DECLARATION

I declare that this thesis is my own work. It is begin submitted foe masters of Philosophy in Mathematical Sciences (Financial Engineering) in the University of Mines and Technology, Tarkwa. It has not been submitted for any master's degree or examination in any other University.

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(Signature of Candidate)

…… day of ……, 2020.

ABSTRACT

In the single period Markowitz model, the investor maximizes the expected return of the portfolio and minimizes the risk, measured by the variance of the portfolio returns. Investor considering investments in sectors under GSE is faced with the problem of choosing from a large number of sectors and how to allocate his funds over this group of sectors due to the reforms that occur at the various sectors. As a result, the recent financial crises which has led to the extinction of many institutions such as GN Bank, UT Bank and more have made old literatures on portfolio optimization on the Ghana stock exchange vulnerable. This is because some equities that were considered in portfolio are no more in existence. This issue is addressed by creating a new portfolio optimization after this financial sector reforms. A historical secondary data of nineteen companies were obtained from Ghana Stock Exchange. Variance-Covariance, Correlation and Standard Market Model methods were employed to determine stock and portfolio performance in the Ghanaian financial market. Models were developed to maximize returns and minimize risk. 38.84 percent stock index were found aggressive and 26.32 percent stocks overperformed. For investor to take up investment decision TOTAL, AYRTN, UNIL and CLYD were efficient and the most efficient stock is UNIL. The study finally recommended that, prospective investors especially the risk averse to take up risky investments via diversification using the research outcome as working tool. Companies on the GSE are also expected to put in place measures that can help improve on performance with respect to dividend payment and returns that the companies pose. Portfolio managers should constantly restructure their portfolio due to reforms that occur at various sectors of the GSE.

DEDICATION

I dedicate this thesis to my parents Mr. and Mrs. Arthur, My Supervisors and my love ones.

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CHAPTER 1

INTRODUCTION

1.1 Background

A portfolio is a collection of financial assets consisting of investment tools such as stocks, bonds, gold, foreign exchange, asset-backed securities, real estate certificates and bank deposit which are held by a person or a group of persons (Darko, 2012). Portfolio analysis is a core problem in the financial investment and plays a vital role in both theory and practice. Financial investment on the other hand is the commitment of funds with the expectation of receiving additional returns in future (Fama, 1970).

The Ghana Stock Exchange (GSE) is a vital link between companies or the Government with capital needs, and the public to invest. GSE was established in July 1989 as a private company and gained recognition under the Stock Exchange Act of 1971 (Act 384) in October 1990. Objectives of GSE includes: to provide the facilities and framework to the public for the purchase and sales of bonds, shares and other securities, to regulate the dealings of members with their clients and other members, and to co-ordinate the stock dealing activities of members and facilitate the exchange of information including prices of securities listed for their mutual advantages and for the benefit of their clients (Anon, 2004).

However, an investor considering investments in securities is faced with the problem of choosing from a large number of securities and how to allocate his funds over this

group of securities. Modern Portfolio Theory suggests that one can maximize returns, given the amount of risk (or volatility) you are willing to take on (Chambers, 2010).

Markowitz (1952) portfolio theory provides the basis for modern theory of portfolio choices. The intuition behind Markowitz theory is maximizing the expected returns while minimizing the risk of a portfolio. From Markowitz concepts, the utility of the investor is mainly a function of the first two moments (mean and variance) of returns. The theory also incorporates diversification effects, investor's preferences and expectations of return and risk of all assets considered. Detail of the mean-variance theory indicates that prefer portfolios of securities with high expected returns have higher risk. With the information on mean, standard deviation and correlation, a set of efficient portfolios can be computed. These portfolios maximize expected returns for various levels of risk. Using Markowitz model, an investor can weigh his portfolio in a way that maximizes the expected return for a given risk (Darko, 2012).

1.2 Statement Problem

Investment has become a prominent aspect of today's business in the Ghanaian financial market, precisely the Ghana Stock Exchange (GSE). An investor considering investments in sectors under the GSE is faced with the problem of choosing from a large number of stocks and how to allocate his funds over this group of sectors. Predicting the rise or fall of the stock market for an investor to maximize profit is compelled to have a choice of buying a basket of stocks in various sectors which is known as diversification and holding them for a long-term. This has led to several studies on the performance and improvement of equities under the Ghana Stock Exchange (Ahiadu, 2015; Manu, 2017; Agyeman, 2010; Mensah *et al.* 2012).

For instance, Ahiadu (2015), researched on the over-performing and under-performing equities under the Ghana Stock Exchange using regression and standard market model. The major findings were that, majority of the stocks exhibit underperforming nature relative to the market.

Again, Manu (2017), examined the effect of stock performance on economic growth in Ghana. The study employed vector error correction model (VECM) and Granger causality test, using market capitalization ratio as stock market indicator and Gross Domestic Product (GDP) as economic growth indicator. Result from VECM regression analysis showed that, the stock market contributes positively to economic growth only in the short-run. In the long-run, the exchange does not affect economic growth in Ghana.

Also, Agyeman (2010), studied how GSE can be improved by interacting with the stock market industry players in Ghana to gain understanding of the challenges impeding the growth of the GSE and solicit their views on how to address them.

Moreover, Mensah *et al.* (2012), investigated the challenges and prospects of the Ghana Stock Exchange. They also sought to ascertain the level of public awareness of the stock exchange and discover the availability of prospects at the Stock Exchange. The study designed was cross sectional. Stratified, simple random and sampling methods were used for the study. The result revealed that the stakeholders expected higher performance compared to the actual performance of the Variance.

Lastly, Gozah (2017), focused exclusively on determining portfolio optimization using vale-at-risk. Variance-Covariance, Correlation and Standard Market Model method

were employed in the study in determining stock and portfolio performance in the financial market.

The recent financial crises that led to the extinction of many institutions such as GN Bank, UT Bank and several others in effect have made some of the old literatures on portfolio optimization on the Ghana stock exchange vulnerable. This is because some equities that were considered in portfolio are no more in existence (Anon. 2018).

Therefore, a new portfolio optimization needs to be built after this financial sector reforms.

1.3 Research Objectives

The objectives of these research are to:

- determine the performance of selected stocks in the Ghanaian financial market.
- evaluate the selected stocks for portfolio creation.
- create an optimal portfolio from the selected stocks.

1.4 Methods Used

The methods to be employed would include:

- Standard Market Model: This will be used as a basis for determining the equity performance in the observable market rate.
- Mean-Variance Model: This will be used to quantify expected portfolio returns and acceptable levels of portfolio risks in portfolio optimization.

- Variance-Covariance method: This will be used to calculate the risk, the mean (expected value) and standard deviation of the equity portfolio by considering price movement of equities and also compute portfolio's maximum loss.
- Correlation Analysis: This measures the association between the various equities used in the study.
- Markowitz model: This measures the optimize returns for a given level of variance across an investment opportunity.

1.5 Structure of the Thesis

This thesis is divided into five chapters. Chapter 1 gives the general introduction of the work. Chapter 2 represent the general literature review of the topic under study and relevant models related to it. Chapter 3 focusses on the methods used in the analysis and their mathematical interpretations. Chapter 4 deals with the analysis and discussion of the data collected. Lastly, Chapter 5 talks about the conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

A well-functioning and efficient financial system is vital in supporting economic growth for the successful development of a strong and dynamic private sector. The capital market, of which the stock exchange plays a key role, is an important part of the financial system, offering effective investment generation, distribution, and corporate governance delivery mechanisms. The stock exchanges also promote the management of government debt, conduct monetary policy, and provide a privatization stream. Before 1999, many Ghana companies were unable to sell their securities on a structured market. Most of the stocks are sold on the counter market. This created a lack of trust in the capital market system.

Stock markets provide an incentive for rising companies to raise capital at lower cos ts. Moreover, companies are less dependent on bank financing in countries with developed stock markets, which can reduce the risk of a credit crunch. Accordingly, stock markets are able to influence economic growth positively by promoting individual investment and providing opportunities for firm funding.

2.2 Historical Review

This chapter reviews literature related to this study and other relevant materials. It highlights what the stock exchange market is, its significance, and the Ghana Stock Exchange (GSE) performance over the years. The chapter also looks at the risk-

return relationship presented to the exchange-listed companies. Lastly, it reviews how stocks on the market are determined.

2.2.1 Stock Exchange

According to Watson (2007), a stock exchange also known as the structured bond exchange is a secondary corporate securities market. The markets have primarily central market places where individuals and members of the company buy and sell orders for shares accepted for trading. The presence of good markets in which investors can buy or sell outstanding securities has an important, though indirect, effect on corporations' ability to raise new capital through securities sales. Much of the secondary corporate securities trading takes place on organized security exchanges such as the Ghana Stock Exchange (GSE), the Johannesburg Stock Exchange (JSE), and the New York Stock Exchange (NYSE), to name just a few. Asamoah (2010), defined a stock exchange as a type of exchange where firms issue equity to finance their new investment. It is a position where companies offer marketable securities as a way of buying other firms or holders of other companies that want to sell their shares at a future date. All of these goals can be achieved through a listing of stocks.

With reference to the World Exchange Federation, a stock exchange or share exchanges is a business or cooperative association which offers services to stock brokers and investors, shares for trading companies and other securities, as well as other financial instruments and asset events including income and dividend payments. Contains shares exchanged in a stock exchange. It has to be listed there in order to be able to trade a security on a certain stock exchange. Normally, there

is at least a central location for record keeping, but trade is less related to such a physical location, as modern markets are digital networks that provide the benefits of speed and transaction costs. The initial offering of stocks and bonds to investors is by definition done in the primary market and subsequent trading is done in the secondary market. A stock exchange is often the most important component of a stock market. Supply and demand in stock markets is driven by various factors which, as in all free markets, affect the price of stocks. The stock exchange fulfills two purposes, according to Bokpin and Isshaq (2008). These are the provision of a capital market where it is possible to bring together investors with companies requiring new financing and a secondary market for existing financial resources already owned by the investor. The main market, however, is inextricably linked to the nature of a secondary market because investors are more likely to acquire new securities if they are certain that their investment will be readily available on the stock exchange. Thus, while in the absence of a stock market a new issue market might exist, in reality new issues are usually subject to the stock exchange regulations as well as the relevant statutory rules.

Manual (1994) also suggests that two key functions are performed by the stock market. First, the stock market brings together companies and investors to allow investors to put risk capital into businesses. The businesses will make use of the funds they raise to invest in new capital projects. Additionally, by providing a ready market, the stock market provides investors with a means of selling their investment by buying and selling secondary shares and loan stock, should they wish to do so.

2.3 The Concept of Efficient Market Hypothesis

The primary assumption for EMH is that stock prices reflect all available information accurately and quickly in such a way that no one can earn abnormal returns. The time for any new information to be changed is considered a vital factor; if the market adjusts faster and more accurately, it is considered more efficient. Dyckman and Dela (1986), state that a security market is generally defined as efficient if (1) the market price of the security behaves as if it completely represents all available information and (2) those prices react instantly or almost unbiasedly to new information. The alternative hypothesis is that the security market is inefficient and the new information is not accurately reflected as a consequence of the stock price. This could result from the following: the investor is unable to correctly interpret the new information; the investors have no access to the new information; the trading security transaction cost is a barrier to free trading; the short sale restriction; and finally, the changes in accounting principles could mislead the investors.

2.3.1 Efficient Market Hypothesis Classification

Fama (1970) coined the phrase "efficient market" to describe the market price which fully reflects all available information. Also, based on information, Fama classifies market efficiency into three levels: Weak, Semi-strong, and Strong.

2.3.2 Weak Form of Market Effectiveness

The weak form occurs when stock prices only reflect information about the previous share price series. Another way of stating this hypothesis by using investors who rely on past share price information to predict future stock prices are worthless. Thus, the chartist's impact on price data is irrelevant in determining future stock prices. This

degree of efficiency means that prices follow a random walk and advice on buying and holding is impounded on such a business investor. Some studies deal with whether or not stock price activity is a random walk. Osborne (1959) finds that a random walk follows a change in stock prices. The random walk hypothesis simply states that, "the size and direction of the next price change is random in relation to the available knowledge at that point in time at a given point in time." Dyckman and Dale (1986) argue that the fact that stock prices randomly change does not mean that stock prices change without any reason; there is a reason for this movement, which has been the subject of empirical research for more than a decade. There were four main methods for measuring time-return dependency (market efficiency weak-form): serial correlation test, filter rule test, cyclic test, and volatility test.

2.3.3 Semi-Strong Efficiency on The Market

In a semi strong form, the market is efficient if the security prices reflect on all information available to the public. This means that the stock price is adjusted to all public announcements in newspapers, corporate forecasts and annual reports quickly and unbiasedly. Through EMH, this degree of efficiency means that investors cannot use data that is publicly available to predict future prices. Therefore, financial analyst's impact is immaterial. Semi-strong form of market efficiency is also known as a weak form of efficiency and is important to the accounting profession since accounting is the primary source of public information via financial reporting issues. If the stock market is semi-strongly effective, investors will not be able to achieve consistently above average returns. On the other hand, if at the time of public announcement of specific information, investors can reliably achieve above average return at investment, then the stock market is inefficient with regard to this data (Osei, 1998).

2.3.4 Strong Market Performance

The strong form of market efficiency exists when all public and private knowledge are expressed in the stock price. This form is the most comprehensive case and it is very difficult to test EMH in strong form because it is difficult to observe private information. In the United States, there is an official Security Exchange Commission (SEC) document containing an official summary of insider trading, recording trading transactions by officers, directors and major stockholders using private information. If the outcome of these trades is an unusual return, then the market is not strongly successful (Winful *et al.*, 2012).

2.4 Empirical Analysis of Market Efficiency

Fama (1991) developed a new market efficiency classification: firstly, a return predictability test rather than a weak-form test; secondly, an activity analysis rather than a semi-strong form test; and thirdly, a private information test rather than a strong-form test. He focuses on predicting returns with other variables such as dividend yields and interest rates, testing asset pricing models and anomalies, and testing for seasonal returns and volatility in security prices for return predictability. On the other hand, event study is the clearest evidence of market effectiveness as it gives new information a picture of the speed of price adjustment. The market efficiency test for information on investment decisions, changes in dividends, changes in capital structure and corporate control transactions is carried out in the event study. Checking business performance with regard to private information can be achieved by checking the activities of corporate executives, adjusting the ratings of price column, analysts' survey and the activities of pensions and mutual funds.

Frimpong (2008) examined in the case of the Ghana Stock Exchange (GSE), the weak form of the Efficient Market Hypothesis (EMH). Daily returns from the Databank Stock Index (DSI) were used for the exercise over a five-year period 1999-2004. The basis for their study is Random walk (RW) and GARCH (1, 1) models. The GSE DSI returns series displays a clustering of volatility, an indication of GSE inefficiency. The GSE hypothesis of the weak-form efficient market (random walk) was rejected, which means the market is inefficient. To investors, both domestic and foreign, the fragmented market has significant implications. Knowledge of profitable market predictability arbitration incentives helps to attract investors to diversify from more efficient markets to invest on the GSE bourse to maximize their returns.

Asamoah (2010) analyzed the effect on share price actions in Ghana of the dividend announcement. The protocol for the analysis of the case was used to achieve the research goal. In addition, the Ranked Test signed by the Wilcoxon Matched-Pair was used to test the null hypothesis. The main finding was that the GSE was not semi-strongly efficient, leading to the conclusion that the GSE had to address three forms of efficiency – operational efficiency, efficiency of allocation, and efficiency of pricing.

Osei (1998) also looked at the Ghana stock exchange's asset pricing and information quality. The study shows that after the announcement week, i.e. week zero, the market continues to trend up or down. This is inconsistent with the hypothesis of an efficient market (EMH). The conclusion is that the Ghana Stock Market is inefficient in the exchange-listed companies reporting annual earnings information.

Healy (1988) expanded to include dividend omission announcements on the study conducted by Asquith and Mullins (1983). They created two sets of samples for the

study – one for announcing the initiation of dividends and the other for announcing the omission of dividends. They took 131 firms in sample A that made a first dividend announcement or firms that announced their dividend after 10 years of gap (1986-1996). While they took 172 companies in sample B, they announced they would no longer provide dividends or decide not to pay dividends after 10 years of continuous payment of dividends. The result was not expected to be very different from what Asquith and Mullins discovered five years before their research. They found a significant change in earnings in the following year due to the announcement of dividends or omissions. They also found, however, that there is a strong correlation between changes in stock prices and changes in the initiation or omission of dividends. They also later conducted another study on the dividend hypothesis information content by emphasizing the share price response due to the company's dividend announcement. To make the study more reliable which was the company's information environment in concern, they took one more variable. Because quantifying the company's information environment is complex, they have taken few proxy variables to quantify the company's information environment.

They used company size, number of analysts following the company trade, number of institutional investors in company equity and percentage of equity held by institutional investors to measure the company's information environment in their study. They took 80 companies from January 1976 to December 1987 to announce the initiation of dividends. They further divided the sample into three other low, medium and high information environment categories. The finding was once again no different from the previous studies on the same subject. They also found out, however, that on the 2nd day after the announcement, the stock price of the company

that made its dividend initiation announcement went abnormally high.

2.5 Existing Stock Exchange Performances

African stock exchanges are recording mixed performance overall. In addition to the mode and well-known Johannesburg Stock Exchange in the world, many of the other stock exchanges are bugged down by numerous issues including: Low demand and supply of financial products, low trade volumes, high taxes, poor macroeconomic conditions, insufficient market infrastructure, etc. Generally speaking, African stock markets are dominated predominantly by equity markets to which some emerging covered bond markets could be added. Stock markets are not well known for derivative instruments and other specialized products and are therefore rare in Africa (Anon. 2010).

2.6 Stock Exchanges in African

The African continent currently has twenty three stock exchanges and is geographically divided as follows: four stock exchanges in northern Africa: Algeria, Casablanca, Cairo and Alexandria (CASE) and Tunis; Four stock exchanges in West Africa: the Bourse Régionale des Valeurs Mobilières (BRVM) bringing together eight West African Economic and Monetary Union (WAEMU), Cape Verde, Ghana and Nigeria; two stock exchanges in Central Africa: Douala and the Bourse des Valeurs Mobilières de l'Afrique Centrale (BVMAC), bringing together the six members of the Central African Economic and Monetary Community (CEMAC); Eight stock exchanges are based in Southern Africa: Botswana, Johannesburg, Malawi, Mozambique, Namibia, Swaziland, Zambia and Zimbabwe. Five stock exchanges in East Africa: Nairobi, Island of Mauritius, Kampala, Tanzania, and Sudan (Alder *et al.*

1998).

2.7 The African Stock Exchange Structure

In compliance with international standards, their regulatory authorities have followed the concept of net separation of the roles and obligations of the different players in order to ensure the effective and efficient functioning of stock exchanges. This preference is usually expressed in the development of two distinct poles of competence on each stock exchange: a public pole that serves as the regulatory and supervisory authority of the stock exchange, whose main role is to control and supervise the stock exchange, and a general private pole with the primary purpose of managing and organizing the stock exchange market. The first pole serves as the state's delegate and works independently. This safeguards the general interest of market actors and guarantees the stock exchange's security and reputation. A specific body is set up in most countries. In a few countries, this function is directly entrusted to a finance department within the Ministry. The pole's roles and duties are divided into the following elements: protecting savings invested in financial instruments and any other investment that may result in public savings invitations, providing detailed and reliable information; To all stakeholders or investors in a fair manner, to ensure the proper functioning of stock exchanges and the smooth conduct of public stock sales, to regulate and control all financial transactions concerning quoted companies (listing, capital increase, public sale or acquisition, merger, etc.) and to develop an organized, fair and efficient stock exchange .

The second pole, commonly referred to as the stock exchange or market place of the business, has the function of managing and coordinating the stock exchange

operations. It is responsible, among other things, for: listing securities, quoting securities, monitoring and managing quotation sessions, disseminating / publishing stock exchange information and laying down rules for negotiation, clearing and delivery of settlements as well as the rules of good practice to be respected by brokers and dealers in order to ensure transparency, impartiality and the proper organization of the stock exchange (Arzac and Bawa, 1977).

Capitalizations measure the stock exchange's financial capacity. It is equal to the total products of listed securities' stock exchange prices and the number of each of the listed shares. Among other things, this indicator is used to assess how secure the investment is through the listed companies' stock exchange weight. From 2002 to 2006, African stock market capitalization increased from $238.4 billion to $955.5 billion, That is to say, an increase of 75.1 per cent made possible by the increase in the number of stock markets, the relatively large size of new listed companies and the flow of foreign investors to African markets such as South Africa and Egypt (Anon. 2015).

The JSE Limited accounted for almost 75% of African stock market capitalization in 2006, led by CASE with 10%. Together, these two exchanges represent about 85% of the capitalization of African stock exchanges, but only 1.5% of the capitalization of the International Federation of Stock Exchanges, where they (i.e. JSE and CASE) are the only two African members. Nevertheless, capitalization of the smaller African stock markets increased significantly between 2002 and 2006, the most important differences were seen in Ghana (+ 1.559%), Zambia (+ 1.196%) and Uganda (+ 1.085%). Swaziland, for its part, was the least diverse with only 57%. According to the IMF, the total capitalization rate was 36 percent for Africa compared to 161.3

percent, 70.6 percent and 25.4 percent for Indonesia, Thailand and Mexico, respectively. This ratio was still 214.1 percent for JSE Limited in 2004 and 51.3 percent for CASE, which reflects the relatively advanced stage of these two financial markets.

North Africa, with four stock markets, had a capitalization of $147 billion in 2006 compared to $37 billion in 2002, a rise of 297%; less than the continent's average of 379% over the same time. Cairo and Alexandria Stock Exchange (CASE) accounts for 63.7% of that region's capitalization. While that of Algiers is still negligible (0.1 percent) mainly because it only recently went into service. In West Africa, stock capitalization increased significantly from 8 billion dollars to 50 billion dollars on the three existing stock markets between 2002 and 2006. This strong growth, 525 percent, is largely due to the dynamism of the Nigerian stock exchange, the fourth most active stock exchange and the continent's largest in 2006, with around 80 percent of the region capitalizing. While being international, the BRVM has a small 5% capitalization, while Ghana's Stock Exchange contributes 15% of the country as a whole. Through five stock markets, East Africa has a total stock capitalization of $4 billion in 2002 and $22 billion in 2006 – a rise of 400 million over five years.

This rise is largely due to innovations on the Kenyan market, above the continental average. The region of Southern Africa has the largest number of African stocks, eight markets and also the most active ones. Consequently, its capitalization rose from $188 billion in 2002 to $736 billion in 2006 following a 290 percent increase, a rate slightly below the continental average. Only Malawi has a market capitalization of more than $10 million in 2006 (12.29 million) from other stocks in the country. The Douala Stock Exchange has a capitalization of about 4 million U.S. dollars in the

Central African region, with a listed company in 2006, while BVMAC began operations with the issuance of a government bond in August 2008. Analysis of the test of feasibility establishment of the pan-African stock exchange (Anon., 2015).

2.8 African Stock Market Development

As diverse and contrasting as the African continent is, so are the capital markets of Africa. The African continent has more than 20 active stock exchanges with 53 countries, including one of the world's only local stock exchanges, linking eight French speaking countries in West Africa. Africa hosts one of the world's largest stock markets with a market capitalization of over US$ 180 billion in South Africa. It contrasts sharply with the other African stock markets with relatively small market capitalizations. Except for the South African market and the North African markets to a limited extent, African stock markets are described as “border markets”. Typically, these markets are characterized by relatively small levels of capitalization and liquidity. As a result, most of these markets are removed from the key indices of the global equity market, thereby drawing few investment funds from Global Emerging Markets (GEM). Nonetheless, in the past two and a half years, many leading African markets such as Botswana have bucked the negative trend and reported solid performance in the midst of the bearish performance of established stock markets.

Smaller African markets, due to their lack of interaction with developed markets, have been relatively immune to international jitters affecting share values worldwide. In terms of risk diversification, this distinctive feature of African equity markets provides positive benefits. Before they could enter a new phase of rapid growth, African stock exchanges face a number of challenges. The most critical issue is to remove existing

barriers to the growth of institutions. These include greater dissemination of information on these markets, rigorous implementation electronic trading and central depository systems adoption. A number of countries have already started to implement necessary changes, particularly in the area of trading and settlement systems and regulatory regimes that will continue to improve. A number of African governments experienced a systematic switch to free-market policies driven by the desire to reduce the burden on government finances in the 1990s. By introducing market-friendly changes, this has been done. The privatization of state-owned companies was a central component of this process. A lot of these privatizations have been impacted by local exchange listing. Several subsidiaries of large international corporations were also encouraged to list their local operations in order to further promote the growth of a regional capital market. Capital raising in African equity markets is lively for both investors and emerging private companies, despite the relatively small size of issues.

Some African governments took advantage of local capital markets development to issue treasury debt instruments listed on the stock exchange. Kenya and Ghana are a case in point where these governments have been able to better manage local debt by issuing long-term instruments. The spin-off of this has been enhanced transparency in local bank lending facilities pricing and increased competition within local banking industries. We are still witnessing rapid development in the African capital markets debt segment. The creation of the local pension fund industry is an increasingly encouraging phenomenon. With the exception of South Africa and to some extent South Africa, traditionally, private and institutional cash flows have been invested mainly in real estate, term bank deposits and treasury bills. As part of broader financial

sector reforms, a number of African countries have introduced new legislation allowing a regional fund management industry to develop. Looking ahead, African capital markets represent international capital's final frontier (Prigent, 2007).

2.9 Institutional Framework: The Sale of Ghana Stock

The Ghana Stock Exchange was formed in 1989 as a private limited company under the 1963 Companies Code of Ghana under guarantee. In October 1990, the Exchange was registered as an official stock exchange under the 1971 Stock Exchange Act (Act 384). Nevertheless, the Exchange changed its status to a limited by guarantee public company in April 1994. Trading began in November 1990 on the Exchange floor. There are currently 42 equities (from 37 companies) and 2 corporate bonds.in the GSE trading takes place under the Continuous Auction Trading System (CAT) on the Exchange Floor. Securities exchanged on the trading floor include ordinary shares (common stock), corporate debt securities and government bonds. Through the Trading Control, non-resident Ghanaians and foreigners were given permission to invest through the Exchange without prior approval. However, only up to 10 percent of any security approved for listing on the Exchange can be held by an external resident portfolio investor (whether individual or institutional). Moreover, the total holdings of all external residents in one security shall not exceed 74%. The original capital is subject to free and complete foreign exchange remittance plus all capital gains, returns and related earnings for all investors, both resident and non-resident, there is a 10% withholding tax (which is also the final tax) on dividend income. Nonetheless, capital gains on listed shares remained tax-free until 2015. There's also no obligation to mark. The output of the GSE has varied

considerably since its inception. The key index, known as the GSE All-Share Index, contains all listings. The GSE was the sixth best index on the emerging stock market in 1993, with a capital increase of 116%. It was the best index in all emerging markets in 1994, gaining 124.3 percent in its index level. The index growth in 1995 was a disappointing 6.3%, partially due to high inflation and interest rates. The 1997 index growth was 42% and it was 868.35 at the end of 1998 (Anon. 2010).

The market is dominated by the manufacturing and brewing industries. The banking sector is a distant third, while other listed companies fall into the insurance, mining and oil sectors. Total market capitalization rose from about 30 460 trillion in November 1990 to about 391 385 million at October 31, 2001. The total volume of shares traded between November 1990 and October 31, 2001 was estimated at 569 million at 558,845.75 million.

The Exchange aims to strengthen its efforts to increase listings and facilitate the mobilization of funds and encourage the sale on the Exchange of State-Owned Enterprises. The exchange will also encourage the listing of corporate and government debt instruments and new products such as collective investment schemes to be developed. Capacity building will be strengthened to improve competence in the sector by improving the expertise of its employees and business participants locally and internationally. However, infrastructure for the automation of trading, clearing, settlement and depository systems will be put in place to increase the efficiency of the securities market for all users (Anon., 2004).

2.10 The Ghana Stock Exchange's Development

Various studies have suggested a number of proposals to address the challenges facing

the GSE. Yartey and Adjasi (2007) discussed policy options to foster stock market development in Africa. They proposed comprehensive electronic trading systems and central depository systems as being very important to tackle the problems of stock exchanges in Africa. Their study findings are useful and may have reinforced the need for an electronic platform and therefore GSE automation. It takes much more to boost liquidity, however, than automation and institutional developments. If the shares of a listed stock are not available, the liquidity rate would still be relatively low, irrespective of how efficient the electronic trading systems are. In order to deal effectively with liquidity, the existence of a large pool of investors and a relatively large float of shares should be among other things. Wallenius (2009) illustrates the Ghana Stock Exchange's securities clearing and settlement in the light of the international standards set for these processes. She compares and contrasts with other emerging markets the GSE clearing and settlement process. She suggests that the modernization of the clearing and settlement processes will provide a good basis for the stock exchange's overall improvement. Clearing and agreement applies to everything that happens to enable the final transfer of property and ownership after that initial declaration. Modernizing the clearing and settlement system would provide real opportunities to drive down transaction costs, shorten the settlement period, and raise successful trading volumes.

There is ample evidence to support the need for daily GSE trading to be automated. This will require strong political will and substantial financial commitment to automate the trading system; GSE's increased efficiency will outweigh its cost. The GSE is now using an electronic trading platform that allows stockbrokers to trade through a secure internet connection from the Ghana Stock Exchange floor, their offices, and any

location. In addition to the trading system, there is an electronic clearing and settlement process. The scheme facilitates T+0 or T+1-based mutual settlement of trade. Shares are automatically moved on settlement days to customer accounts in the depository network and the broker's debited settlement account. There is also securities and depository network that supports the automated trading, clearing and settlement processes of the Exchange with depository services. The platform enables investors to more effectively pass, pledge and access security information.

This revolution is expected to make it easier for investors to trade in listed securities, thereby boosting liquidity and reducing the risks associated with the previous paper certification such as impersonation and forgery. In general, it is expected that electronic trading would boost market efficiency and increased liquidity. The digital software provided direct access to real-time trading information available through the Bloomberg Professional platform to investors around the globe.

2.11 Challenges Facing Ghana Stock Exchange

Since its inception, the GSE's spectacular performance does not mean the GSE is growing like the New York or the London Stock Exchange. Two styles of constraints have impeded the growth of Ghana's stock market from the relevant literature. The first concerns the political and macro-economic set-up and the second directly concerns the trade. Over the past two decades, Ghana's macroeconomic policies have been marked by unstable and generally high unemployment, high interest rates and strong exchange rate fluctuations. These factors prevent investors in the medium to long term from having a clear visibility of the macro environment. Investors are generally willing to face diversifiable risks but unwilling to look at markets that are

characterized by political and macro-uncertainty.

According to Benimadhu (2003), low liquidity, few listed companies and the small size of the exchange as well as performance are among the exchange-specific issues affecting stock markets in Africa. Certain factors that have stifled the GSE's growth are the absence of a strong and active domestic shareholder base, led by institutional investors including pension funds and insurance firms. Clearing and exchange processes and financial networks are also behind the stock market of the world class. Manual trading, clearing and settlement processes were run by the GSE until 2009.

2.12 Ghana Stock Exchange Importance

In theory, it is predicted that the stock market will stimulate economic growth by improving domestic savings and growing investment quantity and value (Singh, 1997). The Ghana stock exchange, one of Africa's leading stock exchanges, was recognized as a stock exchange by executive instrument in October 1990 under the Stock Exchange Act 1971 (Act 384) to improve liquidity on the capital market.

Ghana's stock exchange has been developed with the following objectives: to provide the public with the facilities and mechanism for the purchase and sale of bonds, shares and other securities, to regulate members' transactions with their customers and other members, and to track quotations on the securities market for bonds.

The Ghana stock exchange was set up with the following objectives: To provide the facilities and framework to the public for the purchase and sales of bonds, shares and other securities, regulate the dealings of members with their clients and with other members and to control the granting of quotations on the securities market in respect of bonds.

By supplying investors with an additional financial instrument that can better meet their risk expectations and liquidity needs, the stock market is expected to promote savings. The savings rate may be improved through better savings mobilization (Levine and Servos, 1998).

In addition, stock markets also provide an opportunity for growing firms to raise capital at lower costs. Also, companies are less dependent on bank financing in countries with developed stock markets, which can reduce the risk of a credit crunch. Therefore, stock markets are able to influence economic growth positively by promoting individual savings and providing opportunities for firm financing.

Through means of the takeover process, the stock market should ensure that previous investments are also used most effectively. Theoretically, it is anticipated that the risk of acquisition will provide an opportunity for management to optimize firm value. The assumption is that unless management maximizes firm value, another economic agent can take control of the company, replace management, and reap the profits from the more efficient business. Therefore, a free market in corporate control is supposed to provide the strongest guarantee for performance in the use of resources by providing financial discipline. Likewise, it is expected that the opportunity to trigger improvements in the management of listed companies would ensure effective use of managerial capital (Kumo, 2008).

Critics of the stock market claim that when speculative bubbles emerge on the market, stock market values do not accurately reflect the underlying fundamentals (Binswanger, 1999). In such cases, stock market values are not calculated simply by increasing the projected future cash flows, which should represent, on the basis of an efficient market hypothesis, all the basic information currently available. The stock

market creates its own volatile growth patterns under this scenario, which can be driven by irrational behavior. It is predicted that this irrationality will adversely affect the real economy sector as it is in danger of becoming a casino's by-product.

Some also argue that volatility on the stock market may have a negative impact on corporate governance, as very liquid stock markets can promote myopia of investors. Because investors can sell their shares easily, more liquid stock markets can weaken investors' engagement and opportunity to exercise control of the company. In other words, instant liquidity on the stock market will deter investors having a long-term commitment to businesses whose shares they own and thereby raising possible corporate governance problems with serious economic growth consequences (Bhide, 1994).

Critics also point out that the actual operation of the mechanism for pricing and acquisition in well-functioning stock markets results in short-term and lower long-term investment rates. It also provides perverse incentives, rewarding managers for their performance in financial engineering rather than through organic growth creating new wealth (Singh, 1997). This is because prices respond very rapidly to a variety of information that has an effect on financial markets expectations. Thus, stock market prices tend to be highly volatile and make short-term gains. Therefore, because the stock market undervalues long-term investment, managers are not encouraged to make long-term investments because their efforts are measured by the quality of the financial assets of a company; Which could hurt long-term business prospects (Binswanger, 1999).

However, empirical evidence shows that the takeover process does not serve as a disciplinary role and that competitive choice in the corporate control market takes

place on the basis of scale rather than quality (Singh, 1997). A large, inefficient company therefore has a higher chance of survival than a small, relatively efficient company. These problems are further magnified in developing countries especially sub-Saharan African economies with their weaker regulatory institutions and greater macroeconomic volatility. The higher degree of price volatility on stock markets in developing countries reduces the efficiency of the price signals in allocating investment resources. Such extreme stock market restrictions have led many observers to doubt the system's role in fostering African countries' economic growth.

CHAPTER 3

MATERIALS AND METHODS

3.1 Introduction

This section discusses the procedures used in data analysis and the type of data used for the study. Procedures used includes, sampling technique, data collection, data analysis, statistical procedure and organizational profile. The research also aims to use Jensen alpha ratio to analyze the performance of companies listed on the Ghana stock exchange relative to the output or motion of the market. The ratio has the potential to show either positive or negative qualities of firms that are underperforming or overperforming the market. Theoretically, the Jensen ratio can also be used to validate EMH stock market performance literature, so no investor in an efficient financial market is expected to beat or outperform the market. Lastly, the Capital Asset Pricing Model (CAPM) was also used to estimate the beta (risk) of the under-study listed companies.

3.2 Approach for research

Using a quantitative approach, the performance of companies listed on the Ghana stock exchange was analyzed from 2009 to 2018 to evaluates stocks that surpassed or underperformed the stock exchange market, and identified the returns on the Ghana stock exchange for individual stocks.

3.3 Research population

The study population consists of 42 listed stocks (companies) from 42 firms on the GSE. This has been divided into twelve (12) sectors in Ghana. The sectors are: finance, manufacturing, distribution, insurance, food and beverage, technology, agriculture, education, exchange traded funds, ICT, advertisement and production and mining.

3.4 Sampling technique

Sampling helps a lot in research. It is one of the most important factors which determines the accuracy of your research/survey result. If anything goes wrong with your sample then it will be directly reflected in the final result. There are lot of techniques which help us to gather sample depending upon the need and situation.

3.4.1 Purpose or judgmental Sampling technique

Purpose or judgmental non-probability sampling technique was mainly used for the purposes of this study. Expert sampling and snowball sampling were the two judgmental sampling methods used by the author. Judgmental sampling was used to select the samples used for this research as the researcher needed to ensure that stocks in the sample had the required characteristics, as such, would be suitable for the study and would also ensure a fair representation of the interest population. Expert sampling involves assembling a sample of stocks in a specific area with known or proven experience and expertise. This technique was adopted for the study because it is the best way to obtain the views on stocks with specific expertise in the subject area as well as to provide evidence of the report's validity. Snowball sampling is based on initial subject references to generate additional subjects (Goodman, 1961). This

technique allowed the researcher to select stocks capable of handling the problem being studied. Therefore, focusing on the entire population was impractical for the sample size.

3.4.2 Performing Preliminary Test on the Data

Monthly Returns

The returns of one stock are simply expressed as in Equation (3.1)

$$R_s = \frac{Closed_1}{Closed_0} - 1 \tag{3.1}$$

Where $Closed_0$ is the previous month close of stock and $Closed_1$ is current month close of stock. However, returns were computed in monthly and used where necessary.

3.5 Sample size

Of the forty-two (42) companies listed on the GSE, nineteen (19) firms were sampled using judgmental sampling technique. Industries in this group declared their cash dividend between 2009 and 2018 and were listed on the stock exchange between 2009 and 2018 — industries that were listed in 2009 and did not pay dividends were de-listed. The author has adjusted to the GSE Composite Index (CI), which is the calculation of all listed companies and market performance. For the study, the CI format was more convenient. Monthly data of 10 years, spanning up to 120 monthly cycles were used for the analysis.

3.6 Data collection

Secondary data was used for this study. The data includes trading month-end stock prices, stock dividend yield, selected listed companies' cash dividend, risk-free rates,

and monthly GSE Composite Index from 2009 to 2018 were used for research.

3.7 Beta Values

Valid beta values were calculated using the analyst's most popular single-market model in equation (3.2) due to its simplicity and efficiency. The returns were measured using the month-end share price from 2009 to 2018 before calculating a beta stock.

3.7.1 The standard market model

The standard model state that the return on a security depends on the returns on the market portfolio and the extent of the security's responsiveness as a measure of beta. The return also depends on conditions that are unique to the firm. The standard model of the Single Market used as the basis for estimating the valid beta on a security is as follows:

$$R_{it} = \alpha_i + \beta_i R_{mt} + U_{it} \tag{3.2}$$

where:

R_{it} = actual or realized monthly returns at period t

α_i = constant in regression equation called alpha

R_{mt} = rate of return of the market index in the period t

β_i = slope of the regression equation called the beta value.

U_{it} = distribution term or excess returns

$R_t = R_f + (R_m - R_f)\beta$. *making* β *the subject of the relation yields*

$$\beta = \frac{R_t - R_f}{(R_m - R_f)} \tag{3.3}$$

where:

R_t = actual return of the stock

R_m = return of the market

R_f = risk free rate

β = the beta of the stock (riskiness)

3.7.2 Market returns

This is the return on the overall theoretical market portfolio which includes all assets and having the portfolio weighted for value. The market return is indicated in Equation (3.4) as follows:

$$R_{mt} = \frac{CI_t - CI_{t-1}}{CI_{t-1}} + Div \tag{3.4}$$

where:

CI_t = the current stock composite index of the period t

CI_{t-1} = the previous stock composite index of the period t-1

Div = the average dividend yield of the index over the period t

R_{mt} = rate of return of the market index in period t

3.7.3 Actual or realized monthly returns

The actual or realized monthly returns are given by Equation (3.5)

$$R_{it} = \frac{P_t - P_{t-1} + cash\ div.}{P_{t-1}} \tag{3.5}$$

where:

R_{it} = actual or realized monthly returns of stock i at period t

P_t = price at the end of period t

P_{t-1} = price at the beginning of period t

cash div. = cash dividend over the period t

3.8 Jensen Alpha Model

This is used to determine the abnormal return of a security or portfolio of securities over the theoretical expected return. It is a version of the standard alpha based on a theoretical performance index instead of a market index. The Jensen alpha model is indicated in equation (3.6)

$$\alpha = r_p - \left[r_f + (r_m - r_f) \right] \beta_p \tag{3.6}$$

where:

r_p = actual or realized returns of security p

r_f = risk free rate

β_p = beta of security p

r_m = return of the market

$(r_m - r_f)$ = risk premium thus compensation of the market for diversification

α = Jensen alpha determines the performance of the stock to that of the market

3.9 Portfolio Optimization using Portfolio Theory of Markowitz

Markowitz' model (1952) which consist of mathematical methods that makes it possible for construction of stock portfolio with different combinations of stocks where short sale may be allow or not. Markowitz model is all about simultaneously maximizing returns and minimizing risk. With this model, a single portfolio of risky assets with the least possible risk that is preferred to all other portfolio with the same

level of return must be reached. Our optimal portfolio will be somewhere on the ray connecting risk free environment to risky portfolio and where the ray becomes tangent to our set of risky portfolios. This point has the highest return with least risk. Markowitz used the arithmetic mean, variance and the covariance parameters for return and risk estimation. This method uses the Global Minimum Variance Portfolio (GMV). Now, in construction of portfolio for stocks we first find the Minimum Variance (MV). This means that the model use of mean as the portfolio measure of expected return and the variance or the standard deviation as the measure of risk. In finding the risk and the returns of each sector, there is the need to determine the mean and variance of the portfolio.

Suppose that there are n asset with random rates of returns $X_1, X_2, X_3, \ldots\ldots X_n$. The expected rates of returns are $E(X_1), E(X_2), E(X_3), \ldots\ldots E(X_n)$.

Assuming we have a portfolio that consist of n assets, and W_i is the weight of asset i in the portfolio, such that $\sum_{i=1}^{n} W_i = 1$, then the portfolio return is thus $\sum_{i=1}^{n} W_i X_i$ and the expected portfolio return is $\sum_{i=1}^{n} W_i E(X_i)$.

Risk is normally identified as the variance of the portfolio, σ^2. The variance of a random variable is its second central moment and its mathematical definition is given as Equation (3.7)

$$\sigma^2 = E[(X - \overline{X})^2]$$

$$= E(X^2) - 2E(X)(\overline{X}) + (\overline{X})^2$$

$$= E(X^2) - (\overline{X})^2 \tag{3.7}$$

Given the variance of individual assets, the variance of the portfolio can be calculated using the covariance between assets i and assets j - σ_{ij}. Let σ_i^2 and σ_p^2 be the variance of assets i and the portfolio, respectively. The variance of the portfolio can be calculated as shown in equation (3.8).

$$\sigma^2 = E[(X_p - \overline{X})^2]$$

$$= E[(\sum_{i=1}^{n} W_i X_i - \sum_{i=1}^{n} W_i \overline{X_i})^2]$$

$$= E[(\sum_{i=1}^{n} W_i (X_i - \overline{X_i})^2]$$

$$= E[(\sum_{i=1}^{n} W_i (X_i - \overline{X_i}))(\sum_{i=1}^{n} W_i (X_i - \overline{X_i}))]$$

$$= E[(\sum_{i,j=1}^{n} W_i W_j (X_i - \overline{X_i})(X_j - \overline{X_j}))^2]$$

$$= \sum_{i,j=1}^{n} W_i W_j \sigma_{ij} \tag{3.8}$$

Alternatively, let W be the vector representing the weight of each assets, P be the current portfolio position, and $\sum$ be the variance-covariance matrix. The variance of the portfolio can also be written as σ^2 in Equation (3.9).

$$\sigma_p^2 = W^T \sum W \tag{3.9}$$

In maximum expected return problem for an investor that wishes to attain the maximum expected return, the following problem indicated in Equation (3.10) maximizes the expected return of the portfolio.

$$\text{maximize } \sum_{i=1}^{n} W_i E[X_i]$$

$$\text{subject to } \sum_{i=1}^{n} W_i = 1 \tag{3.10}$$

In minimum variance portfolio, maximizing the returns of the portfolio might sound tempting, but the model given in equation (3.8) does not take into account the risk of the portfolio. Instead of maximizing the returns of the portfolio, risk-averse investors might want to minimize the risk (variance) of the portfolio instead. The following problem gives the portfolio with the minimum variance.

$$\text{Min} \quad \sum W_i W_j \sigma_{ij}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} W_i = 1 \tag{3.11}$$

Mean-Variance Efficient Portfolio can be calculated in two ways: the geometric methods and arithmetic method or the sample mean approach (Huang, 2008). But seek of this study we are using the arithmetic approach or the sample mean approach because the sample mean of returns of returns are calculated using historical data. However, what happened in the past for the investor is not as important as what happens in the future because all the investors' decisions are focused to the future or inspected result from the investment. Of course, no investor knowns the future but he/she can use pass information and historical data as well as to use this knowledge

and practical experiences to make some estimates about it. This may be maximizing the returns, minimizing the risk and minimizing the total distance travelled or minimizing the total time to complete a project. For a given project we formulate a mathematical description called a mathematical model to present the situation. The arithmetic mean of the list of numbers (observations) is the sum of all the members in the list divided by all the number of items in the list. Mathematically the mean and the average returns can be express as

$$\mu_r = \sum_{i=1}^{n} \left(\frac{r_i}{n}\right)$$
$$R_i = (r_i - r_{i-1}) / r_{i-1} \qquad (3.12)$$

where:

r = the historical monthly return for i^{th} month;

i = the month of the stock returns;

n = the number of observed months;

The variance of a portfolio is the risk or the deviation from the mean (Markowitz, 1952). Risk can be defined as a chance that the actual outcome from an investment will differ from the expected outcome (Huang, 2008). It is obvious, that most investors are concerned that the actual outcome will be less than the expected outcome. The more variable the possible outcomes that can occur, the greater the risk. Risk is associated with the dispersion in the likely outcome and dispersion refers to variability. So, the total risk of investments can be measured with such common absolute measures used in statistics as;

- Variance
- Standard deviation.

The variance of the returns on the stocks can be calculated as a sample variance as;

$$\sigma_r^2 = \sum_{i=1}^{n} \left((r_i - \mu)^2 / (n-1) \right) \tag{3.13}$$

where:

r = the historical monthly return for i^{th} month;

i = the month of the stock returns;

n = the number of observed months;

μ = the mean of the monthly stock returns.

Th standard deviation can be determined by taking the square root of the variance,

Thus, standard deviation, STD = $\sqrt{\sigma^2}$

3.10 Sharpe Ratio

The Sharpe ratio computed based on the realize return is as follows:

$$S_i = \frac{\hat{\mu}_i - r_f}{\sigma_i} \tag{3.14}$$

Where $\hat{\mu}_i$ is the return on the asset i, r_f is the return on risk free asset, σ_i is the semi deviation of $\hat{\mu}_i$. The portfolio that maximizes the Sharpe ratio is known as the Sharpe optimal portfolio and is given as:

$$S^* = \arg\max \frac{\hat{\mu}_i - r_f}{\sigma_i} \text{ where } \left(I : \sum_{i=1}^{d} i = 1 \right) \tag{3.15}$$

3.11 Efficient Portfolio

In Markowitz theory what happens to a portfolio when relevant parameters such as initial invested capital C_0 and risk aversion γ change when considering risk free asset. It is observed that the optimal solution changes when the above parameters change. This as a result leads to change in investment which promote diversification.

The minimum variance portfolio is considered if C_0 is increased by one, the investment in each asset of the minimum variance portfolio is raised with the small change in the minimum variance portfolio with respect to small increment of the initial investment capital to realize the inverse of covariance of the single asset investment.

$$\frac{\partial \theta_{mv}}{\partial C_0} = \frac{\partial \sum^{-1} \bar{I} \frac{C_0}{c}}{\partial C_0} = \sum^{-1} \bar{I} \frac{1}{c} \tag{3.16}$$

This means that it does not matter how much money an investor is able to invest, the proportion invested in each asset always remain the same. For tangency portfolio the factor C_0 is linearly present but portfolio fractions are independent of C_0. This is as a result that, the tangency allocation and invested capital C_0 linearly depend on each other. Optimal portfolio does not have linear relationship because it occurs with and without risk-free asset. Optimal portfolio without risk-free asset is given by:

$$\theta_{opt} = \frac{b}{C_0 \gamma} \theta_{tg} + \left(1 - \frac{b}{C_0 \gamma}\right) \theta_{mv} \tag{3.17}$$

where:

θ_{opt} = optimal portfolio

C_0 = initial investment capital

γ = risk aversion

θ_{mv} = minimum variance portfolio

θ_{tg} = tangency portfolio

b = asset

ln terms of minimum variance and tangency portfolio it is realized that if C_0 is approaching infinity, the optimal portfolio also approaches minimum variance portfolio. So, if an investor has very small money to invest, the investor should become more risk averse by investing great amount of money in the minimum variance portfolio. The proportion investing in the tangency portfolio must decrease because there is no change when the invested capital is increased. However, there is a turn around when investment made in tangency portfolio occurs when $C_0 = \frac{b}{\gamma}$. Investment becomes marginally small or weird (strange) when $C_0 \simeq 0$.

For investor to achieve maximum utility the amount be invested in the tangency portfolio should move upward and even approaching upward infinity whiles the amount, that should be invested in minimum variance portfolio should move downward and also approaches downward infinity implying that when an investor has very small money ($C_0 \prec 0$) to invest the optimal Markowitz portfolio does not seem usable. The same analysis holds for the parameters of risk aversion γ.

Every risk averse investor whose aversion is high is prudent to invest much in minimum variance portfolio but $\gamma = b / C_0$ then much of investor investment must be geared to tangency portfolio. However, if the investor is risk loving then parameter risk aversion should be close to zero and below, then optimal portfolio becomes very long in tangency portfolio and very short in the minimum variance portfolio. In the case of market portfolio, the allocation is proportional to initial invested capital, the fraction invested in each asset are the same for all values for C_0. If the risk-free asset is zero then market portfolio is identical to the tangency portfolio also if risk-free asset is $\frac{b}{c}$ then the denominator goes to zero, the optimal moves away from the c tangency portfolio along the efficient frontier and the allocation becomes:

$$\lim_{u_f \to \frac{b}{c}} \frac{C_0}{b - c\mu} \sum^{-1} \left(\mu - \mu_f \bar{I}\right) = \lim_{x \to 0} \frac{b}{x} \left(\theta_{tg} - \theta_{mv}\right) \tag{3.18}$$

So, the allocation becomes proportional to $\left(\theta_{tg} - \theta_{mv}\right)$

Optimal portfolio with risk-free asset shows that risky part does not depends on initial invested capital which imply heavy amount is invested in risk-free securities whereas lesser amount is invested in risky assets. Risk averse investor invest much in risk-free while $\gamma = b - c\mu_f / C_0$ means that there is nothing invested in risk-free asset and that everything is invested in risky asset. Optimal portfolio is identical to the market portfolio when risk averse gain value but parameter of risk averse is close to zero then investor borrows much at risk-free rate and invest it in risky assets. The performance here is not nominal amount but are measured in percentages.

3.12 Global Minimum Variance Portfolio (GMV)

The global minimum variance portfolio is the portfolio with highest return with the least risk. In other words, MV is the portfolio with the maximum mean with minimum variance or standard deviation (Zhou *et al.,* 2003). There are a lot of methods use in determining the global minimum variance portfolio. (This study uses the means variance approach using excel and R-software computations). Optimization problems are real world problems we encounter in many areas such as mathematics, engineering, science, business and economics. In these problems, we find the optimal, or most efficient, way of using limited resources to achieve the objective of the situation.

The global minimum variance portfolio is obtained using the historical monthly closing prices of the individual stock indices. In the in formulation of the matrix from the historical monthly closing prices, it is necessary to determine the excess returns on individual stocks by

$$R_i = (r_i - r_{i-1}) / r_{i-1} \tag{3.19}$$

where:

R_i = excess returns

r_i = the historical monthly return;

i = the month of the stock returns.

A square matrix, $n \times n$ matrix is then found using the excess returns as

$$R_{ij} = \begin{pmatrix} r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots \\ r_{nj} & \cdots & r_{nn} \end{pmatrix} \tag{3.20}$$

where r_{ij}'s are returns serving as the entries of the matrix R.

Again, expected return (mean) and the variance or standard deviation (risk) open investor with the idea about the nature of probability distribution associated with one asset and the return and risk are the characteristics of particular asset. But in the same portfolio, how does one asset having a particular feature between return and risk effect the other one with diverse characteristics of return and risk? And be the influence of this relationship to the investor's portfolio? The statistics that can provide the investor with information that can answer these questions are covariance and correlation coefficient. Covariance and correlation are related and they generally measure the same phenomenon – the relationship between two variables. Both the concept if would be best understood by looking at the mathematics behind them.

3.12.1 Covariance matrix

Covariance matrix is a matrix that indicates the extent to which two variables move together over a period of time. Covariance between two stocks or assets A and B can be defined mathematically as follows;

$$Cov_{(A,B)} = \sum_{r=1}^{n} ((r_{At} - \mu_A)(r_{Bt} - \mu_B) / (n-2)) \tag{3.21}$$

where:

n = The sample size of the returns

r_{At} = return for asset A in time t

μ_{At} = mean return for asset A during period t to n.

Positive covariance between two assets suggests that when one asset produces returns above its mean, the other asset tends to also produce above its means. Negative covariance, on the other hand, tells us that when one asset produces returns above its mean the other tends to be below its mean and vice versa. Zero covariance would suggest that there is no consistent relationship between the movements of returns for the two assets. Diversification effects improve as the covariance measure moves from positive to negative between assets (Zhou *et al.,* 2003). This means that negative covariance between two stocks decrease the risk involved when the two stocks are in the same portfolio.

3.12.2 Correlation Matrix

A correlation matrix is a square matrix which leading diagonal entries are ones. Correlation measures the degree of relationship between two stocks or assets. The correlation coefficient (ρ) between two stocks A and B can be defined mathematically as follows;

$$\rho_{A,B} = Cov_{(A,B)} / \sigma_A \sigma_B \qquad (3.22)$$

where:

σ_A = Standard deviation of stock A

σ_B = Standard deviation of stock B

The correlation coefficient between two stocks range from -1 to + 1. The closer the coefficient to (+1), the stronger the relationship between the returns of the two stocks. Two stocks are also said to be perfectly positively correlated when the correlation coefficient between them is +1 and vice versa. However, when the correlation coefficient is zero (0), it indicates that there is no relationship between the stocks (Pirvu and Zitkovic, 2014).

The correlation matrix is also given as:

$$\rho_{ij} = RR_{ij}^{T} \tag{3.23}$$

Thus

$$\rho_{ij} = \begin{pmatrix} r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots \\ r_{nj} & \cdots & r_{nn} \end{pmatrix} \begin{pmatrix} r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots \\ r_{nj} & \cdots & r_{nn} \end{pmatrix}^{T} \tag{3.24}$$

Again, the coefficient of determination (β) can be defined as square of the correlation coefficient between two stocks A and B. Thus $\beta_{(A,B)} = \rho^2$

where ρ = the correlation coefficient between stocks A and B. The coefficient of determination shows how much variability in the returns of one stock can be associated with variability in the returns of the other.

3.12.3 Portfolio Variance Components

The portfolio variance components of the stocks are given as:

$$V_{ij} = W_{ij}^{T} \rho_{ij} W_{ij} \tag{3.25}$$

where:

W_{ij} = vector which represent the weight of the portfolio.

Again, since this study makes use of excel solver in determining the global minimum variance portfolio, it is therefore important to determine the portfolio's variance in returns, σ_p^2 and is given by the sum of all the entries in the portfolio variance component matrix

Thus

$$\sigma_p^2 = \sum_{i=1,\, j=1}^{n} V_{ij} \tag{3.26}$$

Therefore, in calculating the portfolio variance leads to the portfolio standard deviation of returns, and is given by portfolio STD = $\sqrt{\sigma_p^2}$

3.13 Portfolio Investment Problem Model

Portfolio investment problem model is the model design to aid investor take well informed investment decision at any point in time. Achievement of the objective function of the investor is characterized by two constraints (i) minimizing risk given return and (ii) invest the capital the investor had at that moment, so that the amount invested on each asset sum up to C_o.

The parameters of the problem are as follow:

μ = expected return

θ = portfolio

$\bar{I}$ = single asset

C_o= initial capital invested

$E(R_p)$= expectation on the total portfolio return

σ = variance and σ_{ij} = covariance of assets i and j

C_T= capital at the end of the period

γ = parameter of absolute risk aversion

Below shows the constraint when:

(1) risk is minimized

(2) expected return is maximized

For a given choice of target mean return μ, choose the portfolio θ to:

$$\text{Min}\{\theta^T \sum \theta \mid A^T\theta = B\} \tag{3.27}$$

where A = (μ $\bar{I}$) and B = $\begin{pmatrix} \mu_p \\ C_0 \end{pmatrix}$

Applying Lagrangian multipliers to the convex optimization (minimization) problem is subject to linear constraints:

$$L(\theta,\ \lambda_1,\ \lambda_2) = \tfrac{1}{2}\theta^T \sum \theta + \lambda_1(\mu_0 - \theta^T\mu) + \lambda_2(1 - \theta^T\bar{I}) \tag{3.28}$$

Deriving the first-order conditions:

$$\tfrac{\partial L}{\partial \theta} = 0 = \sum \theta - \lambda_1\mu - \lambda_2\bar{I}$$

$$\frac{\partial L}{\partial \lambda_1} = 0 = \mu_0 - \theta^T \mu \tag{3.29}$$

$$\frac{\partial L}{\partial \lambda_2} = 0 = 1 - \theta^T \bar{I}$$

Solve for θ in terms of λ_1, λ_2 $\qquad \theta_0 = \lambda_1 \sum^{-1} \mu + \lambda_2 \sum^{-1} \bar{I}$

Solve for λ_1, λ_2 by substituting for θ

$$\mu_0 = \theta_0^T \mu = \lambda_1 (\theta^T \sum^{-1} \mu) + \lambda_2 (\mu^T \sum^{-1} \bar{I})$$

$$I = \theta_0^T \bar{I} = \lambda_1 (\mu^T \sum^{-1} \bar{I}) + \lambda_2 (I^T \sum^{-1} \bar{I}) \tag{3.30}$$

$$\Rightarrow \begin{bmatrix} \mu_0 \\ 1 \end{bmatrix} = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Simplifying Equation (3.28) produces the following formulation at Equation (3.29)

$$a = \mu^T \sum^{-1} \mu$$

$$b = \mu^T \sum^{-1} \bar{I} = \bar{I} \sum^{-1} \mu \tag{3.31}$$

$$c = \bar{I} \sum^{-1} \bar{I}$$

$$d = ac - b^2$$

It shows that parameters of *a,b,c* and *d* in Equation (3.30) are positive. The assumption is that the covariance matrix $\sum$ is positive definite, the inverse matrix $\sum^{-1}$ is also positive definite. This means that $X^T \sum^{-1} X \succ 0$ for all nonzero (N$\times$1)-vectors X, so it clear that $a \succ 0, c \succ 0$.

The second constraint is the expected return maximization, for a given choice of target return variance σ_p^2, choose the portfolio θ to:

$$\text{Max} \quad E(R_p) = \theta^T \mu$$

$$\text{Subject to:} \quad \theta^T \sum \theta = \sigma_p^2 \tag{3.32}$$

$$\theta^T \bar{I} = 1$$

Solving using Lagrange application equation applied in Equation (3.30) the maximum expected return in the portfolio is obtained as:

$$\mu_p = \frac{b}{c} C_0 \pm \sqrt{\frac{d}{c}} \sigma_p \tag{3.33}$$

Minimum variance and efficient frontier on the portfolio are obtained as:

$$\sigma_p = \frac{1}{d}(\mu_p \quad c_0)\begin{pmatrix} c & -b \\ -b & a \end{pmatrix}\begin{pmatrix} \mu_p \\ C_0 \end{pmatrix} = \frac{1}{d}(c\sigma_p^2 - 2C_0\mu_p + aC_0^2) \tag{3.34}$$

$$\theta_{EF} = \frac{1}{d}\sum^{-1}\left((a\bar{I} - b\mu_p)C_0 + (c\mu_p - b\bar{I})\mu_p\right) \tag{3.35}$$

The importance of this model is to enable the investor to be aware of the amount that must be invested in the single asset to achieve the expected return and risk on the efficient frontier. The minimum variance for the investor to choose from in the efficient frontier of the portfolio is obtained as:

$$\sigma_{mv}^2 = \theta^T \sum \theta = \frac{C_0}{c}\left(\sum^{-1}\bar{I}\right)^T \sum \frac{C_0}{c}\sum^{-1}\bar{I} = \left(\frac{C_0}{c}\right)^2 \bar{I}^T \sum^{-1}\bar{I} = \left(\frac{C_0}{c}\right)^2 C = \frac{C_0}{c} \tag{3.36}$$

whereas the minimum variance for the expected return is as:

$$\mu_{mv} = \mu^T \theta = \mu^T \sum^{-1} \bar{I} \frac{C_0}{c} = b \frac{C_0}{c} = \frac{b}{c} C_0 \tag{3.37}$$

CHAPTER 4

ANALYSIS AND DISCUSSION

4.1 Introduction

The chapter presents the performance analysis using the Jensen alpha ratio of nineteen (19) stocks listed on the Ghana stock exchange. The Jensen alpha measures individual stock performance relative to the performance or movement of the market. Again, the ratio has the ability to reveal either positive or negative values to companies that are underperforming or overperforming the market. Theoretically, EMH literature on stock market performance can also be verified using the Jensen ratio, so no investor is expected to beat or outperform the market in an efficient financial market. In addition, the Single Market Model was also used to estimate the beta (risk) of the stocks being studied.

4.2 Performances of Stocks Under Ghana Stock Exchange

A stock's systematic risk (beta), is defined in the literature, is the variability of returns on a stock or portfolio that is associated with changes in return on the entire market. The beta of a stock that is a measure describing the relationship between the return of a stock and the return of the entire market is that index that systematically measures the risk of a stock. Of the companies analyzed by the researcher as shown in Table 2, the companies listed on the Ghana Stock Exchange are less risky than the entire market. Their argument was based on the fact that although most of the 19 Ghana Stock Exchange listed companies have low beta values, a significant amount of stocks

varies with the entire market to some extent. The analysis starts with monthly stock estimation and market returns prior to further analysis.

Table 4.1 Beta and Returns of Stocks

STOCK	AVERAGE RETURNS	BETA
UNIL	2.40	2.87
TOTAL	2.32	2.35
PZC	0.26	0.56
CAL	0.72	0.48
SIC	0.17	0.49
ETI	0.12	0.52
ALW	0.06	-2.28
AYRTN	0.05	-0.56
CLYD	0.02	-0.05
PBC	0.03	-0.04
SWL	0.03	-0.03
CMLT	0.08	0.02
GCB	3.04	2.92
AADs	0.410	0.35
SCB	14.93	14.8
GOIL	1.75	1.67
GSR	1.06	0.99
AGA	22.84	22.7
BOPP	4.06	4

4.2.1 Discussion of Beta

Beta is a quantitative measure of a given stock's volatility relative to the market as a whole. Table 2 above shows that seven firms or stocks had their beta above that of the market. TOTAL, UNIL, GCB, SCB, GOIL, AGA and BOPP were thus considered

as an aggressive stock. Based on the calculated beta index, stocks AADs, ALW, AYRTN, CAL, CLYD CMLT, ETI, GSR, PBC, PZC, SIC and SWL averagely have their betas below the market beta of 1. Thus, these betas are defensive betas. This means that they are defensive beta and thus reflect the risk value of GSE-listed stocks. Thus, when market performance is increasing, such betas should be encouraged.

4.2.2 Beta Greater than 1

A stock with beta value greater than 1 is considered an aggressive stock. This is because the excess return of the stock is greater than the excess return of the portfolio of the company. In general, that stock has a chance that is more unavoidable than the entire market. Table 2 reveals that the aggressive betas are TOTAL, UNIL, GCB, SCB, GOIL, AGA and BOPP. When the economy is growing, these stocks are better purchased. For example, the beta of 9.87 implies that if market performance increases by 10%, UNIL's stock returns will rise by 98.7%, but if the market falls by 10%, UNIL's stock will fall by 98.7%.

4.2.3 Beta less than 1

A stock with a beta of less than one means the excess return on stocks varies less than proportionally with the market portfolio's excess return. These stocks: AADs, ALW, AYRTN, CAL, CLYD CMLT, ETI, GSR, PBC, PZC, SIC and SWL are referred to as defensive stocks. When the market falls, these stocks will be the best purchase. For example, PZC's beta of 0.56 indicates that if the market falls by 100%, PZC's returns will fall by 56%, but if the market rises by 100%, PZC's stock will rise by only 56%. This confirms the theory of trade off risk-return, hence the higher the risk the higher the returns. In Table 1 above, the majority of stocks are defensive stocks.

4.2.4 Beta equal to 1

A stock with one beta implies that excess stock returns vary proportionally with excess market portfolio returns. This type of stock carries the same systematic risk as the entire market. This stock rises and falls along with the movement or performance of the market. This form of beta was not shown by any of the companies or stocks in table 1

4.2.5 Negative beta

A stock with a negative beta indicates an inverse relationship between excess return on stock and excess return on the business portfolio. That is, its stock returns are falling as the market rises. The returns of AYRTN, ALW, CLYD, PBC, and SWL correlated inversely with those of the market. Research by Shapiro and others has shown that high-beta firms have done much better in a rising market than low-beta firms and much worse in a falling market, just as the capital asset pricing model predicts (Lakonishok and Shapiro, 1984). That is, there were negative betas on about 39 percent of the stocks listed on the GSE.

Table 4.2 Summary of Beta Values of Stocks

Types of Beta Value	Stock's Nature	Percentage Pulled
Beta > 1	Aggressive Stocks	36.84
Beta = 1	Neither gain or loss	0
Beta < 1	Defensive Stocks	36.84
Beta = Negative Values	Inversely to Market Return	26.32

From the above table given a period from January 2009 to December, 2018 shows that about 36.84 percent of the stocks sampled on the GSE have beta values greater than one and 36.84 percent have their beta values less than one, whereas 26.32 percent have beta values equal to negative. None of the stocks were equal to one of their betas.

4.3 Performance Assessment

GSE's performance evaluation was successful in monitoring the exchange-listed over-performance rate of stocks. Other scholars, including Frimpong (2008), used other sophisticated and nuanced approaches to conclude that the Ghana Stock Exchange's performance was low. Jensen Alpha model was used by the study to determine the performance of certain selected stocks on GSE. The table below shows GSE's evaluation of the stocks picked.

Table 4.3 Performance of stocks relative to the market performance

STOCK	MARKET RETURNS	JENSEN ALPHA	RANK
UNIL	-4.23	13.848	1ST
TOTAL	-4.23	12.449	2nd
PZC	0.32	0.373	3rd
CAL	0.73	0.295	4th
SIC	-0.3	0.244	5th
ETI	-0.21	0.165	6th
ALW	0.2	0.052	7th
AYRTN	0.37	0.038	8th
CLYD	2.4	-0.021	9th
PBC	2.4	-0.033	10th
SWL	2.4	-0.035	11th
CMLT	2.4	-0.098	12th
GCB	1.31	-0.517	13th
ADDs	2.45	-0.532	14th
SCB	1.2	-0.896	15th
GOIL	1.9	-1.333	16th
GSR	2.45	-1.383	17th
AGA	2.45	-29.92	18th
BOPP	2.4	-5118	19th

From table 4.3, UNIL, TOTAL, PZC, SIC and ETI shown in Table 4.3 have their Jensen alpha value (J_n) greater than the market return (Rm) meaning that the equity

exceeds the GSE performance which indicates overperforming. In the market, however, equities such as GCB, SCB, GOIL, AGA, BOPP AADs, ALW, AYRTN, CAL, CLYD CMLT, GSR, PBC and SWL are underperforming because the calculated alpha value of Jensen is far less than the market return. 26.32 percent are overperforming under the stock index and 73.68 percent under the GSE. This analysis is consistent with Frimpong and Oteng-Abayie (2007) that no investor is expected to consistently outperform or beat market returns but contradicts Degutis and Novickyte's findings (2014) position of stock returns is often found at random and investors who are not constantly able to earn access returns because market prices often deviate from their intrinsic value and market capitalization dynamism as well.

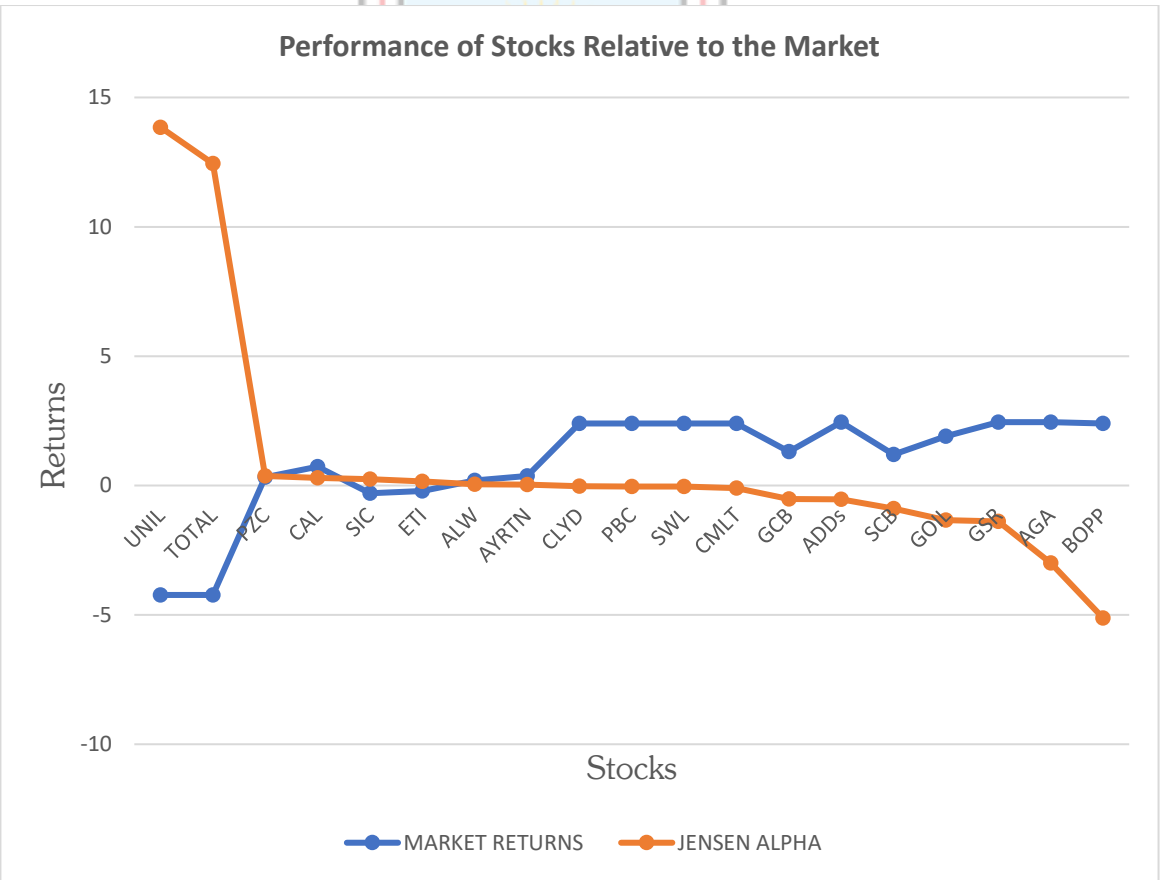

Figure 4.1 Performance of stocks relative to the market

4.4 Portfolio Optimization using Portfolio Theory of Markowitz

In this section, data collected from Ghana Stock Exchange on historical yearly returns from 2009 to 2018 are considered. First ten (10) equities out of the nineteen (19) were selected based on their Jensen alpha values and these equities include; Unilever Ghana Limited, Total Petroleum Ghana, PZ Cussons Ghana Limited, CAL Bank Limited, SIC Insurance Company Limited, Ecobank Transnational Inc, Aluworks Limited, Ayrton Drug Manufacturing Limited, Clydestone Ghana Limited and Produce Buying Company Limited.

These equities will be analyzed to determine the following;

1. Average returns
2. Standard deviation
3. Excess returns
4. Covariance matrix
5. Efficient frontier portfolio
6. Tangency portfolio
7. Global Minimum Variance portfolio
8. Covariance and Correlation matrix
9. Portfolio weight

This will enhance the determination of which equities are advisable for an investor to invest in.

4.4.1 Average return and Standard Deviation or the Risk

The average returns of the sectors indicate the value in which an investor will be expecting at the end of the month using the historical monthly returns of the sectors.

The standard deviation also measures the risk in which an investor should expect in investing into a particular sector.

Table 4.4 Security Statistics

SECTORS	AVERAGE RETURN	STANDARD DEVIATION OF RETURNS
Unilever Ghana Limited	2.400	0.060
Total Petroleum Ghana	2.320	0.721
PZ Cussons Ghana	0.260	0.125
CAL Bank Limited	0.720	0.103
SIC Insurance Company Limited	0.080	0.194
Ecobank Transnational Inc	0.077	0.112
Aluworks Limited	0.060	0.111
Ayrton Drug Manufacturing Limited	0.420	0.030
Clydestone Ghana Limited	0.470	0.032
Produce Buying Company Limited	0.030	0.126

From Table 4.4, Unilever Ghana Limited has the highest average returns of 2.400 with a risk of 0.060 representing 240 % and 6% respectively, followed by Total Petroleum Ghana with average return of 2.320 representing 232 % and 0.721 standard deviation representing 72.1 % level of risk. Produce Buying Company Limited has the least return of 0.030 representing 3 % with a standard deviation of 0.126 representing a risk of 12.6 %. Also, and investor might be tempted to select Unilever Ghana Limited to invest since it has the highest return and leave Produce

Buying Company Limited due to its lowest average returns without considering the risk involved.

4.5 Sharpe Ratio

The Sharpe ratio is a measure of the performance of a fund stock. By definition it is the ratio of the excess return of an asset to their volatility. It is also known as the variable-reward ratio. The Sharpe ratio measured on the basis of achieved returns is as follows Total Petroleum Ghana recorded the highest Sharpe ratio of 1.666 followed by Unilever Ghana Limited with a Sharpe ratio of 1.537. The Sharpe ratio simply indicate the excess returns of a given equity over the risk-free instrument Total Petroleum Ghana and Unilever Ghana Limited outperform the risk-free instrument (182 Treasury Bill) more compared to the performance of PZ Cussons Ghana, CAL Bank Limited, SIC Insurance Company Limited, Ecobank Transnational Inc, Aluworks Limited, Ayrton Drug Manufacturing Limited, Clydestone Ghana Limited and Produce Buying Company Limited.

Table 4.5 shows some selected equities and their corresponding Sharpe Ratio, and Figure 4.2 shows the bar chart of the equities and their corresponding Sharpe Ratio.

Table 4.4 Selected Equities and Their Corresponding Sharpe Ratio

Sectors	Sharpe Ratio
Unilever Ghana Limited	1.537
Total Petroleum Ghana	1.666
PZ Cussons Ghana	0.390
CAL Bank Limited	0.709
SIC Insurance Company Limited	0.617
Ecobank Transnational Inc	0.578
Aluworks Limited	0.556
Ayrton Drug Manufacturing Limited	0.509
Clydestone Ghana Limited	0.311
Produce Buying Company Limited	0.505

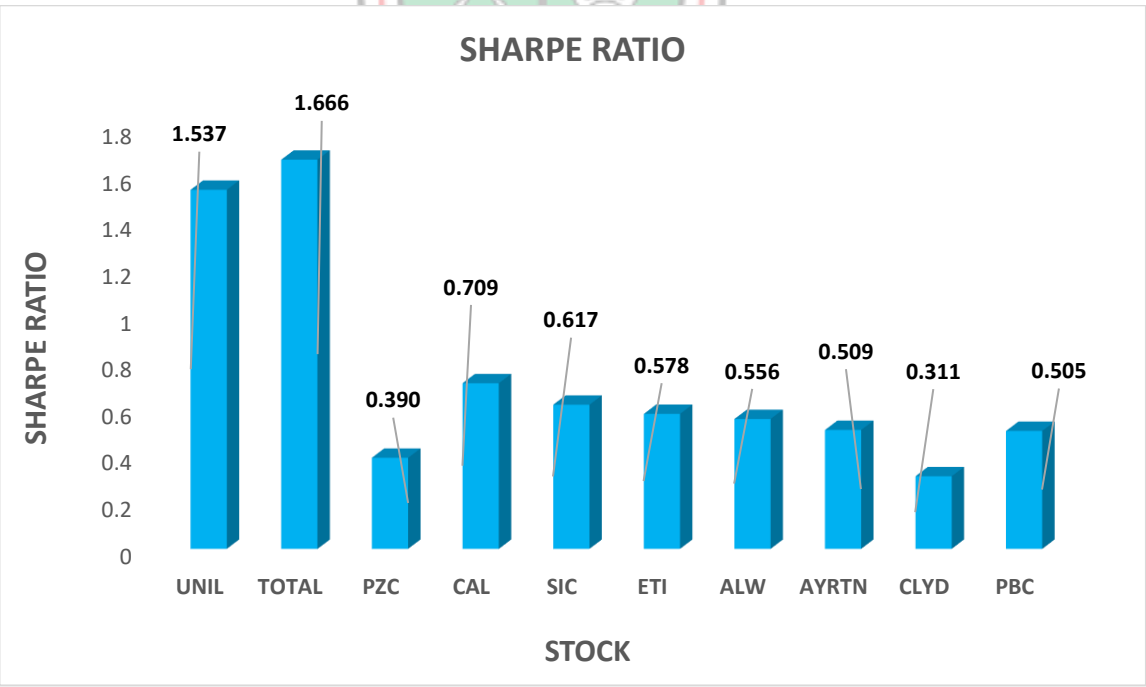

Figure 4.2 Bar Graph of Sharpe Ratio of Stocks

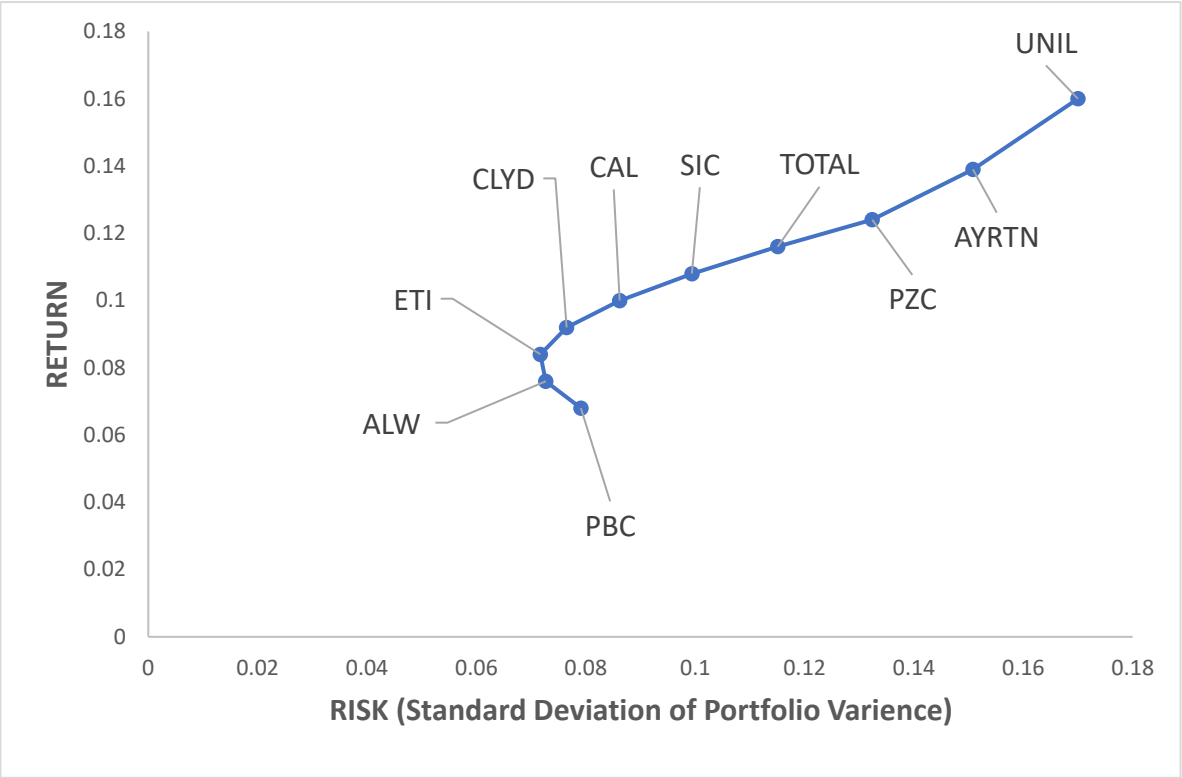

Figure 4.3 Efficient Frontier and Optimal Portfolio

Figure 4.3 above indicates the efficient portfolios and the optimal portfolio. From Figure 4.3 clearly illustrates the global minimum variance portfolio at the point where risk is 0.070 and return is 0.077. This implies that risk averse investors will be interested to invest in ETI since it is the point with the minimum returns and minimum risk. Figure 4.3 also describes the maximum possible stocks. Or on the other hand, it describes the minimum amount of risk that one must live with for any given amount of expected return. From figure 4.3 the point indicated Total shows the tangent portfolio or the market portfolio or the maximum portfolio that an investor can go for. This market portfolio has the highest return with the highest risk. This means that all the stocks that lie above the global minimum variance portfolio are called the efficient set of portfolios, meaning an investor can be advised to invest in without losing. On the other hand, all the stocks that lie below the global minimum variance

portfolio are inefficient portfolio and it is not advisable for an investor to select such portfolio as part of his or her portfolio allocation. For instance from Figure 4.3, considering CLYD and ALW of 0.09 and 0.07 expected returns respectively shows that CLYD is more efficient to invest in since it lies above the global minimum portfolio and ALW on the other hand is inefficient for an investor to invest in due to it expected returns lying below the global minimum portfolio.

4.6 Tangency Portfolio

The tangent portfolio is always necessary in portfolio analysis since any investor who wishes to maximize his or her portfolio return will always invest in the tangency portfolio. Furthermore, if an investor desires a minimum amount of risk, then the investor must take the portfolio that has the minimum variance portfolio.

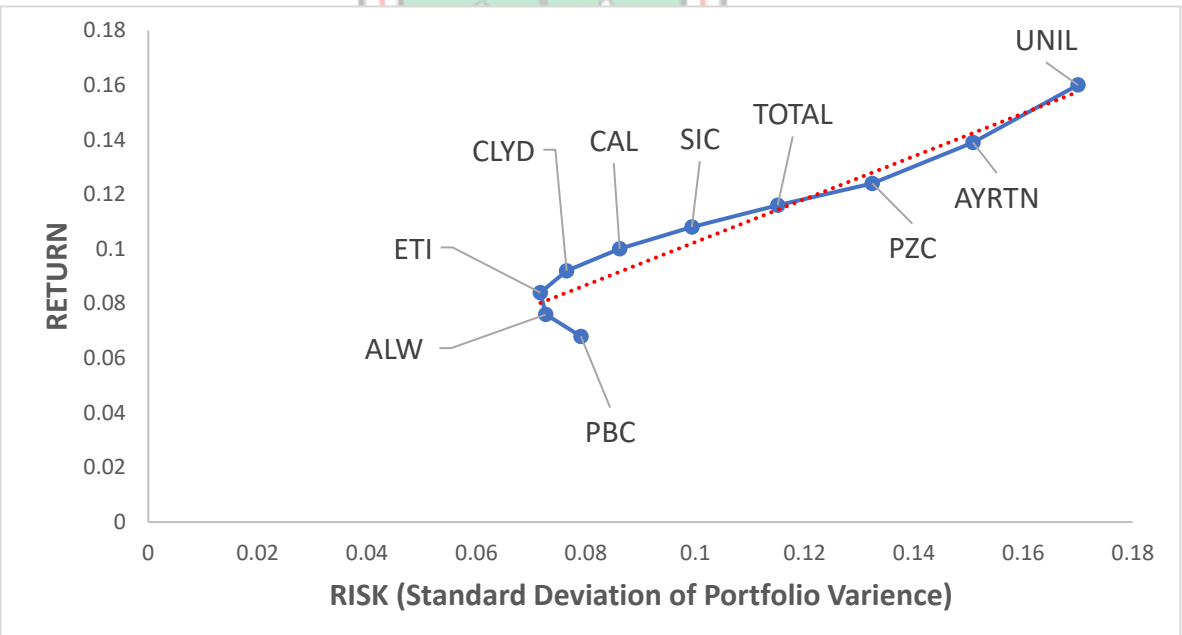

Figure 4.4 Tangency Portfolio

Figure 4.4 describes the tangency portfolio of the ten stocks, the gradient of the tangent line and the risk-free rate of the portfolio. From figure 4.4, the TOTAL indicates the point on the efficient frontier where the line is tangent to the efficient frontier. This is the point where any aggressive investor will invest more of his or her money because it has the highest expected return of 0.11 with a risk of 0.12. Again, the intersection of the line and return axis is the risk-free rate of the portfolio and at this point, all conservative investors will invest more of their money into it. But every portfolio manager will advise his or her client to invest more in the tangency portfolio. Figure 4.4 clearly illustrates efficient stocks and inefficient ones. This confirms the results in table 4.1. Also, the global minimum variance portfolio has a risk of 0.070 and return of 0.077 as indicated in figure 4.4. Also, to determine both the efficient and inefficient portfolios, the study compares the returns of the global minimum variance portfolio to the average returns of the individual stocks.

4.7 Descriptive Statistics

To ascertain whether the results in figure 4.3 and figure 4.4 are valid, statistical test was performed using Kurtosis and Skewness. Table 4.6 shows the normality test performed for the data.

Table 4.6 Kurtosis and Skewness Equities on the Ghana Stock Exchange

Stock	Mean	Kurtosis	Skewness
CAL	6.270667	3.034	1.860
ETI	5.645083	6.768	2.498
SIC	9.588833	4.481	2.036
TOTAL	6.752167	1.565	1.247
PBC	5.739833	6.893	2.631
AYRTN	1.389583	8.885	2.794
PZC	4.147333	26.655	4.671
UNIL	9.23725	3.960	1.339
ALW	4.726833	13.389	3.009
CLYD	0.950667	21.491	4.457

Table 4.6 indicates the listed equity kurtosis. The normal distribution, with a kurtosis coefficient of 3, is the standard reference for kurtosis in return distribution. The higher the kurtosis coefficient, the higher the kurtosis degree, so that a kurtosis coefficient of 4 implies a relatively high distribution of returns, while a kurtosis coefficient of 2 indicates a relatively flat distribution of the return. Except for TOTAL, kurtosis of 3 and above was recorded by all other equities, indicating that returns do not follow normal distribution.

4.8 Global Minimum Variance portfolio (GMV)

The global minimum variance portfolio is the portfolio with the highest returns with a minimum risk or deviation. In this study, this portfolio is calculated mainly on the

sector's average returns, standard deviations, covariance matrix, correlation matrix and the weights of the portfolio by optimizing the objective function.

Table 4.7 Summary of Efficient and Inefficient Portfolio

SECTORS	AVERAGE RETURN	GMV
Unilever Ghana Limited	2.40	0.077
Total Petroleum Ghana	2.32	
PZ Cussons Ghana	0.26	
CAL Bank Limited	0.72	
SIC Insurance Company Limited	0.11	
Ecobank Transnational Inc	0.01	
Aluworks Limited	0.06	
Ayrton Drug Manufacturing Limited	0.42	
Clydestone Ghana Limited	0.47	
Produce Buying Company Limited	0.03	

The Table 4.7 above indicates that Aluworks Limited and Produce Buying Company Limited stocks are inefficient portfolios whiles the efficient portfolios are the set of Unilever Ghana Limited, Total Petroleum Ghana, Ayrton Drug Manufacturing Limited, Clydestone Ghana Limited, PZ Cussons Ghana, CAL Bank Limited, SIC Insurance Company Limited and Ecobank Transnational Inc. Among these efficient portfolios, the most efficient is the Unilever Ghana Limited since it has the highest

returns with a certain level of risk but every aggressive investor will like to invest in Ecobank Transnational Inc because it has a return closer to tangency portfolio which all portfolio managers will encourage their clients to select. Again, as some investors are "Aggressive investors", some are also risk averse or conservative investors. And every conservative investor will have selected Ecobank Transnational Inc his or her investment portfolio because is just a little greater than the GMV.

4.9 Creating a portfolio

The analysis of the first ten (10) selected stocks based on their ranking from table 4.3 indicated that not all stocks under GSE are efficient for an investor to invest into. The analysis showed that before an investor can select any of these stocks under GSE, there is a need to know the average returns of the individual stocks based on their historical returns. Though the average returns are what the investor will be expecting as his or her returns but since there is risk in every business, standard deviation which represent risk was also determined, and it was realized that, as the return increased, the risk also tends to increase for most of the stocks.

Table 4.8 Efficient Stocks

STOCKS	EFFICIENCY	
	AVERAGE RETURN	GMV
Unilever Ghana Limited	2.40	0.077
Total Petroleum Ghana	2.32	
PZ Cussons Ghana	0.26	
SIC Insurance Company Limited	0.11	
Ecobank Transnational Inc	0.01	
CAL Bank Limited	0.72	
Ayrton Drug Manufacturing Limited	0.42	
Clydestone Ghana Limited	0.47	

4.9.1 Covariance Matrix

Diversification is the process of allocating capital in a way that reduces the exposure to any one particular asset or risk. A common path towards diversification is to reduce risk or volatility by investing in a variety of stocks. The covariance also talks about the deviation which measures the level of risk involved when two stocks are paired. The covariance also indicates that the smaller the risk, the better they are paired.

Table 4.9 Covariance Matrix

	CAL	ETI	SIC	TOTAL	PBC	AYRTN	PZC	UNIL	ALW	CLYD
CAL	77.47	70.76	70.76	12.07	53.47	6.73	22.73	-10.90	35.14	4.42
ETI	30.67	46.45	46.45	13.37	25.22	8.39	23.66	-18.66	31.84	7.51
SIC	70.76	239.67	239.67	13.78	72.17	9.91	61.81	-19.40	30.06	11.46
TOTAL	12.07	13.78	13.78	16.29	8.83	3.82	3.42	-5.35	11.44	2.28
PBC	53.47	72.17	72.17	8.83	128.10	12.54	28.83	-7.02	30.41	1.08
PZC	22.73	61.81	61.81	3.42	28.83	0.12	111.90	-5.27	12.24	-1.36
UNIL	-10.90	-19.40	-19.40	-5.35	-7.02	-4.40	-5.27	35.80	5.01	-5.42
ALW	35.14	30.06	30.06	11.44	30.41	3.52	12.24	5.01	71.77	2.35
CLYD	77.47	11.46	11.46	2.28	1.08	1.45	-1.36	-5.42	2.35	9.27

Table 4.9 indicates how the risk of one stock affects the other when they are paired. The positive covariance between two stocks produces returns above it mean the other stock also tends to produce above it mean. But for a negative covariance value indicates that as one stock move below it mean the other stock tends to move above it mean which reduce the risk involved in such portfolio. A better diversification, can only be achieved when the covariance coefficient moves from positive to negative. The covariance values indicated that some paired equities were moving together and others were moving in the opposite direction. This implies some of these equites can be included in the same portfolio whiles others cannot be in the same portfolio. Thus, an investor can distribute his or her investment portfolio percentage wide across all the stocks selected. It was realized that some of the stocks were not advisable for the investor to allocate some of his or her money due to their performances

Therefore, from Table 4.9 it commonly recommended for an investor to combined UNIL and SIC to achieve diversify portfolio when trying to invest in any two stocks among these stocks. From the table, it also shows that when an investor wishes to allocate his/her investment across three stocks, it is recommended to choose UNIL, SIC and ETI since ETI Also combined with UNIL to give the next strong pair as far as diversification is concern.

4.9.2 Correlation Matrix

A correlation matrix is an important tool used in diversification. It assists an investor to select the best pair of these stocks by taking into consideration the strength of the correlation between the individual equities. Equities with higher correlation coefficient

are paired. Correlation coefficient can range from -1.0 to +1.0. Table 4. 10 is the correlation matrix between equities.

Table 4.10 Correlation Matrix

	CAL	ETI	SIC	TOTAL	PBC	AYRTN	PZC	UNIL	ALW	CLYD
CAL	**1.000**	0.358	0.519	0.339	0.537	0.282	0.244	0.207	0.471	0.165
ETI	0.358	**1.000**	0.308	0.340	0.229	0.318	0.229	0.321	0.386	0.254
SIC	0.519	0.308	**1.000**	0.221	0.412	0.236	0.377	0.209	0.229	0.243
TOTAL	0.339	0.340	0.221	**1.000**	0.193	0.349	0.080	-0.222	0.335	0.186
PBC	0.537	0.229	0.412	0.193	**1.000**	0.408	0.241	0.104	0.317	0.031
AYRTN	0.282	0.318	0.236	0.349	0.408	**1.000**	0.004	-0.271	0.153	0.176
PZC	0.244	0.229	0.377	0.080	0.241	0.004	**1.000**	0.083	0.137	0.042
UNIL	0.207	0.321	0.209	-0.222	0.104	-0.271	0.083	**1.000**	0.099	-0.297
ALW	0.471	0.386	0.229	0.335	0.317	0.153	0.137	0.099	**1.000**	0.091
CLYD	0.165	0.254	0.243	0.186	0.031	0.176	0.042	-0.297	0.091	**1.000**

From Table 4.10, the correlation matrix is symmetric and therefore its diagonal entries are all ones which indicates that when a sector is paired with itself its perfect correlation or 100%. But the correlation is less than 1 when two different equities are paired. The positive values also show that the individual sectors being paired with each other moves in the same direction indicating that they are positively correlated.

The negative values also suggest that the individual equities paired are moving in opposite direction.

In view of that stocks such as TOTAL, AYRTN, UNIL and CLYD were selected because both have their coefficient of covariance and correlation to be negative. Portfolio weights were assigned to the selected stocks. The portfolio expected returns and the portfolio weights are indicated in Table 4.11 and Figure 4.5 respectively.

Table 4.11 Portfolio Weight

Stocks	Portfolio A		Portfolio B		Portfolio C		Portfolio D	
	weight (%)	Portfolio Expected Return (%)	weight (%)	Portfolio Expected Return (%)	weight (%)	Portfolio Expected Return (%)	weight (%)	Portfolio Expected Return (%)
TOTAL	36.84	85.46	5.19	12.03	7.58	17.59	50.40	116.92
AYRTN	7.58	3.18	50.40	21.17	36.84	15.47	5.19	2.18
UNIL	50.40	120.95	7.58	18.19	5.19	12.45	36.84	88.41
CLYD	5.19	2.44	36.84	17.31	50.40	23.69	7.58	3.56

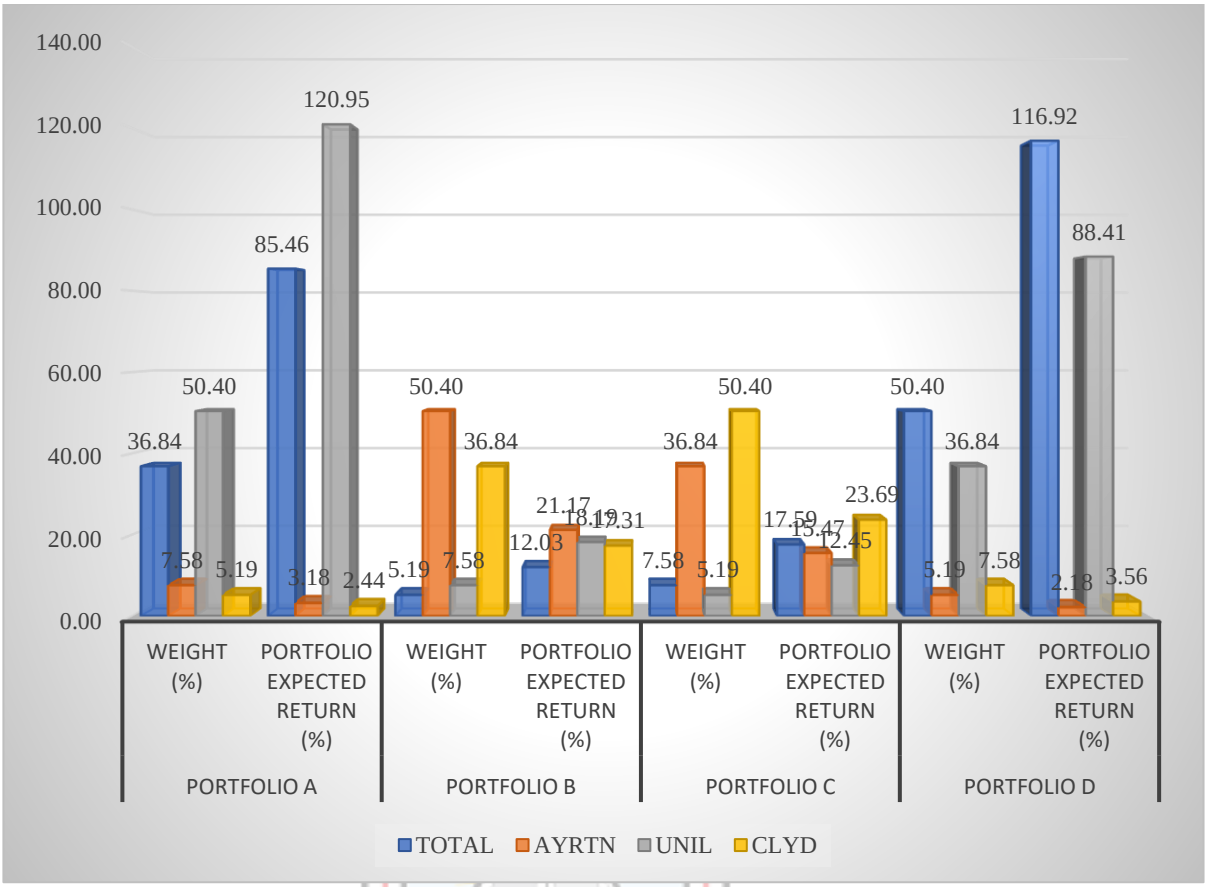

Figure 4.5 Weight and Returns of Selected Equities

Table 4.6 Portfolio Selection

Portfolio	Portfolio weight	portfolio expected return	portfolio Risk
A	0.37 TOTAL + 0.08 AYRTN + 0.50 UNIL + 0.05 CLYD	0.691	0.220
B	0.05 TOTAL + 0.50 AYRTN + 0.08 UNIL + 0.37 CLYD	0.590	0.196
C	0.08 TOTAL + 0.37 AYRTN + 0.05 UNIL + 0.50 CLYD	0.598	0.189
D	0.50 TOTAL + 0.05 AYRTN + 0.37 UNIL + 0.08 CLYD	0.687	0.217

From Table 4.12, portfolio selection was made available for all the risk type of investors. Portfolio A is suitable for risk lover's investors because it has its returns and risk higher than the rest of the portfolios. Risk neutral investors had portfolio D to be suitable for investment because their returns and risk are slightly lower than portfolio A. Due to the lower risk level and returns of portfolio B and C, risk averse investors were advised for any of such portfolios.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATION

5.1 Conclusions

The study was conducted in building a new portfolio optimization after the financial sector reforms. This was achieved by applying standard market model, Variance-Covariance and Correlation to deduce Markowitz theory for portfolio maximization. With respect to the individual stock performance Jensen alpha has established UNIL, TOTAL, PZC, SIC and ETI as overperforming whilst GCB, SCB, GOIL, AGA, BOPP AADs, ALW, AYRTN, CAL, CLYD CMLT, GSR, PBC and SWL were underperforming. Also, the study revealed that the most efficient stock at GSE is Unilever Ghana Limited and the most inefficient is Ayrton Drug Manufacturing Limited. From the analysis, a diversified portfolio of selected equities on the Ghana Stock Exchange has been generated for different classes of investors. The optimal portfolio is given by:

Risk Lover's Investor

Min Var. Portfolio = 0.37 TOTAL + 0.08 AYRTN + 0.50 UNIL + 0.05 CLYD, with 0.691 returns at 0.220 risk level

Risk Neutral Investor

Min Var. Portfolio = 0.50 TOTAL + 0.05 AYRTN + 0.37 UNIL + 0.08 CLYD, with 0.687 returns at 0.217 risk level

Risk Averse Investor

Min Var. Portfolio = 0.05 TOTAL + 0.50 AYRTN + 0.08 UNIL + 0.37 CLYD, with 0.590 returns at 0.196 risk level

OR

Min Var. Portfolio = 0.08 TOTAL + 0.37 AYRTN + 0.05 UNIL + 0.50 CLYD, with 0.598 returns at 0.189 risk level

5.2 Recommendation

The study finally recommended that, prospective investors especially the risk averse to take up risky investments via diversification using the research outcome as working tool. Companies on the GSE are also expected to put in place measures that can help improve on performance with respect to dividend payment and returns that the companies pose. Portfolio managers should constantly restructure their portfolio due to reforms that occur at various sectors of the GSE.

REFERENCES

Agyeman, C. A (2010), "How Ghana Stock Exchange can be Improved", Journal of Finance, Vol 2, No. 6, pp. 24-27.

Alder, R., Feldman, R. and Taqqu, M. S. (1998), "A practical guide to heavy-tails: Statistical techniques for analyzing heavy-tailed distributions", Journal of Finance, Vol 6, No. 3, pp. 439-501.

Anon. (2004), "Output, the Stock Market and Interest Rates", Databank Brokerage Ltd, 9 pp.

Anon. (2004), "Output, the Stock Market and Interest Rates", Databank Brokerage Ltd, 9 pp.

Anon. (2010), "Stock-market Development in Sub-Saharan-Africa Trend-and characteristics", Directorate for Financial and Enterprise Affairs, Competition Committee DAF/COMP, Ghana Stock Exchange Bulletin, 94 pp.

Anon. (2015), "Performance Evaluation of Comparing Listed on the GSE", Thesis submitted to the KNUST` School of Business, KNUST, Kumasi, pp. 56-58.

Anon. (2018), "The Stock Market and Interest Rates", Directorate for Financial and Enterprise Affairs, Competition Committee, 94 pp.

Arzac, E. R. and Bavva, V.S. (1977), "Portfolio Choice and Equilibrium in Capital Markets with Safety-first lnvestors", Journal of Financial Economics, Vol. 4, No.3, pp. 277-288.

Asamoah, G.N. (2010), "The Impact of Dividend Announcement on Share price

Behavior in Ghana", Journal of Business and Economics Research-April 2010, Vol 8, No. 4. pp. 47.

Asquith P. and Mullins D. (1983), " The impact of initiating dividend payments on shareholders wealth", Journal of Business, Vol. 56, No. 2, pp. 77-96.

Bally, W., Stulz, R. and Yen, S. (1990), Properties of Daily Stock returns from the Pacific Basin stock market: evidence and implications, S G Rhee and R. P. Chang(eds.), Pacific-Basin capital markets research, North-Holland, Amsterdam, Vol. 32, No. 4, pp. 155-171.

Benimadhu, S. (2003). "Challenges facing African capital markets". Presentation at UNECA Forum. Johannesburg.

Bhide, A. (1994), " The hidden costs of stock market liquidity", Journal of Financial Economics, Vol. 16, No. 34, pp. 31-51.

Binswanger, M., (1999), "Stock Markets, Speculative Bubbles and Economic Growth", Edward Elgar Publishing Limited, Vol. 11, No. 13, pp. 149-175.

Bokpin, G. A. and Isshaq, Z. (2008), "Stock market development and financing decisions of listed firms in Ghana", *African Journal of Business Management,* Vol. 2, No. 9, pp. 209-216.

Chambers, D.R. (2010), "Diversification and the market portfolio", AIER, Modern Portfolio Theory, Vol. 5, No. 3, pp 1 – 4.

Darko, S. (2012), "Constructing Optimal Stock Portfolio with Markowitz Model", *Unpublished Master's thesis*, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana, Vol. 8, No. 2, pp. 6–10.

Degutis, A. and Novickyte, L. (2014), "The efficient market hypothesis: A critical review of literature and methodology", African Economic Research, Vol 1, No. 3, pp. 72-86.

Dyckman, T. R. and Dale M. (1986), "Efficient Capital Markets and Accounting: A Critical Analysis", Journal of Finance. Vol. 10, No. 2, pp. 15-36.

Fama, E. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work", Journal of Finance, Vol. 6, No. 3, pp. 383-417.

Fama, E. (1991), "Efficient Capital Markets II", Journal of Finance. Vol. 2, No. 5, pp. 1575-1617.

Frimpong, J. M. and Oteng-Abayie, E. F. (2007), "Market returns and weak-form efficiency: The case of Ghana Stock Exchange", Journal of finance, Vol. 2, No. 1, pp. 72-96.

Frimpong, J. M. (2008)," Capital Market Efficiency: An Analysis of Weak-form Efficiency on the Ghana Stock Exchange", Journal *of Money, Investment and Banking*, Vol. 5, No. 2, pp. 16-28.

Gozah, K. A. (2017), "Portfolio Optimization: Using Value-at-Risk", Journal of Financial Engineering, Vol 1, No. 1, pp. 22-90.

Healy, P. (1988), "Earnings Information Conveyed by Dividend Initiations and Omissions", Journal of Financial Economics, Vol. 7, No. 3, pp. 149-175.

Huang, X. (2008), "Mean-Semi variance Model for Fuzzy Portfolio Selection", Journal of Computational and Applied Mathematics, Vol. 217, No. 1, pp. 1-8.

Kumo, W. (2008), "Stock exchange in Africa; prospects and challenges", Journal of African Economics, Vol. 17 No. 5, pp. 527-549.

Lakonishok, J. and Shapiro, A.C. (1984), "Stock returns, beta, varience and size: empirical analysis", Financial analysts journal, Vol. 40, No. 4. Pp. 36-41.

Levine, R. and Zervos, S. (1998), "Capital control liberalization and stock market development," World Development, Vol. 26, No. 7, pp. 1169-1183.

Manu, F. (2017), "Performance of Ghana Stock Exchange and Economic Growth", Journal of Finance, Vol 2, No. 39, pp. 16-26.

Manual B. P. P. (1994), "Stock Market Patterns and Financial Analysis: Methodological Suggestions", Journal of finance, Vol. 14, No. 1, pp.1-10.

Markowitz, H. M. (1952), "Portfolio Selection", The Journal of finance, Vol. 7, No. 1, pp. 77-91.

Mensah, M., Awunyo-Vitor, D., and Sey, W. (2012), "Challenges and Prospects of The Ghana Stock Exchange", ISSN 2225-0565 (Online), Vol. 2, No.10, pp. 224-607.

Osborne, M. (1959), "Brownian Motion in the Stock Market", Journal of Finance, Vol. 15, No. 2, pp. 145-173.

Osei, K. A. (1998), "Asset Pricing and information Efficiency of the Ghana Stock Market", AERC Research paper Vol. 8, No. 1, 115 pp.

Pirvu, T.A and Zitkovic, G. (2014), "Maximizing Portfolio Growth Rate under Risk Constraints", Mathematical Finance, Vol. 19, No. 1, pp. 423-455.

Prigent, J. L. (2007)," Portfolio Optimization and Performance Analysis", CRC Press, Vol. 11, No. 5, 301pp.

Singh, A. (1997), "Liberalization, the Stock Market and the Market for Corporate Control: A Bridge Too Far for the Indian Economy ", Accounting and Finance Discussion, Vol. 16, No. 8, pp. 97-35.

Wallenius, L. (2009), "Securities Clearing and Settlement on Ghana Stock Exchange: A Study of an Emerging Stock Market", Vol. 10, No. 2 pp. 15-36.

Watson, D. (2007), "Corporate Finance; Principles and Practice.", Pearson Education Limited, Edinburg Gate, Harlow Essex CM20 2JE. Vol. 4, No. 2, pp. 29-33.

Winful, E. C., Sarpong, D. J. and Kumi-Koranteng, P. (2012), "The performance of Ghana stock exchange for the period 2000 to 2009", African Journal of Business Management, Vol. 6, No. 38, pp. 10340-10359.

Yartey, C. and Adjasi, C. (2007)", "stock market development in Sub-Saharan Africa", Critical Issues and Challenges IMF working paper, Vol. 7, No. 18, pp. 10-19.

Zhou, X. Y., Yin, G. and Markowitz, H. (2003), “Mean-Variance Portfolio Selection with Regime Switching: A continuous-time model”, SIAM J Control Optimal, Vol. 42, No. 3, pp. 1466-1482.

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