reverse side of the contract is referred to as the provider of the insurance. Theorists generally define a speculator as someone who purchases an asset with the intent of quickly reselling it or sells an asset with the intent of quickly repurchasing it, Stout

(1998). Speculative trading behaviour incorporates two motives in the activity; risk hedging and information arbitrage.

2.8.1 Risk Hedging

The risk-averse investors pay to avoid taking risk (like through insurance policies), while investors with greater tolerance to risk reap some profit through accepting the risk rejected by the risk-averse investors. In the risk-hedging model of speculation, speculators are relatively risk-neutral traders. For instance, a risk-averse rice farmer in Abakiliki, Ebonyi State, Nigeria, whose crops will soon be ready for harvest, and as a risk averse farmer, is more worried about the fall in price of rice during the harvest than the possible rise in price, might prefer to sell his crops now at a slight discount (forward derivative) to deliver it in, say forty days' time. On the contrary a more risk-neutral rice speculator might purchase the contract since the price discount creates for him a "risk-premium" that compensates him for accepting the changes of future price of rice within the forty days.

This risk hedging model implies that speculative traders generally involve "hedgers" on the one side of the transaction, and "speculators" on the other side. The risk-averse (hedgers) like the rice farmer is therefore happy to pay, to avoid the price variation (presumably downwards) inherent in holding the asset(s) (rice product), while a more risk-neutral speculator is happy to be paid a premium to assume the risk. Risk management that reduces return volatility is frequently termed hedging, while risk management that increases the return volatility is called speculation.

2.8.2 Information Arbitrage

The other model of speculative trading different from risk hedging is the information arbitrage model. The information arbitrage approach describes speculators as traders who through financial research are able to predict future changes in prices of assets and liabilities. They are equipped with superior knowledge of market information that permits them to trade on favourable terms with less-informed buyers and sellers who are trading for other reasons. As an illustration, a major dealer in Nigerian rice who collects data about other rice farmers in several regions like Lafia, Gboko, Nassarawa, Ugbawka and Kano, all in different rice producing areas of Nigeria that might show a low harvest yield in the regions which will necessitate price increase, may profit form the strategy of buying and storing rice from less well-informed farmers and stakeholders in the rice industry.

On a larger spectrum, Smith and Stulz (1985) demonstrate that when a risk-averse manager owns a large number of firm's shares, his expected utility of wealth is significantly affected by the variance of the firms expected profits. The Manager will direct the firm to hedge when he believes that it is less costly for the firm to hedge the share price risk than it is for him to hedge the risk on his own account. Consequently, Smith and Stulz predict a positive relation between managerial wealth invested in the firm and the use of derivatives. Thus, for speculation to be a profit-making activity in rational markets, either a firm must have an information advantage related to the prices of the instruments underlying the derivatives, or it must have economies of scale in transactions costs allowing for profitable arbitrage opportunities.

However, Hentschel and Kothari (2001) state that public discussion regarding corporate use of derivatives focuses on whether firms use derivatives to reduce or increase firm risk. They opine that in contrast, empirical academic studies of corporate derivatives usually take it for granted that firms hedge with derivatives. Their findings are consistent with Stulz (1984) argument that firms primarily use derivatives to reduce the risks associated with short-term contracts.

Stulz (1984); Smith and Stulz (1985); Froot *et al.* (1993) construct models of corporate hedging that could be useful to investors in Nigeria when the derivative products take off fully in Nigeria. These models predict that firms attempt to reduce the risks they face if they have poorly diversified and risk-averse investors face progressive taxes, suffer large costs from potential bankruptcy or have some funding needs for future investment projects in the face of strongly asymmetric information.

2.9 Chapter Summary

The history and performance of the Ghana Stock Exchange was captured in this chapter. Also, the overview and literature of Efficient Market Hypothesis and Geometric Brownian Motion was also discussed. Finally, the Geometric Fractional Brownian Motion, its empirical evidence from the world and in Africa was also elaborated.

CHAPTER 3

METHODOLOGY

3.1 Overview

Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. The notion of stochastic processes is very important in mathematical finance as it can be used to model various phenomena where the quantity or the variable varies continuously over time. Many processes are often modelled by a stochastic process of which stock price is no exception. Any collection of random variables $X(t)$ depending on time t where time can be discrete, $t = 0,1,2...$ or continuous $t \geq 0$, can be said to follow a stochastic process.

3.2 Stock Price

Following Feng (2018), assets in finance can be divided into risk-free assets and risky assets. The former includes a bank deposit or a bond issued by the government or other financial institutions. These kinds of assets are given with fixed returns and their future values are known beforehand. The risky assets can be gold, foreign currency and other virtual assets of which their future price is unknown currently. A stock is a typical risky asset held by different investors. Its price can represent the unit value of a stock in the stock market. Since return of a risky asset can be considered as random, the price of a stock has unpredictability in the sense that we can not tell for sure its future prices.

Since stock prices are unpredictable, there are many properties in the returns of stock prices which still attract many mathematicians and financier. In GFBM modelling of stock prices, the stock prices at a small-time scale and a large time scale share a common property of fractional behaviour determined by a specific parameter, a Hurst exponent. For GFBM, such property is denoted as self-similarity.

3.3 Logarithmic Return

For a risk-free asset with compounding interest payment, its value at time t is

$$X(t) = X_0\left(1+\frac{r}{m}\right)^{mt}, t = 0, \frac{1}{m}, \frac{2}{m}, \ldots \tag{3.1}$$

where m represents a compounding frequency. The constant X_0 is the initial price and r is also a constant, a periodic compounding interest rate. If the compounding frequency increases to infinite, then

$$X(t) = X_0 e^{rt}, t > 0 \tag{3.2}$$

Feng (2018), observe that, for the risk-free asset with price (3.2) and time span Δt,

$$\ln\frac{X(t+\Delta t)}{X(t)} = r \tag{3.3}$$

since logarithm is additive and convenient in mathematics, it is defined and use in financial models based on equation (3.3).

Definition 3.3.1. For a given asset with price $X(t)$ at time t, the logarithmic return of such asset during time $(t+\Delta t)$ is

$$R(t) = \ln\frac{X(t+\Delta t)}{X(t)} \tag{3.4}$$

3.4 Stationarity

A stationary process is a stochastic process whose unconditional joint probability distribution does not change over time. Consequently, parameters such as mean and variance also do not change over time. Stationarity is an assumption underlying many statistical procedures used in time series analysis, non-stationary data are often transformed to become stationary. The most common cause of violation of stationarity a trend in the mean, which can be due to either the presence of unit root or a deterministic trend. A stationary process is not strictly stationary, but can be easily be transformed into stationary process by removing the underlying trend, which is solely a function of time.

Similarly, processes with one or more of the unit roots can be made stationary through differencing. An important type of non-stationary process that does not include a trend-like behavior is a cyclo-stationary process, which is a stochastic process that varies cyclically with time. For applications, strict-sense stationarity is too restrictive so the wide-sense stationarity is employed.

Definition 3.4.1. A stochastic process $\{X(t), t \in \mathbb{R}\}$ is said to be wide-sense stationary (WSS) if its mean is constant and the correlation function only depends on the time lag, that is,

$$\mathrm{E}\left[X(t)\right] = \mu \tag{3.5}$$

for all t and for some constant μ and

$$Cov\left[X(t), X(t+\tau)\right] = r(\tau) \tag{3.6}$$

for all t and τ for some function $\tau \mapsto r(\tau)$.

3.5 Gaussian Distribution and Gaussian Process

From Miller and Childers (2012) and Zhou *et al.*, (2008), the Gaussian distribution or normal distribution is frequently used in financial models.

Definition 3.5.1. A continuous random variable X for which its probability distribution function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], x \in \mathbb{R} \tag{3.7}$$

is said to be Gaussian distributed, denoted as $X \sim N(\mu, \sigma^2)$.

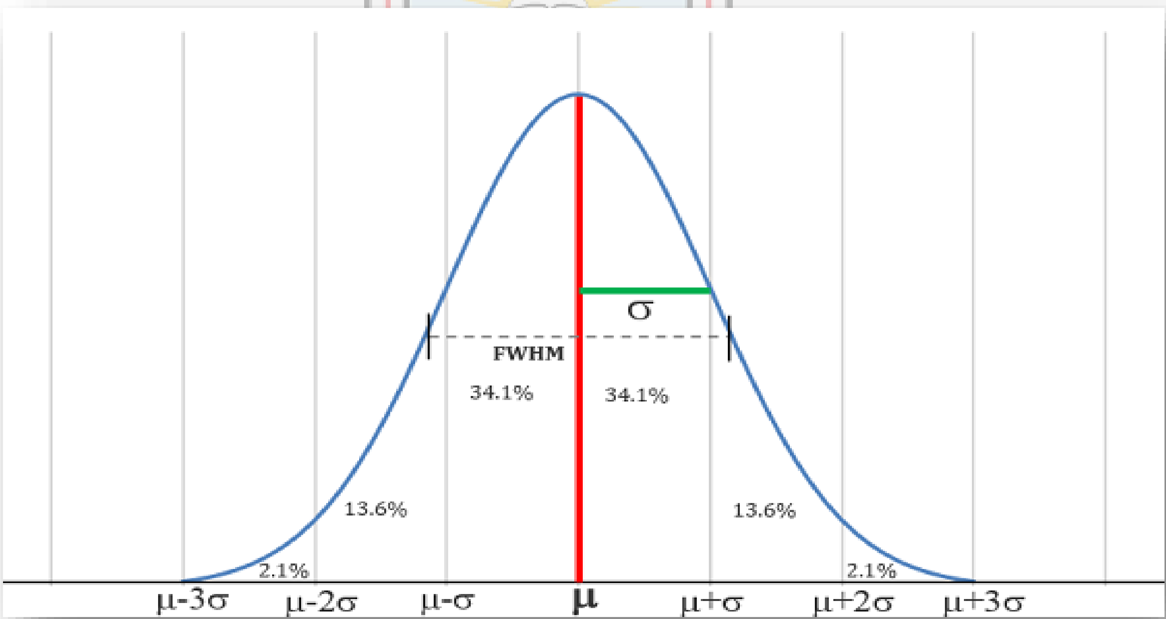

Figure 3. 1 Normal Distribution Curve

An illustration of the Gaussian density also known as a normal curve can be seen in Figure 3.1. Such probability distribution figure is symmetric and has maximum value at $x=\mu$. The standard deviation σ represents a mean deviation between the distribution and the mean. The Gaussian distribution has the property that if two Gaussian random variables are not correlated then they are also independent.

Definition 3.5.2. A random vector $\vec{X}=(X_1,\ldots,X_p)'$ with mean vector $\vec{\mu}$ and covariance matrix $\sum$ given by equations (3.8) and (3.9).

$$\vec{\mu}=\begin{pmatrix} \mathrm{E}[X_1] \\ . \\ . \\ . \\ \mathrm{E}[X_p \end{pmatrix} \tag{3.8}$$

$$\sum=\begin{pmatrix} Cov[X_1,X_1] & . & . & . & Cov[X_1,X_p] \\ . & & & & . \\ . & & & . & . \\ . & & & & . \\ Cov[X_p,X_1] & . & . & . & Cov[X_p,X_p] \end{pmatrix} \tag{3.9}$$

is multivariate Gaussian distributed if its probability density function satisfies

$$f(\vec{x})=\frac{1}{\sqrt{(2\pi)^p}\,|\Sigma|^{\frac{1}{2}}}\exp[-\frac{(\vec{x}-\vec{\mu})'\,\Sigma^{-1}(\vec{x}-\vec{\mu})}{2}],\vec{x}\in\mathbb{R}^p \tag{3.10}$$

and it is denoted by $\vec{X}\sim N_p(\vec{\mu},\Sigma)$.

3.6 The Brownian Motion Process

The Brownian motion $B(t)$ is used to capture the uncertainty in the future behaviour of a stochastic process and has the following properties.

i. $B(t)-B(s)$, for $t>s$ is independent of the past (independent increment).

ii. $B(t)-B(s)$ has normal distribution with mean 0 and variance $t-s$. If $s=0$ then $B(t)-B(s) \sim N(0,t)$.

iii. $B(t), t \geq 0$ are continuous functions of t.

3.6.1 Derivation of the Geometric Brownian Motion

According to Wilmott (2000), investors main concern will be the return on investment which is referred to as the percentage growth in the value of an asset. The quantity X_t is the asset value on the tth day and return from day t to day $t+1$ is given by:

$$R_t = \frac{X_{t+1} - X_t}{X_t} \tag{3.11}$$

Rate of return can be explained as the rate of profit or loss in investment. For instance, if yesterday the price in counter A is GHS 0.50 and today it is GHS 0.55, then the rate of return is GHS 0.1. Meaning that, if investor invest in counter A, the rate of return will be 10% increase in capital investment. The positive value of rate of return indicates increase of profit, while a negative value, means that the investor will run at a loss. By knowing the rate of return, the mean return distribution of drift, μ can be estimated as;

$$\mu = \overline{R} = \frac{1}{N} \sum_{t=1}^{N} R_t \tag{3.12}$$

where N is the number of returns in the sample and the standard deviation (volatility, σ) is;

$$\sigma = \sqrt{\frac{1}{(N-1)dt} \sum_{t=1}^{N} \left(R_t - \overline{R}\right)^2} \tag{3.13}$$

Volatility refers to the fluctuations of the stock prices, that is the price at which a security moves up and down (Pathak, 2013). Volatility is computed by calculating the annualized standard deviation of the daily change in price where standard deviation is a statistical measure of dispersion around a central tendency.

High volatility refers to share prices rapidly moving up and down over a short period of time. In simple words, it refers to the risk level, since the fluctuation of the prices is unpredicted and uncertain. Investing in stock market is risky. Investors will face either loss or profit after investment. Therefore, volatility of the rate of return can be used as the measurement of the risk level so that higher volatility will be termed higher risk level (Chen *et al.*, 2009).

Wilmott (2000) posit that, the returns can be written as a random variable, drawn from a normal distribution with a known constant, non-zero mean and a known constant and non-zero deviation since the return is closed enough to normal distribution. The usage of normal distribution is attributed to the fact that return value changes in one unit of time by an amount that is normally with mean and standard deviation. The normal distribution is a good choice because the return variable is being affected additively by many independent random variables. The standardize normal distribution of asset return by entering the standard normal variable ϕ into the asset return model is given as:

$$R_t = \frac{X_{t+1} - X_t}{X_t} = \mu + \sigma \times \phi \tag{3.14}$$

Time step for one day denoted as dt. Mean of scale follows the size of the time step. By assuming μ to be constant, it can be written as:

$$mean = \mu dt \tag{3.15}$$

Let the standard deviation of the asset return over time steps, will be written below by letting σ to be some parameter measuring the amount of randomness.

$$r = \sigma dt^{\frac{1}{2}} \tag{3.16}$$

The mean and the standard deviation over the time step by assuming μ and σ as constant will be:

$$R_t = \frac{X_{t+1} - X_t}{X_t} = \mu dt + \sigma\phi dt^{\frac{1}{2}} \tag{3.17}$$

Equation (3.17) can be simplified as:

$$X_{t+1} - X_t = \mu X_t dt + \sigma\phi X_t dt^{\frac{1}{2}} \tag{3.18}$$

The left-hand side shows the changes of the asset price, while in the right-hand side shows the random walk model in discrete time step. According to Wilmott (2000), stock markets are changing continuously over very small intervals of time which follows the Brownian motion (BM). BM refers to the limiting process for a random walk as the time step go to zero. This change on the asset price is being altered by random amount called the BM which is the fundamental tool to describe the mathematical model on all the financial asset pricing. This was strongly supported by Asuquo and Akpan (2013), who stated that the behavior of the stock market's price is unpredictable and follow the random walk in GBM.

A GBM model is a continuous time stochastic process explained by Ladde and Wu (2009), in which the logarithm of the randomly varying quantity follows a BM also known as Wiener process. Wiener process or BM process can be defined as a stochastic process $\{X(t), t \geq 0\}$. By using BM process notation, asset price model in continuous time limit, can be written as in Equation (3.21), where dX refers to the change in the asset price. At a limiting value, a small change in time will be. Thus $dB(t)$ will be a random variable, from normal distributions with mean zero and variance dt.

$$E[dB(t)] = 0 \tag{3.19}$$

$$E\left[dB(t)^2\right] = dt \tag{3.20}$$

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t) \tag{3.21}$$

A unique solution to Equation (3.21) is obtained as follows. Let $f(t) = \ln X(t)$, then $f'(t) = \frac{1}{X(t)}$ and $f''(t) = -\frac{1}{X(t)^2}$. By Ito formula for BM process:

$$d(\ln X(t)) = f'dX(t) + \frac{1}{2} f''\sigma^2(t)dt \tag{3.22}$$

hence;

$$d(\ln X(t)) = \frac{1}{X(t)} dX(t) + \frac{1}{2}\left(-\frac{1}{X(t)^2}\right)\sigma^2(t)X(t)^2 dt \tag{3.23}$$

$$d(\ln X(t)) = \frac{1}{X(t)}(\mu X(t) + \sigma X(t)dB(t)) - \frac{1}{2}\sigma^2 dt \tag{3.24}$$

$$d(\ln X(t)) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dB(t) \tag{3.25}$$

integrating both sides of Equation (3.25) gives;

$$\int d\left(\ln X(t)\right) = \int\left(\mu - \frac{1}{2}\sigma^2\right)dt + \int \sigma dB(t) \tag{3.26}$$

$$\ln X(t) = \ln X(0) + \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dB(t) \tag{3.27}$$

where $\ln X(0)$ is the constant of integration also known as initial stock price;

$$\ln\left(\frac{X(t)}{X(0)}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dB(t) \tag{3.28}$$

$$\frac{X(t)}{X(0)} = \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dB(t)\right] \tag{3.29}$$

$$X(t) = X(0)\exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dB(t)\right] \tag{3.30}$$

if we let $\xi(t)$ be the randomness captured by the Brownian process, then $\xi(t) = \frac{dB(t)}{dt}$, which implies that:

$$dB(t) = \xi(t)dt \tag{3.31}$$

therefore, Equation (3.30) becomes:

$$X(t) = X(0)\exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\xi(t)dt\right] \tag{3.32}$$

in a discrete form in the short time period Δt, Equation (3.32) can be written as:

$$X(t) = X(0)\exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\xi(t)\sqrt{t}\right] \tag{3.33}$$

since the Brownian process is normally distributed with mean 0 and standard deviation 1, the $\xi(t)$ is also normally distributed with mean 0 and standard deviation 1. Therefore, $\xi(t) \sim N(0,1)$ changes Equation (3.33) to:

$$X(t) = X(0)\exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma N(0,1)\sqrt{t}\right] \tag{3.34}$$

Equation (3.34) is the GBM model for predicting stock price.

3.7 Derivation of the Geometric Fractional Brownian Motion (GFBM)

Mishura *et al.* (2008) apply fractional Brownian motion $B_H(t)$ to replace the classical Brownian motion $B(t)$ and further incorporate the existence of long memory in financial market. The suggested model is as follows:

$$dX(t) = \mu X(t)dt + \sigma dB_H(t), H \in \left(\frac{1}{2},1\right) \tag{3.35}$$

3.7.1 Analytical Solution of GFBM

Since $H \neq \frac{1}{2}$, $B_H(t)$ is not a semi martingale; thus, the general theory of stochastic calculus cannot be applied on $B_H(t)$ instead, Wick calculus is used. Now, we assume the initial condition $X(0) = X_0$. The stochastic differential equation in Equation (3.35) can be written as:

$$\frac{dX(t)}{dt} = \mu X(t) + \sigma X(t)\frac{dB_H(t)}{dt} \tag{3.36}$$

applying Wick calculus, Equation (3.36) can be rewritten in $(S)_H^*$ as:

$$\frac{dX(t)}{dt} = \mu X(t) + \sigma X(t) W_H(t) \tag{3.37}$$

or

$$\frac{dX(t)}{dt} = (\mu + \sigma W_H(t)) X(t) \tag{3.37}$$

integrating both sides of Equation (3.37) gives:

$$X(t) = X_0 \exp\left(\mu t + \sigma \int_0^t W_H(u) du\right) \tag{3.38}$$

given $\frac{d}{dt} B_H(t) = W_H(t)$ in $(S)_H^*$, Equation (3.38) can be written as:

$$X(t) = X_0 \exp(\mu t + \sigma B_H(t)) \tag{3.39}$$

now, we introduce Definition 1, Lemma 1, and Definition 2 from Biagini *et al.* (2008) to make further deduction.

Definition 3.7.1. If $Y : \mathbb{R} \to (S)_H^*$ is a given function provided that $Y(t)W_H(t)$ is integrable in $(S)_H^*$, then we can define the fractional Wick-Ito integral of a function as:

$$\int_{\mathbb{R}} Y(t) dB_H = \int_{\mathbb{R}} Y(t) W_H(t) dt \tag{3.40}$$

$$\int_0^t B_H(S) dB_H(S) = \frac{1}{2} B_H^2(t) - \frac{1}{2} t^{2H} \tag{3.41}$$

Definition 3.7.2. Let $f \in L_H^2(\mathbb{R})$, then;

$$\exp(\langle w, f \rangle) = \xi(f) = \exp\left(\int f dB_H - \frac{1}{2} \|f\|_H^2\right) \tag{3.42}$$

applying Definition 3.7.2 and Equation (3.42), Equation (3.39) can be represented as:

$$X(t) = X_0 \exp\left(\mu t + \sigma B_H(t) - \frac{1}{2}\sigma^2 t^{2H}\right) \tag{3.43}$$

$$X(t) = X_0 \exp\left(\mu - \frac{\sigma^2}{2}\right) t^{2H} + \sqrt{\sigma^2}\xi(t)\sqrt{t^{2H}} \tag{3.44}$$

substituting $\xi(t) \sim N(0,1)$, we get;

$$X(t) = X_0 \exp\left(\mu - \frac{\sigma^2}{2}\right) t^{2H} + \sqrt{\sigma^2} N(0,1)\sqrt{t^{2H}} \tag{3.45}$$

Equation (3.45) is the analytical solution of the GFBM based on Wick calculus and it is used for predicting asset pricing in financial market.

3.8 Testing for ARCH Effect

3.8.1 Ljung-Box Test

The Ljung-Box test was used as test for the presence Heteroscedasticity in the model residuals. The test statistic is given by:

$$Q_m = n(n+1)\sum_{k=1}^{n}(n-k)^{-1} r_k^2 \approx \chi_{m-r}^2 \tag{3.46}$$

where r_k^2 is the residual autocorrelation at lag k, n is the number of residuals and m is the number of lags included in the test. The test was performed at the 1% level of significance.

3.8.2 ARCH-LM Test

The ARCH-LM test is Lagrange Multiplier (LM) test for Autoregressive Conditional Heteroscedasticity (ARCH) in the residuals. The test statistic is computed from an auxiliary test regression. To test the null hypothesis that there is no ARCH effect up to order in the residuals, the regression is run as:

$$e_t^2 = \beta_0 + \beta_1 e_{t-1}^2 + \ldots + \beta_q e_{t-q}^2 + v_t \tag{3.47}$$

where e_t is the residual. This is the regression of the squared residuals on constant and lagged residuals up to order. The LM test statistic is asymptotically distributed as a Chi-Square with degrees of freedom.

3.9 Mean Absolute Percentage Error (MAPE)

The mean absolute percentage error also known as the mean absolute percentage deviation is a measure of the prediction accuracy of a forecasting method in statistics, for example in trend estimation and also used as a loss function for regression problems in machine learning. It usually expresses the accuracy as ration define by the formula:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right|(100) \tag{3.48}$$

where A_t is the actual values and F_t is the forecasted values;

3.9.1 Scale of Judgement of MAPE

The scale of judgement of the mean absolute percentage error as proposed by Fildes *et al.* (2009) is shown in Table 3.1.

Table 3.1 Scale of Judgement of MAPE

MAPE	Decision
0% to 10%	Accurate
11% to 20%	Good
21% to 50%	Moderate
>50%	Inaccurate

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Overview

The chapter presents the analysis of results and discussions on the performance of sampled stocks listed on the Ghana Stock Exchange. Normality test was performed on the daily time series data of the logarithmic returns on the stocks. In the preliminary analysis Hurst exponent of the individual stocks is estimated and the stocks that exhibit the long memory is determined. To boost investors' confidence on stock price prediction, the Geometric Fractional Brownian motion model is developed to simulate price path for the selected stocks.

4.2 Preliminary Analysis of Data

The daily closing price of some selected stocks is collected from the Ghana Stock Exchange from January 2018 to December 2018 which constitute 247 trading days. Time series plot of the logarithmic returns of the data shows that some stocks are highly volatile whiles others show less volatility. Some of these plots are shown in Figure 4.1 to Figure 4.4.

4.2.1 Performing Normality Test

To validate the data a normal distribution statistical test was performed using Shapiro-Wilk test, Kurtosis and Skewness which have consistently been proven valid by many literatures. Table 4.1 show the normality test performed for the data. The results from Table 4.1 shows that none of the stocks considered exhibit normal distribution. The computed p-value obtained were not significant at a significance level of 5%. The kurtosis which is also a measure of normality conforms to the p-value because in Table 4.1 all the stocks have kurtosis greater than 3.

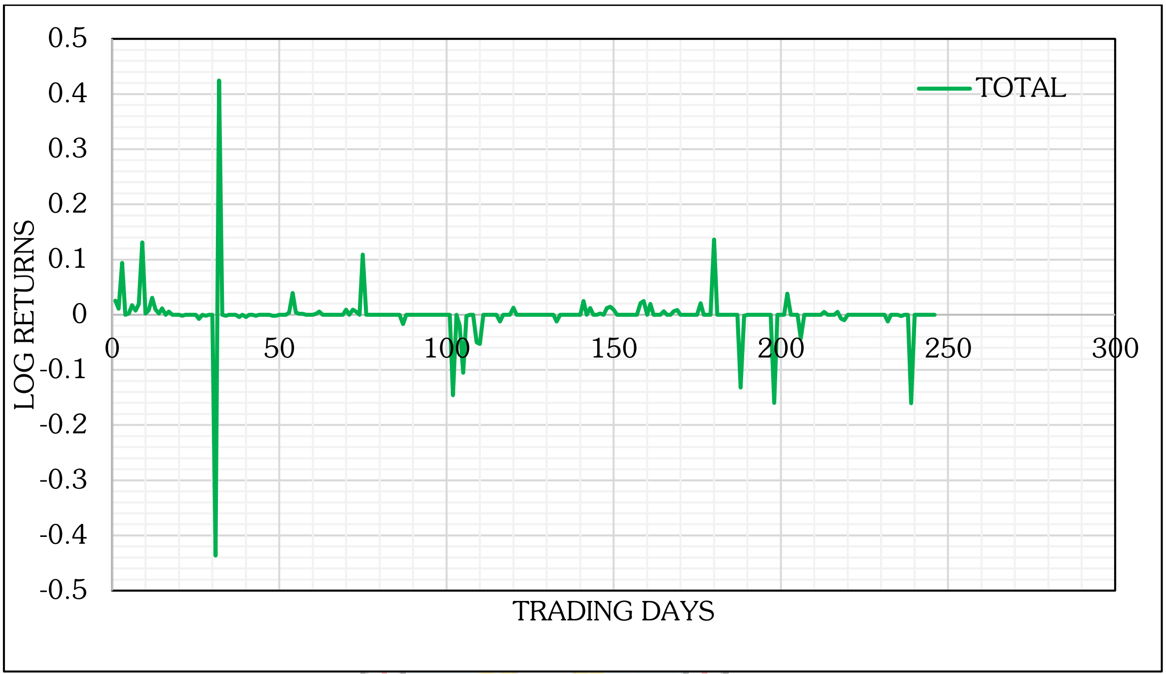

Figure 4.1 Logarithmic Returns TOTAL

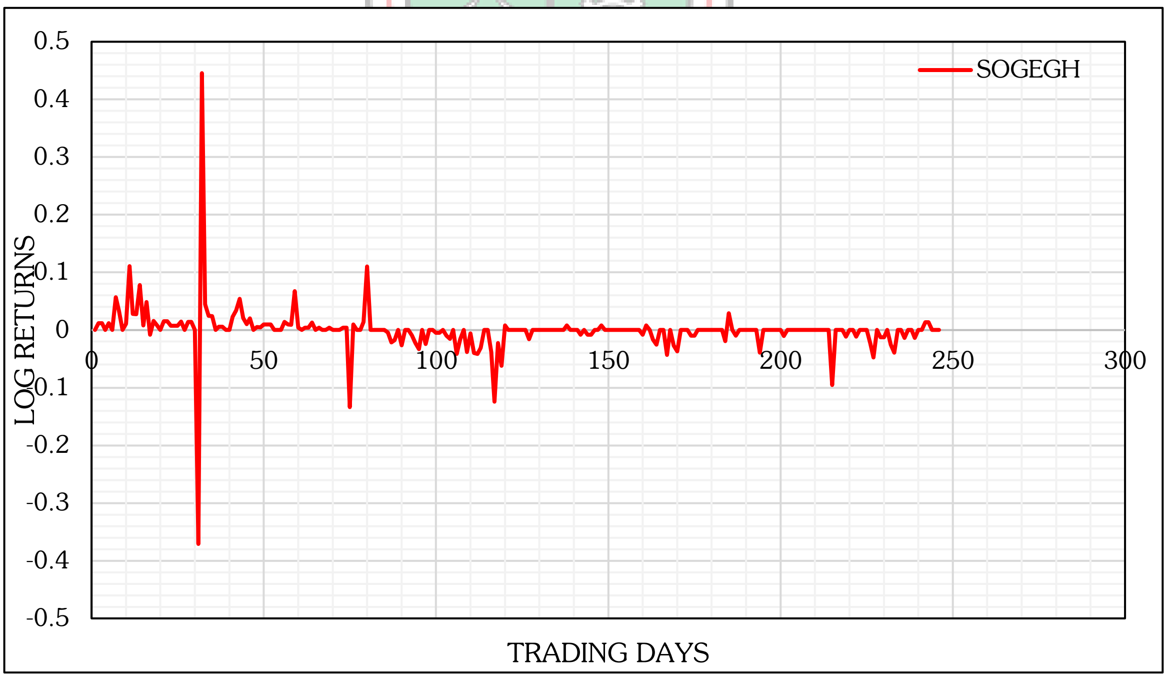

Figure 4.2 Logarithmic Returns SOGEGH

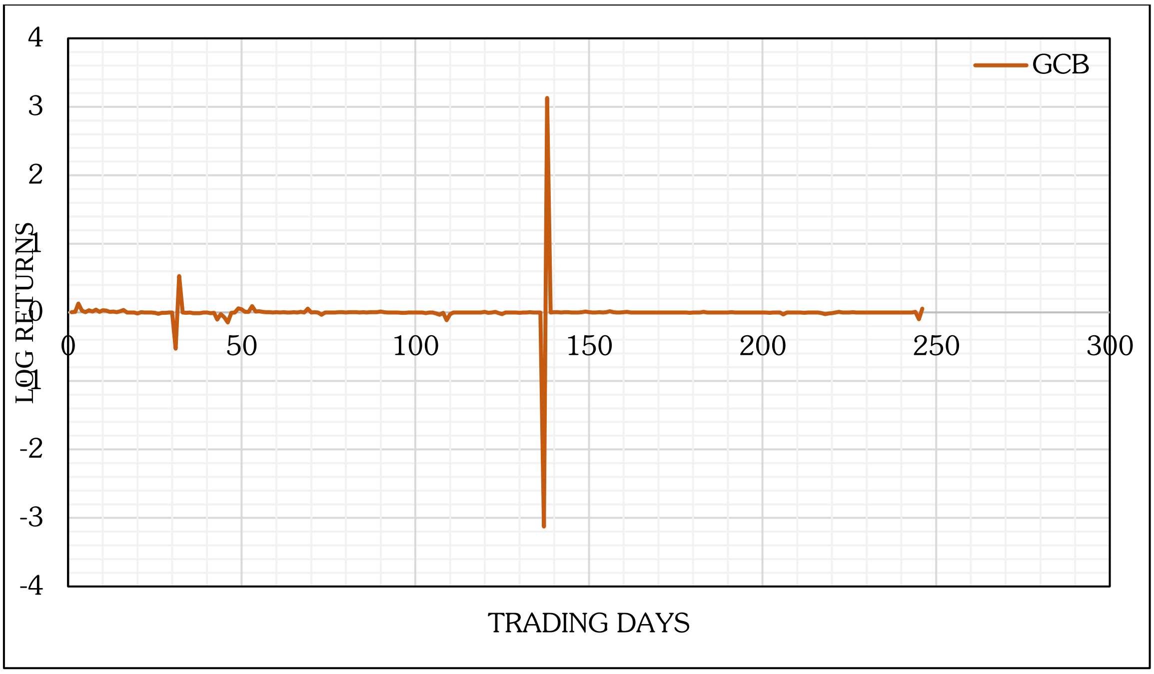

Figure 4.3 Logarithmic Returns GCB

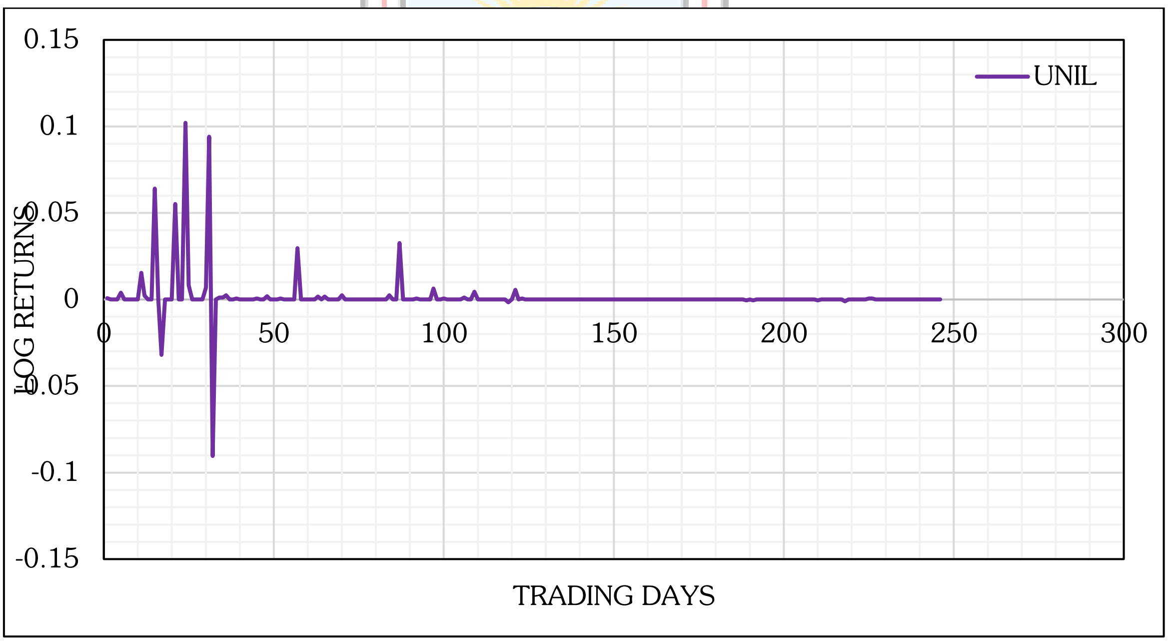

Figure 4.4 Logarithmic Returns UNIL

Table 4.1 Descriptive Statistic of the Data

Stocks	Mean	Shapiro-Wilk	Kurtosis	Skewness
ACCESS	0.0136	0.0048***	8.1267	-0.2776
ADB	0.0001	0.0000***	114.8568	10.2958
BOPP	0.0056	0.0041***	89.2052	-0.5079
EGH	0.0125	0.0060***	69.1591	0.0761
EGL	0.0127	0.0092***	114.0185	2.0148
GCB	0.0378	0.0362**	113.9066	0.0237
GGBL	0.0028	0.0012***	25.1948	0.3152
GOIL	0.0101	0.0040***	46.0792	0.1858
SCB	0.0109	0.0061***	79.7195	-0.1133
SOGEGH	0.0142	0.0055***	65.2453	1.7717
TOTAL	-0.0002	0.0059***	57.3257	-0.5532
UNIL	0.0034	0.0016***	45.4749	3.1780

Note: ***, indicate no normality at significant level at 1% and ** at 5%

Apart from ACCESS, BOPP, SCB and TOTAL that have negative skewness indicating an increase in probability at the higher quantiles (heavy left tails), the remaining stocks have positive skewness depicting increase in probability at the higher quantiles (heavy right tails).

From Table 4.1 it is apparent that GCB security has 0.0378 as the highest expected logarithmic returns followed by SOGEGH, ACCESS, EGL, EGH, SCB, GOIL, BOPP, UNIL, GGBL and ADB whiles TOTAL made the least expected logarithmic returns of -0.0002 analysed over 247 trading days.

4.3 Estimation of the Hurst Exponent

To estimate the Hurst exponent from the selected stocks on the GSE, the rescaled range and the periodogram method was adopted. The results are shown in Figure 4.5 and Table 4.2.

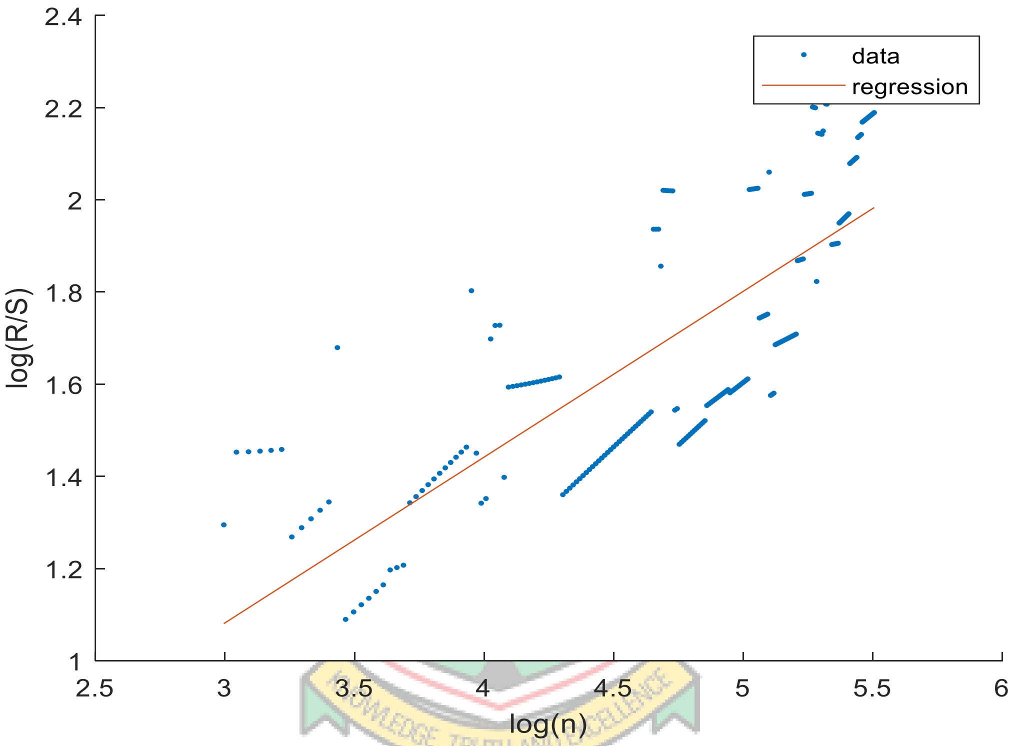

Figure 4.5 Hurst Exponent Estimation

4.3.1 Interpretation of Hurst Exponent

The criteria for the selection of the Hurst exponent were, $0 < H < 0.5$ indicate a time series with long-term switching between high and low values in adjacent pairs, meaning that a single high value will probably be followed by a low value and that the value after that will tend to be high, with this tendency to switch between high and low values lasting a long time into the future. Another criteria is $H = 0.5$ indicating a completely uncorrelated series, but this value is applicable to series for which the autocorrelation at small time lags can be positive or negative and where the absolute values of the autocorrelation decay exponentially quickly to zero which is mostly referred to as random walk process. The last criteria which is $0.5 < H < 1$ indicates a time series with long-term positive autocorrelation, meaning that a high value in the series will probably be followed by another high value and that the values a long time into the future will also tend to be high.

KNOWLEDGE, TRUTH AND EXCELLENCE

Table 4.2 Hurst Exponent Estimate

Stocks	Rescaled range Analysis	Periodogram Method	P-Value
ACCESS	0.3597	0.2593	0.1070
ADB	0.3933	0.3719	0.5052
BOPP	0.6752	0.6389	0.0000***
EGH	0.5856	0.5877	0.0000***
EGL	0.4739	0.1975	0.0000***
GCB	0.4159	0.1405	0.0000***
GGLB	0.3907	0.3779	0.0000***
GOIL	0.8002	0.7989	0.0000***
SCB	0.7519	0.6414	0.0000***
SOGEGH	0.7540	0.6732	0.0000***
TOTAL	0.7111	0.6331	0.0000***
UNIL	0.3593	0.6220	0.0004***

Note: ***, indicate significant level at 1%

4.3.2 Selection of Stocks based on Hurst Exponent

From Table 4.2, ACCESS and ADB were the negatively autocorrelated stocks with rescaled range value of 0.3597 and 0.3933 respectively. This is evident in Figure 4.4, where the slope to both curves are relatively less steep. The value of the periodogram is also in line with the rescaled range value with ACCESS having a periodogram value of 0.2593 and 0.3719 for ADB.

The Hurst value from both the periodogram and the rescaled range analysis for ACCESS and ADB was confirmed by the heteroscedasticity test with p-value of 10.7% and 50.5% for ACCESS and ADB respectively. This means that at 5% significant level, inferences from past data of these stocks will be relatively inaccurate. On the same Table 4.2, values from the rescaled range analysis and the periodogram method shows that, EGL, GCB and GGLB are also negatively autocorrelated.

One unique equity on Table 4.2 is the UNIL stock. The value from the rescaled range analysis (0.3593) shows that, it is negatively autocorrelated whiles the periodogram (0.6220) suggest otherwise. Normally, stocks of such characteristics are seen to posses' random walk process and no inferences can be drawn from their past values. From Table 4.2, BOPP, EGH, SCB, SOGEGH and TOTAL are highly persistent with Hurst exponent of 0.6752, 0.5856, 0.7519, 0.7540 and 0.7111 and also 0.6389, 0.5877, 0.6414, 0.6732 and 0.6331 for rescaled range and periodogram respectively. The test statistics was 0% which was significant at 5% significance level. But the most persistent or positively autocorrelated stock was the GOIL stock with rescaled range value of 0.8002 and a periodogram value of 0.7989. Stocks with high Hurst exponent can be predicted using their past values.

In summary from Table 4.2, out of the 12 equities considered, only 3 of them (ACCESS, ADB and UNIL) don't exhibit the long memory hypothesis, the remaining (BOPP, EGH, EGL, GCB, GGLB, GOIL, SCB, SOGEGH and TOTAL) all have long memory characteristics. This means that 75% of the stocks considered can be modelled using the geometric fractional Brownian motion.

4.4 Parameter Estimation

Parameters of the developed model which constitute the drift, volatility and the Hurst exponent was estimated and the results are shown in Table 4.3.

Table 4.3 Parameter Estimate

Stocks	Drift	Volatility	Hurst Exponent
ACCESS	0.04451	0.35257	0.4997
ADB	0.01872	0.00011	0.3933
BOPP	-0.05670	0.25517	0.6752
EGH	0.26557	0.55764	0.5877
EGL	0.14875	0.30122	0.4739
GCB	-0.09333	0.08260	0.4159
GGBL	0.06740	0.02156	0.3907
GOIL	0.27436	0.25214	0.8002
SCB	0.10309	0.57479	0.7519
SOGEGH	0.14403	0.46651	0.7540
TOTAL	0.23550	0.54604	0.7111
UNIL	0.34433	0.13764	0.4907

4.4.1 Annual Performance of Stocks Relative to Drift and Volatility

From Table 4.3, the best performing stock was the UNIL with expected annual return of 34.4% and annual risk of 13.8%. Also, stocks such as EGL, EGH, GGBL, GOIL, SOSEGH and TOTAL performed averagely with annual risk of 30.1%, 55.8%, 2.2%, 25.2%, 46.7% and 54.6% and annual returns of 14.9%, 26.6%, 6.7%, 27.4%, 14.4% and 23.5% respectively. The least performing stock was the BOPP with annual risk of 25.5% and annual returns of -5.6%.

4.5 Model Development

In the model development, the estimated parameters (drift, volatility and Hurst exponent) of the individual stock was substituted into Equation (3.33) and Equation (3.45) and the results are shown in Table 4.4 and Table 4.5.

Table 4.4 Geometric Brownian Motion

Stocks	Model
ACCESS	$X_t = X_0 e^{0.04451t+\sqrt{0.35257}\sqrt{t}N(0,1)}$
ADB	$X_t = X_0 e^{0.01872t+\sqrt{0.00011}\sqrt{t}N(0,1)}$
BOPP	$X_t = X_0 e^{-0.05670t+\sqrt{0.25517}\sqrt{t}N(0,1)}$
EGH	$X_t = X_0 e^{0.26557t+\sqrt{0.55764}\sqrt{t}N(0,1)}$
EGL	$X_t = X_0 e^{014875t+\sqrt{1.30122}\sqrt{t}N(0,1)}$
GCB	$X_t = X_0 e^{-0.09333t+\sqrt{0.08260}\sqrt{t}N(0,1)}$
GGBL	$X_t = X_0 e^{0.06740t+\sqrt{0.02156}\sqrt{t}N(0,1)}$
GOIL	$X_t = X_0 e^{0.27436t+\sqrt{0.25214}\sqrt{t}N(0,1)}$
SCB	$X_t = X_0 e^{0.10309t+\sqrt{0.57479}\sqrt{t}N(0,1)}$
SOGEGH	$X_t = X_0 e^{0.14403t+\sqrt{0.46651}\sqrt{t}N(0,1)}$
TOTAL	$X_t = X_0 e^{0.23550t+\sqrt{0.54604}\sqrt{t}N(0,1)}$
UNIL	$X_t = X_0 e^{0.34433t+\sqrt{0.03764}\sqrt{t}N(0,1)}$

From Table 4.4, the drift of the UNIL stock is 0.34433 and that of the volatility is 0.03764 so the model specification is given as:

$$X_t = X_0 \exp\left[\left(0.34433t + \sqrt{0.03764}N(0,1)\sqrt{t}\right)\right] \tag{4.1}$$

This model specification differs in the various stocks, because stocks have different drift (return) and volatility (risk). The model specification for the remaining stocks are shown in Table 4.5.

Table 4.5 Geometric Fractional Brownian Motion

Stocks	Model
ACCESS	$X_t = X_0 e^{0.04451t^{2H}+\sqrt{0.35257}\sqrt{t^{2H}}N(0,1)}$
ADB	$X_t = X_0 e^{0.01872t^{2H}+\sqrt{0.00011}\sqrt{t^{2H}}N(0,1)}$
BOPP	$X_t = X_0 e^{-0.05670t^{2H}+\sqrt{0.25517}\sqrt{t^{2H}}N(0,1)}$
EGH	$X_t = X_0 e^{0.26557t^{2H}+\sqrt{0.55764}\sqrt{t^{2H}}N(0,1)}$
EGL	$X_t = X_0 e^{0.14875t^{2H}+\sqrt{1.30122}\sqrt{t^{2H}}N(0,1)}$
GCB	$X_t = X_0 e^{-0.09333t^{2H}+\sqrt{0.08260}\sqrt{t^{2H}}N(0,1)}$
GGBL	$X_t = X_0 e^{0.06740t^{2H}+\sqrt{0.02156}\sqrt{t^{2H}}N(0,1)}$
GOIL	$X_t = X_0 e^{0.27436t^{2H}+\sqrt{0.25214}\sqrt{t^{2H}}N(0,1)}$
SCB	$X_t = X_0 e^{0.10309t^{2H}+\sqrt{0.57479}\sqrt{t^{2H}}N(0,1)}$
SOGEGH	$X_t = X_0 e^{0.14403t^{2H}+\sqrt{0.46651}\sqrt{t^{2H}}N(0,1)}$
TOTAL	$X_t = X_0 e^{0.23550t^{2H}+\sqrt{0.54604}\sqrt{t^{2H}}N(0,1)}$
UNIL	$X_t = X_0 e^{0.34433t^{2H}+\sqrt{0.03764}\sqrt{t^{2H}}N(0,1)}$

From Table 4.5, the drift of the UNIL stock is 0.34433, the volatility is 0.03764 and the Hurst exponent is 0.4907 so the model specification is given as:

$$X_t = X_0 \exp\left[\left(0.34433t^{0.4907} + \sqrt{0.03764}N(0,1)\sqrt{t^{0.4907}}\right)\right] \tag{4.2}$$

The remaining model specification are shown in Table 4.5

4.6 Simulation of Price Path

After the model specification, simulation was done for the different values of Hurst exponent in the short-run and the long-run and the results are shown in Figure 4.6 to Figure 4.8.

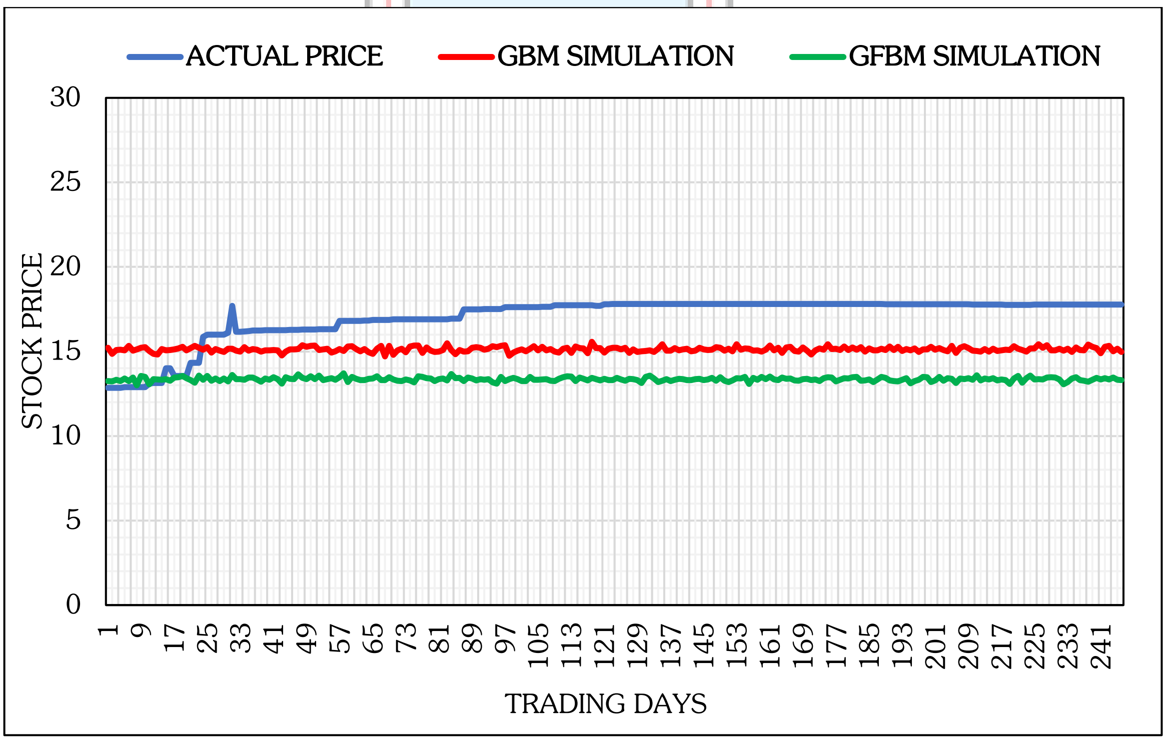

Figure 4.6 Short-Run and Long-Run Simulation of UNIL Stock with H<0.5

From Figure 4.6, it can be seen that, the short-run and the long-run simulation were made for Hurst exponent less than 0.5 and under both cases, the GBM model simulated values were closer to the actual than it was to the GFBM model. The Figure

4.6 is the simulation for only the UNIL stock and the simulation were performed for the remaining stocks considered and the results are presented in Table 4.6 and Table 4.7. In Figure 4.7, simulations for the short-run and the long-run were made at Hurst exponent of 0.5. From the model development, it was deduced that at $H = 0.5$, the GBM model and the GFBM model are the same. This argument holds for the simulation since the predictability of the two models were the same.

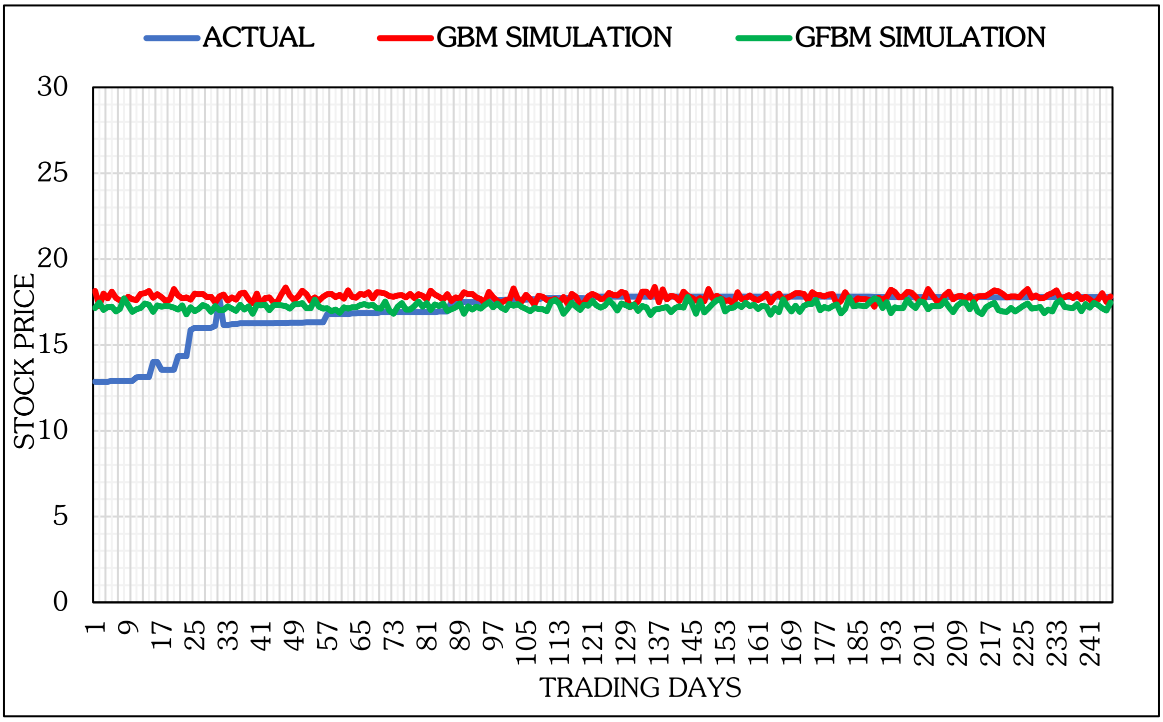

Figure 4.7 Short-Run and Long-Run Simulation of UNIL Stock for H=0.5

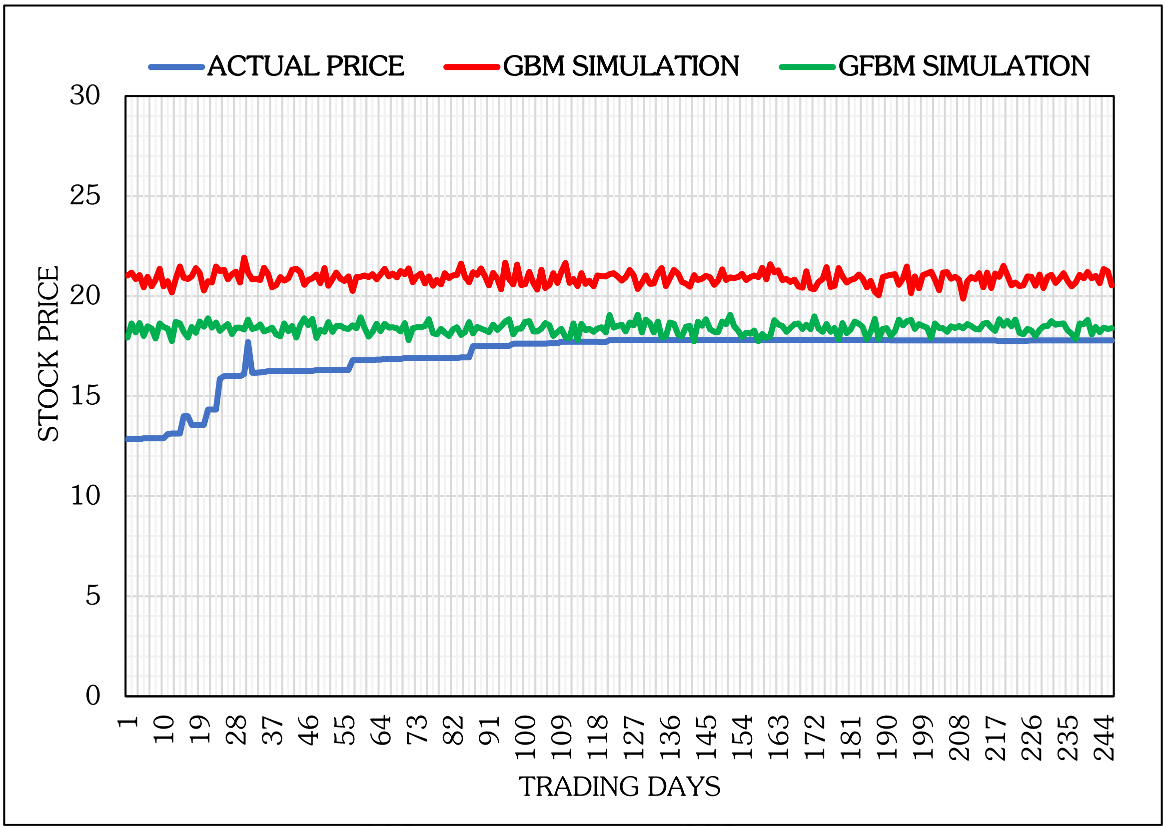

Figure 4.8 Short-Run and Long-Run Simulation of UNIL Stock for H>0.5

For Hurst exponent greater than 0.5, the stock price is persistent with time and for that, the GBM model decay exponentially to zero. Meanwhile the GFBM model at this Hurst exponent (0.5) give the best prediction. This is evident in Figure 4.8, where the simulation of the GFBM model predict far better than the GBM model. The Figure 4.8 is the prediction for only the UNIL stock, the remaining equities were also simulated and the results is shown in Table 4.6 and Table 4.7.

Table 4.6 Short-Run Predicted Stock Prices

Stock	Actual Price (GHS)	Predicted Price (GHS)		Percentage Error	
		GBM Model	GFBM Model	GBM Model	GFBM Model
UNIL	17.0354	15.1118	13.3704	13.18	21.34
ACCESS	3.7637	3.7985	3.9847	6.51	7.78
SCB	27.7921	23.0129	24.6936	18.62	15.96
ADB	5.9474	5.8951	5.8534	0.9	1.58
BOPP	6.6162	5.5936	5.9913	17.66	14.87
EGH	9.3365	7.5656	7.5838	17.88	17.91
EGL	3.514	2.9006	3.5037	18.49	20.93
GOIL	3.7083	2.9681	2.7983	19.05	22.72
GGBL	2.4206	2.1181	2.0733	12.22	13.99
TOTAL	4.6767	3.5525	3.6187	22.85	21.47
GCB	5.6555	5.0377	5.1133	29.34	29.39
SOGEGH	1.4153	0.7854	0.8122	37.88	36.17
Average				17.88	18.68

Table 4.7 Long-Run Predicted Stock Prices

Stock	Actual Price (GHS)	Predicted Price (GHS)		Percentage Error	
		GBM Model	GFBM Model	GBM Model	GFBM Model
UNIL	17.0354	20.9281	18.4112	23.74	9.85
ACCESS	3.7637	3.3128	3.5136	12.54	9.82
SCB	27.7921	19.2346	20.7699	28.85	13.97
ADB	5.9474	6.0059	5.9619	0.98	0.24
BOPP	6.6162	4.6502	4.9970	28.48	15.46
EGH	9.3365	7.4501	7.4844	19.15	18.91
EGL	3.5140	1.7534	2.1479	47.42	15.76
GOIL	3.7083	3.4206	3.2261	15.68	16.32
GGBL	2.4206	2.2456	2.1935	8.30	9.55
TOTAL	4.6767	3.4290	3.4769	25.26	24.31
GCB	5.6555	4.6381	4.8659	33.82	31.22
SOGEGH	1.4153	0.7191	0.7421	42.68	10.77
Average				23.91	14.68

The Mean Absolute Percentage Error (MAPE) of the GBM and GFBM model was estimated. The average MAPE of the GBM model was 17.88% which indicates a good prediction (see Table 3.1) and the average MAPE for GFBM model was 18.68% which also indicate a good prediction. This prediction was all in the short-run, but in the long-run, the average MAPE for GFBM was 14.68% indicating a good prediction whilst the GBM was 23.91% indicating a moderate prediction.

Comparing the predictability of the GBM and GFBM models, it can be seen from Table 4.6 and Table 4.7 that, in the short-run and long-run the GFBM model gave a good prediction whilst the GBM was not good in both cases. To test for the difference in the predictability of both models, a two-sample t-test was performed on the errors of the models and the results is shown in Table 4.8.

Table 4.8 Testing the Predictability of the Models

Stock	Percentage Error (Short-Run)		Percentage Error (Long-Run)	
	GBM MODEL	GFBM MODEL	GBM MODEL	GFBM MODEL
UNIL	13.18	21.34	23.74	9.85
ACCESS	6.51	7.78	12.54	9.82
SCB	18.62	15.96	28.85	13.97
ADB	0.9	1.58	0.98	0.24
BOPP	17.66	14.87	28.48	15.46
EGH	17.88	17.91	19.15	18.91
EGL	18.49	20.93	47.42	15.76
GOIL	19.05	22.72	15.68	16.32
GGBL	12.22	13.99	8.3	9.55
TOTAL	22.85	21.47	25.26	24.31
GCB	29.34	29.39	33.82	31.22
SOGEGH	37.88	36.17	42.68	10.77
Average	17.88	18.68	23.91	14.68
P-Value (2-tailed)	0.8481		0.0483**	

In the short-run, there was no difference in the predictability of the two models. The p-value of 0.8481 tested at 5% significance level confirms the test of no difference.

But in the long-run, the p-value of 0.0483 tested at 5% significance level indicated a significant difference in the predictability of the two models.

4.8 Model Selection for individual Stocks

The MAPE from the GBM and the GFBM model was used to select stocks that predict better on the two models, the result is shown in Table 4.9.

Table 4.9 Model Selection

Stocks	Short-Run	Long-Run
	Model	Model
UNIL	GBM	GFBM
ACCESS	GBM	GFBM
SCB	GFBM	GFBM
ADB	GBM	GFBM
BOPP	GFBM	GFBM
EGH	GBM/GFBM	GFBM
EGL	GBM	GFBM
GOIL	GBM	GFBM
GGBL	GBM	GFBM
TOTAL	GFBM	GFBM
GCB	GBM/GFBM	GFBM
SOGEGH	GFBM	GFBM

In the short-run, stocks such as UNIL, ACCESS, ADB, EGL, EGH, GOIL, GGBL and GCB all predicted better when the GBM and the GFBM models were applied whiles in the long-run, all the stocks considered in this research predicted better when the GFBM model was applied with none of them predicting well in the GBM.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The research was conducted purposely to develop a model that will take into account the Hurst exponent to model stock price in Ghana. From the findings of the research, the following conclusions were drawn in relation to the objective of the study.

A stochastic differential equation model that takes into account the Hurst exponent has been established and this is represent $X(t) = X_0 \exp\left(\mu - \frac{\sigma^2}{2}\right)t^{2H} + \sqrt{\sigma^2}N(0,1)\sqrt{t^{2H}}$ where H is the Hurst exponent.

Parameters of the proposed model such as expected return, risk and relationship index has been estimated. From Table 4.3, the drift parameter was estimated using the constant term $\left(\mu - \frac{\sigma^2}{2}\right)$ from the proposed model where μ is the expected return and σ^2 is the risk on returns, the volatility was also estimated by the expression $\sqrt{\sigma^2}N(0,1)$ where σ^2 is the risk on returns and $N(0,1)$ is the random fluctuation captured in the Brownian process and the Hurst exponent was derived by fitting a regression line to the plot of $\log\left[\frac{R(n)}{S(n)}\right]$ against $\log n$. The slope from the regression line is the Hurst exponent.

Finally, in order to validate the proposed model, numerical simulations of the model were carried out base on the time constraint (long-run and short-run) and was used to compare another theoretical model. From Table 4.6 and Table 4.7, the average mean absolute percentage error of the predictability of the GFBM model was 16.68% which indicates a good prediction from the scale of judgement while the GBM model average error was 20.90% indicating moderate prediction. Therefore, the GFBM model predict stock prices better than the GBM model on the Ghana Stock Exchange.

5.2 Recommendations

The following recommendations based on the finding and challenges of the work are made for decision making and future research.

i. The proposed model could be used by investors in Ghana to predict prices of stocks in the short-run and the long-run.

ii. Artificial Neural Network (ANN) model should also be tested on prices on the GSE due to its ability to predict both the short-run and the long-run in a less volatile market, although it has some validity issues when used in the NASDAQ.

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APPENDIX

APPENDIX A MATLAB CODES FOR SIMULATIONS AND PARAMETERS ESTIMATION

```
FBM figure
fbm=zeros(n,3); for i=1:3
fbm0=cumsum(fgn(: i));fbm(:i)=fbm0;
subplot(3,1,i); plot(1:n, fbm0)
ll=legend("H="+h(i));
set(ll,'Fontsize',14); ylabel('fbm. value')
end
Stock; mu=0.006; sigma=0.027; S0=10;
figure S sim=zeros(n,3); stocks
S sim(1,:)=S0; for i=1:3
f=fbm(:,i); for t=1:n
S sim(t+1,i)=S0*exp(mu*t-sigma*sigma*t/2+sigma*f(t));
end
subplot(3,1,i) plot(0:n, S sim(:,i))
ll=legend("H="+h(i)); set(ll,'Fontsize',14);
ylabel('stock price')
end
Return
rs=zeros(n,3); figure
for i=1:3 r0=diff(log(S sim(:,i)));
rs(:,i)=r0; subplot(3,1,i)
plot(1:n, rs(:,i)); ll=legend("H="+h(i));
set(ll,'Fontsize',14); ylabel('logarithmic return')
end
% Periodogram Method
function [hurst]=PE(X)
```

```
N=length(X);
% separate (-0.5,0.5) into 601 parts
len=601;
lags=linspace(-1/2,1/2, len);
% estimated spectral density
sdf=zeros(1,len);
for f=1: len
tau=0:N-1;
xl=X-mean(X);
xr=exp(-1i*2*pi*lags(f)*tau);
cum=xr*xl;
sdf(f)=(abs(cum)^2)/N;
end
mid=(len+1)/2;
% during low-frequency part
len=fix(mid^(4/5));
x=lags(mid+1: mid+1+len);
y=sdf(mid+1: mid+1+len);
logx=log(x);
logy=log(y);
% spectral exponent
gamma=polyfit(logx,logy,1);
% Hurst exponent
hurst=(1-gamma(1))/2;
end

% input a series of FGN {X t, t>0}
function [hurst]=RS(X)
N=length(X);
% skip the initial 20 points
n=20;
```

```
xvals=n: N;
logx=log(xvals);
yvals=zeros(1, length(xvals));
for t=n: N
tmpX=X(1:t);
% deviation series with means
Y=tmpX-mean(tmpX);
% cumulative series
Z=cumsum(Y);
% range deviation series
R=max(Z)-min(Z);
% standard deviation series
S=std(tmpX);
yvals(t-(n-1))=R/S;
end
logy=log(yvals);
p=polyfit(logx,logy,1);
% Hurst exponent is the slope of linear-fit plot
hurst=p(1);

% the scatter figure of linear regression
scatter(logx,logy,'.')
hold on
plot(logx, logx*hurst+p(2))
xlabel('log(n)')
ylabel('log(R/S)')
legend('data','regression')
hold off
end
% raw GFBM
unnamed;
```

```
%% Stocks
S=flipud(unnamed);
nn=length(S); % with 0
n=nn-1; % without 0
R=diff(log(S));
X=(R-mean(R))/std(R);
h1=A.PE(X); % estimated hurst exponent
h=A.RS(X);
XX=A.cholesky(n,h); % new FGN
BB=cumsum(XX); % new FBM
dt=1; % lag t
std0=A.fgn volatility(R,dt,h); % estimated volatility
mu0=A.fgn drift(R,dt,std0); % estimated drift
S0=S(1);
SS=zeros(nn,1); SS(1)=S0;
for t=1:n % new stocks
SS(t+1)=S0*exp(mu0*t-std0*std0*t/2+std0*BB(t));
end
figure
plot(0:n,S,"r--",0:n,SS,"k");
ll=legend("real.","sim.");
% title("Stock prices");
set(ll,'Fontsize',20);
set(gca,'xtick',[],'xticklabel',[])
set(gca,'ytick',[],'yticklabel',[])
grid on
xlim([0 n])
%% Probability density
figure
[f,xi1]=ksdensity(X);
[f1,xi2]=ksdensity(XX);
```

```
plot(xi1,f,"r--",xi2,f1,"k",xi2,normpdf(xi2),"b.-")
ll=legend('real.','sim.','thm.');
set(ll,'Fontsize',20);
xlim([-5,5]);
disp("Simulated stocks are not a Gaussion process: "+ jbtest(X));
%% Return
RR=diff(log(SS));
figure
plot(1:n,R,"r--",1:n,RR,"k")
grid on
ll=legend('real.','sim.');
% title("Log returns");
set(ll,'Fontsize',20);
xlim([1 n])
%% parameter estimation
std1=A.fgn volatility(RR,dt,h); % test volatility of S sim
mu1=A.fgn drift(RR,dt,std1); % test drift of S sim
%% Autocovariance
[cov real, lags]=A.ecov(X);
cov sim=A.ecov(XX);
cov thm=A.tcov(n,h);
figure
re=20;
hold on
stem(lags(n-re:n+re), cov real(n-re:n+re),"r^-");
stem(lags(n-re:n+re), cov sim(n-re:n+re),"k*-");
stem(lags(n-re:n+re), cov thm(n-re:n+re),"bo-");
ll=legend('real.','sim.','thm.');
% title("Autocovariance");
grid on
xlabel('tau');
```

```
set(ll,'Fontsize',20);
hold off
%% Spectral density function
sdf real = A.dftSDF(cov real);
[sdf sim, xval]= A.dftSDF(cov sim);
[sdf thm]=A.dftSDF(cov thm);
figure
plot( xval, sdf real,"r--", xval, sdf sim,"k", xval, sdf thm,"b.-")
ll=legend('real.','sim.','thm.');
% title("Spectral density");
set(ll,'Fontsize',20);
```

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