THE EFFECT OF RADIATION OF A SODIUM DROPLET IGNITION DELAY AND LIMIT OF IGNITABILITY

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Abstract
The effect of the radiation on the droplet ignition was investigated. The Rosseland approximation was employed in the modelling of the radiation heat transfer and temperatures of the droplet. The governing equation was expressed in dimensionless form and was solved analytically according to Makino [3]. The MAPLE algebraic computation package was employed to implement the analytical solution to generate the numerical results. The results showed that radiation contributes to the ignition delay and increase the temperature at which the thermal ignition will occur.

Keywords: Radiation, droplet temperature, ignition delay, Rosseland approximation.

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Nomenclatures

r_s	=	droplet radius
T	=	droplet temperature
T_0	=	an oxidizing atmosphere
$Y_{0\infty}$	=	oxygen mass fraction
λ	=	thermal conductivity
F	=	sodium Na subscript
W_F	=	molecular weight

V_i = stoichiometric coefficient

n = reaction order

B_s = frequency factor of the "global" surface reaction

c = specific heat

h = heat transfer coefficient

ρ = density

q^0 = heat generated per unit mass of Na,

R = the universal gas constant

E = the activation energy,

β = the radiation parameter

α and γ are heat loss parameters.

1. Introduction

There are many investigations on thermal ignition in the literature. The one of utmost concern is that of ignition of sodium droplet because of its extensive use as a working fluid in the design of nuclear facilities. Although basic research on droplet combustion has been conducted, some questions that arose from these previous works suggest the need for further studies. (Grosan and Pop [2], Morewitz, *et al* [6]). The problem of interest focuses on the ignition of a single sodium droplet with initial radius r_s and temperature T_0 in an oxidizing atmosphere with $T_\infty \left(\leq T_0\right)$. The behaviour of the droplet radius squared is still linear with time under certain conditions. The tendency of the square of the droplet radius is reproduced by imposing the quasi-steady regime (QSR) for the gas phase. Sodium droplet temperature T is assumed spatially uniform but temporally varying because of its high thermal conductivity λ which is characterized by low vapour pressure.

However, at low vapour pressure, the thermal inertial region close to the droplet in the gas phase is much smaller than that of the liquid phase. The spray spatial scale is specified by the boundaries of the combustion chamber and vapourization scale is characterized by the droplet sizes, therefore, the difference in these scales is of several orders of magnitude kind of problem, but they are not the focus of this work Fachinl [1]. Many fluids involved in industrial applications have a non-Newtonian behaviour. On the other hand, if the processes take place at high temperature, radiation effects cannot be neglected Modest [5].

Makino and Fudaka [4] studied ignition and combustion of a single sodium droplet experimentally by using a falling droplet. They discovered that the ignition delay time increases first gradually and then rapidly, with decreasing initial temperature and / or oxygen concentration, and researches the limit of ignitability. In another development, Sazhim *et al* [9] considered initial heat flux between fuel droplet and a gas using a new approach to application of the shell model foe simulation of diesel fuel autoignition. Their problem was rectified by replacing the gas thermal conductivity. Consequently, it was assumed that the effect of thermal radiation on heating and evaporation of semi-transparent Diesel fuel droplets is shown to be considerably smaller when compared with the case of block opague

In this study, radiation effect on the ignition time of the droplet temperature equation as it is dependent on time using asymptotic expansion of Frank- Kermanestki temperature parameter was investigated.

2. Mathematical Formulation

The governing equation is given as,

$$\frac{4}{3}\pi r_s^3 \rho_F c_p \frac{dT}{dT} = 4\pi r_s^2 q^0 V_F W_F \left(\frac{\rho_g Y_0}{W_0}\right)^n B_s e^{-\frac{E}{RT}} - 4\pi r_s^2 h(T - T_\infty) - 4\pi r_s^2 \sigma(T^4 - T_\infty^4) \tag{2.1}$$

$T(0) = T_0$,

This is the extension of Makino [3]

Using the following dimensionless variables,

$$\theta = \frac{T - T_0}{\in T_0} \text{ and } \in = \frac{RT_0}{E} \tag{2.2}$$

with asymptotic expansion, $T = T_0\left[1 + \in\theta + \theta(\in^2)\right]$ and $\tau = t\Delta$, equation (2.1) becomes,

$$\frac{4}{3}\pi r_s^3 \rho_F c_\rho \Delta \in T_0 \frac{d\theta}{d\tau} = 4\pi r_s^2 q^0 V_F W_F \left(\frac{\rho_g Y_0}{W_0}\right)^n B_s e^{-\frac{E}{R}\left(\frac{T-T_0}{TT_0}\right)} - 4\pi r_s^2 h(T_0 + \in T_0\theta - T_\infty)$$
$$-4\pi r_s^2 \sigma\left[(T_0 + \in T_0\theta)^4 - T_\infty^4\right] \tag{2.3}$$

$$\frac{4}{3}\pi r_s^3 \rho_F c_\rho \Delta \in T_0 \frac{d\theta}{d\tau} = 4\pi r_s^2 q^0 V_F W_F \left(\frac{\rho_g Y_0}{W_0}\right)^n B_s e^{-\frac{E}{RT_0}\theta(1+\in\theta)^{-1}} - 4\pi r_s^2 h(T_0 + \in T_0\theta - T_\infty)$$
$$-4\pi r_s^2 \sigma\left[(T_0 + \in T_0\theta)^4 - T_\infty^4\right] \tag{2.4}$$

Simplifying equation (2.4) gives,

$$\frac{d\theta}{d\tau}=\frac{3q^0V_FW_F\left(\rho_gY_0\right)^n}{r_s\rho_Fc_\rho\in T_0W_0^{\ n}}B_se^{-\frac{E}{RT_0}}\frac{1}{\Delta}e^{\theta}-\frac{3h\left[T_0-T_\infty\right]}{r_s\rho_Fc_\rho\in T_0\Delta}-\frac{3h}{r_s\rho_Fc_\rho\Delta}\theta-\frac{3\sigma\left[T_0^4-T_\infty^4\right]}{r_s\rho_Fc_\rho\in\Delta}$$

$$-\frac{3\sigma T_0^3}{r_s\rho_Fc_\rho\Delta}\left[4\theta+6\in\theta^2+4\in^2\theta^3+\in^3\theta^4\right] \tag{2.5}$$

Therefore, equation (2.5) becomes,

$$\frac{d\theta}{d\tau}=e^{\theta}-\gamma-\alpha\theta-\beta(4\theta+6\in\theta^2+4\in^2\theta^3+\in^3\theta^4) \tag{2.6}$$

$$\theta(0)=0,$$

where, $\frac{3q^0V_FW_F\left(\rho_gY_0\right)^n}{r_s\rho_Fc_\rho\in T_0W_0^{\ n}}B_se^{-\frac{E}{RT_0}}.\frac{1}{\Delta}=1$, in which, $\frac{1}{\Delta}=\frac{(2r_s)\rho_F}{6B_sY_0c_\rho}.\frac{W_0^{\ n}}{V_FW_F}\left(\frac{RT}{\rho\bar{w}}\right)^n\left(\frac{c_\rho T_0}{q^0}\right)\frac{RT_0}{E}e^{\frac{E}{RT_0}}$,

$$\alpha=\frac{3h}{r_s\rho_Fc_\rho\Delta},\ \beta=\frac{3\sigma T_0^3}{r_s\rho_Fc_\rho\Delta},\sigma=\frac{2\pi^5k^4}{15c^2h^3}\ \text{and}\ h=\frac{Nu\lambda_g}{2r_s}.$$

Also, $\gamma=\gamma_1+\gamma_2=\frac{3h\left[T_0-T_\infty\right]}{r_s\rho_Fc_\rho\in T_0\Delta}+\frac{3\sigma\left[T_0^4-T_\infty^4\right]}{r_s\rho_Fc_\rho\in\Delta}$,

where, $\gamma_2=\frac{16\pi^5k^4r_s^{\ 2}\left[T_0^4-T_\infty^4\right]}{5c^2N_u^3\lambda_g^3\rho_Fc_\rho\in T_0\,\Delta}$ and $\gamma_2=\frac{16\pi^5k^4r_s^{\ 2}\left[T_0^4-T_\infty^4\right]}{5c^2N_u^3\lambda_g^3\rho_Fc_\rho\in T_0\,\Delta}$,

The equation (2.6) is the generalised equation of droplet temperature with the radiation term.

Existence of Unique Solution

The aim of this section is to show that there exists a unique of problem (2.6).

From equation (2.6), let $f(\theta,\tau)=\frac{d\theta}{d\tau}$

Let D = {(θ, τ); 0 ≤ τ ≤ 1, and 0 < θ ≤ L} and

H = {(θ, τ); 0 < α ≤ M, 0 ≤ β ≤ N, and 0 < γ ≤ L

Theorem: Let D and H hold where M, and L are real constants. Then, for ε > 0, there exists a unique solution of problem (2.3).

Let $f(\theta,\tau)$ be Lipschitz continuous with constant K, such that 0 ≤ K ≤ ∞, then, $f(\theta,\tau)$ has a unique solution if it satisfies Lipschitz condition given as,

$$\left|\frac{\partial f}{\partial \tau}\right| \le K\,,$$

where, 0 < K is Lipschitz constant.

Therefore, f(θ,τ) is Lipschitz continuous. Hence, there exists a unique solution of the problem (2.6).

<u>Proof:</u>

From equation (2.6), we have,

$$f(\theta,\tau) = \frac{d\theta}{d\tau} = e^{\theta} - \gamma - \alpha\theta - \beta(4\theta + 6\in\theta^2 + 4\in^2\theta^3 + \in^3\theta^4)\,,$$

Therefore, $\left|\frac{\partial f}{\partial \theta}\right| = \left|e^{\theta} - \alpha - \beta(4 + 12\in\theta + 12\in^2\theta^2 + 4\in^3\theta^3)\right| \le K < \infty$

and $\left|\frac{\partial f}{\partial \tau}\right| \le 0\,.$

where, 0 < K is a Lipschitz constant.

Thus, f(θ, τ) is Lipschitz continuous. Hence, there exists a unique solution of the problem (2.6).

3. Method of Solution

We are interested in a case where, $\beta \ne 0$, $\in \to 0$ (high activation energy). therefore, the generalized equation (2.6) is reduced to,

$$\frac{d\theta}{d\tau} = e^{\theta} - (\alpha + 4\beta)\theta - \gamma \qquad (3.1)$$

$$\theta(0) = 0$$

Note that when $\beta = 0$ and $\in \to 0$ equation (3.1) reduces to Makino [1] and without gamma parameter gives the well-known equation for the non-steady, non-adiabatic "thermal Explosion" the problem. (Makino [1]).

Comparing equations (3.1) with Marino[1], α = (α + 4β). Adopting method of solution of Makino [1] by replacing α with (α + 4β) and using the series expansion, $\left[\gamma + (\alpha + 4\beta)\theta\right]e^{-\theta} < 1$, , we obtained,

$$\tau_{ig} = \sum_{k=1}^{\infty} \frac{\gamma^{k-1}}{k} \left\{ 1 + \left(\frac{\alpha + 4\beta}{\gamma} \right) . \frac{(k-1)}{k} + \left(\frac{\alpha + 4\beta}{\gamma} \right)^2 \frac{(k-1)(k-2)}{k^2} + \left(\frac{\alpha + 4\beta}{\gamma} \right)^3 \frac{(k-1)(k-2)(k-3)}{k^3} + ... + \left(\frac{\alpha + 4\beta}{\gamma} \right)^{k-1} \frac{(k-1)(k-2)...2.1}{k^{k-1}} \right\} \tag{3.2}$$

When, $(\alpha+4\beta) >> \gamma(T_0=T_\infty)$, we obtain,

$$\tau_{ig} = 1 + \frac{1}{4}(\alpha+4\beta) + \frac{2}{27}(\alpha+4\beta)^2 + \frac{3}{128}(\alpha+4\beta)^3 \cdots + \frac{(k-1)!}{k^k}(\alpha+4\beta)^{(k-1)} . \tag{3.3}$$

While when $\gamma >> (\alpha+4\beta)$ $(T_0 > T_\infty)$,

$$\tau_{ig} = \frac{1}{\gamma} In\left(\frac{1}{1-\gamma} \right). \tag{3.4}$$

A computer symbolic algebra package (MAPLE) was employed to obtain numerically solutions of equation (3.2) when radiation parameter β varies for different values of heat loss parameter α , and when heat loss parameter α varies for different values of radiation parameter β. The results are shown on tables (1) and (2) and represented in figures (1) and (2).

Table 1: Ignition time for various values of radiation parameter β at different values of heat loss parameter α

β	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
0	1	1.0257	1.0532	1.0824	1.1135	1.3578
0.1	1.1136	1.147	1.1829	1.2216	1.2634	1.6028
0.2	1.2634	1.3086	1.3578	1.4112	1.4695	1.9559
0.3	1.4695	1.5331	1.6028	1.6792	1.7629	2.4705
0.4	1.7629	1.8549	1.9559	2.0669	2.1889	3.2192
0.5	2.1889	2.3231	2.4705	2.6325	2.8103	4.2965
0.6	2.8103	3.0053	3.2192	3.4535	3.7099	5.8223
0.7	3.7099	3.9903	4.2966	4.6307	4.9949	7.945
0.8	4.9949	5.3913	5.8224	6.2906	6.7986	10.8443
0.9	6.7987	7.3491	7.9450	8.5892	9.2850	14.7348
1.0	9.285	10.0355	10.8443	11.7148	12.6508	19.869

Table2: Ignition time for various values of heat loss parameter α at different values of radiation parameter β

α	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$
0	1	1.1136	1.2634	1.4695	1.7629	2.1889
0.1	1.02576	1.147	1.3086	1.5332	1.8549	2.3231
0.2	1.05316	1.1829	1.3577	1.6028	1.9559	2.4705
0.3	1.08237	1.2216	1.4112	1.6792	2.0669	2.6325
0.4	1.1135	1.2634	1.4695	1.7629	2.1889	2.8103
0.5	1.1471	1.3086	1.5332	1.8549	2.3231	3.0054
0.6	1.1829	1.3577	1.6028	1.9559	2.4705	3.2192
0.7	1.2216	1.4112	1.6792	2.0669	2.6325	3.4535
0.8	1.2634	1.4695	1.7629	2.1889	2.8103	3.7099
0.9	1.3086	1.5332	1.8549	2.3231	3.0054	3.9903
1.0	1.3577	1.6028	1.9559	2.4705	3.2192	4.2965

4. Discussion of Solution

The computations have been carried out for various values of α and β. In order to validate our method, we have compared the ignition time for various values of α and β with Makino [1] and found excellent agreement.

Figure 1 shows the result of variation of radiation parameter β for different values of heat loss parameter α. In figure 1, it is seen that the ignition time is lowest at $\alpha = 0$, which implies that, if no heat loss is allowed the ignition time is small and ignition occurs faster. At $\alpha = 0.1$ the ignition time is higher than that of at $\alpha = 0$. Also considering $\alpha = 1.0$ and even higher, the ignition occurs very late, this leads to ignition delay. Therefore, it is discovered that heat loss causes ignition delay.

Figure 2 represents the result of variation of heat loss parameter α for different values of radiation parameter β. Fixing the values of β and varying values of α between 0 and 1. We observed that ignition takes place very fast with a limited time, when the value of β is increased the ignition is delayed.

The lower the value of α the lower the ignition time, that is, ignition is faster when alpha is small and vice versa. When β (the radiation term) = 0, no radiation, the effect of radiation is seen when $\beta \neq 0$. Therefore, it is observed that when the radiation term β is increasing the ignition time is delayed which will necessitate the increase in the ignition temperature for ignition to occur not only these but also that more time will be needed for ignition point to be reached .

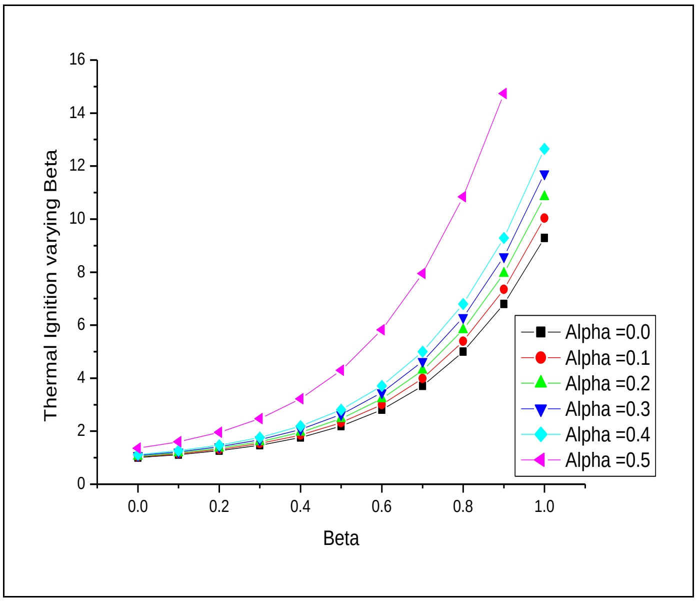

Figure 1: Variation of radiation parameter β for different values of heat loss parameter α .

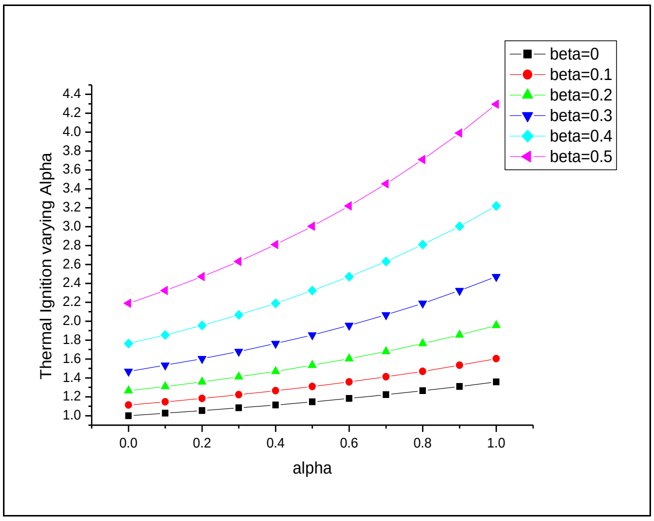

Figure 2: Variation of heat loss parameter α for different values of. radiation parameter β.

5. Conclusion

This paper studied the effect of radiation on the ignition time of a single sodium droplet, taking into consideration the effect of radiation in the presence of heat losses. The governing equation is reduced to dimensionless equation by various values of dimensionless parameters of the problem which is then solved analytically. Comparison with previously published work (Makino [1]), it agrees with the results obtained which presented graphically.

From the results, conclusion is drawn that radiation has effect on ignition time and temperature. It is responsible for increase in ignition time because more time will be needed for ignition to occur and therefore, radiation contributes to the ignition delay.

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