Determination of the Local Geoid Ellipsiod Separation - A Case Study of Tarkwa and its Environs

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Abstract

Although satellite based positioning techniques, especially Global Positioning System (GPS) is used extensively in Ghana together with the conventional methods of positioning fixing, there is little information about the geoid-ellipsoid separation (N), a parameter required to transform measured ellipsoidal heights to the locally used geoidal heights for constructional purposes. This deficiency compels users of GPS in the country to accept the default value (zero) offered by their respective software developers during post processing of GPS data, which results in relatively lower accuracy in height measurement. To overcome this challenge and to enable easy conversion of heights from orthometric to ellipsoidal height, this study sought to determine the actual value of N for the study area. Ellipsoidal and geoidal heights were determined for selected points using GPS receivers and trigonometric leveling from which the average separation between the two surfaces was generated from the dataset. The determination of the separation was accomplished with the aid of the Earth Gravitational Model 1996 (EGM96). A plot of the two heights showed strong correlation with the intercept representing the value of N. The most probable value (MPV) of the height differences between the ellipsoidal and the geoidal height yielded 8.40 m where as the line of best fit from a regression analysis of the scatter plot gave 9.07 m with 95% confidence interval. A sensitivity test was carried out using the determined value of N instead of the default value previously used for reprocessing of the GPS data yielded better accuracy of 0.05 mm.

1. Introduction

The geographic location of a person in relative and absolute terms may be accomplished by either conventional survey techniques or by satellite based navigational techniques for instance the use of the GPS. Mapping any part of the earth surface requires the definition and realization of a reference surface usually the Geoid or a selected ellipsoid. The geoid refers to the equipotential surface of the Earth's gravity field that closely approximates the mean sea level and is perpendicular to the direction of gravity at all points. The ellipsoid on the other hand, is a mathematical surface obtained by rotating an ellipse about its semi-minor axis. It serve as a geometrical model of the earth and its dimensions and orientation of the ellipsoid are usually chosen to give a "best fit" to the geoid over a given area, which in this case is the whole Earth. Heights obtained from GPS are measured with respect to an ellipsoid are not generally useful for control surveys and for engineering work due to discrepancies with heights of ground controls measured above the geoid (orthometric height). It is therefore imperative to unearth a means of transforming heights from ellipsoidal to orthometric and Vis versa. Earlier studies conducted used increased gravity measurements which made it possible to compute gravimetrically the undulation N of the geoid with regard to a given referenced ellipsoid as well as the absolute deflection of the vertical components (Heiskanen, 1967). A model of the geoid at any point on the earth surface requires real gravity measurements and knowledge of the variation of topographic mass density (Anon, 2010b). With the advent of GPS its suitability for height measurements together with other methods to evaluate N from gravimetry was explored and found to be appropriate (Kearsley, 1988). In another study undertaken in Iraq due to the absence of gravity data orthometric heights were determine by integrating GPS data with the Earth Gravitational Model 1996 (EGM96) which yielded appreciable results (Sabah, 2006). This research sought to determine the value of the undulation of the geoid by integrating spot heights obtained by GPS and trigonometric leveling with EGM96.

In Ghana, little is known about the actual value of geoid ellipsoid separation (geoidal undulation), thence a default value of zero (0) is used during the processing and post processing of GPS data which yields relatively low accuracy heights compared to the easting and northing coordinates. In an earlier comparative studies conducted (Boye, & Issaka, 2009) a consistent variation was observed between the heights determined by the GPS and those determine by the conventional methods. This discrepancy was attributed to the unknown value of the Geoid ellipsoid separation for the study area and the country as a whole, the motivation for this research. For a developing country like Ghana where means of obtaining gravity data is difficult to come by the use of the Earth Gravitational Model which suggest a global geoid with errors 1-2m was considered (Sabah, 2006).

In this study ellipsoidal and geoidal heights of selected points in the study area were determined by means of the GPS and trigonometric leveling techniques from which statistical methods were employed to determine the most probable value of the undulation, with the assumption that the vertical deflection was negligible for each station. The volume computed from the two interpolated surfaces generated from the respective spot heights were used to determine the local Geoid ellipsoid separation for the study area. This was found to compare favourably with the linear regression analysis on the scatter plot of the data.

2. Brief Information About the Study Area

The study area is Tarkwa and its immediate environs, a place where there are active mining activities (Figures 1&2).

2.1 Location, Climate and Regional Geology

Tarkwa, the capital of the Tarkwa Nsuaem Municipal Assembly of Ghana, a mining town in the Western Region of Ghana. It is located between latitude 4° 00' to 5° 40' N and longitude 1°30' to 2° 00'W (Anon., 2009) and it is about 85 km from Takoradi the Regional capital and about 322 km from Accra the national capital. Figure 1 is Map of Tarkwa Nsuaem Municipal Assembly showing the location of the study area. It lies in the tropical rain forest zone of Ghana and has a characteristic temperature range of 24 ºC to 35 ºC. There is no complete dry season in Tarkwa since the period supposed to be dry, that is, December through February has intermittent rains. Torrential rains are the norm with bright sunshine and a high humidity throughout the year. Tarkwa area experiences a double maxima rainfall. The first and more pronounced rainy season extends from late March to the end of July and the second season is from October to mid-November. The average annual rainfall is about 180 cm. Annual maximum and minimum temperatures are 37 °C and 26 °C respectively (Issaka *et al*., 2010).

The rock system in the area is known as the Tarkwaian System believed to be approximately 2.1 billion years old. The Tarkwaian System includes the Banket Series in which gold deposits occur as reefs or conglomerate beds. Favourable gold bearing areas are found around the contacts of the Tarkwaian System with the Birimian System (Kesse, 1985). The Tarkwaian strata have been folded into a series of anticlines and synclines. Large thrust faults also exist. As a result of seasonal fluctuations in the water table over time, contrasting weathering types are developed (Kesse, 1985).

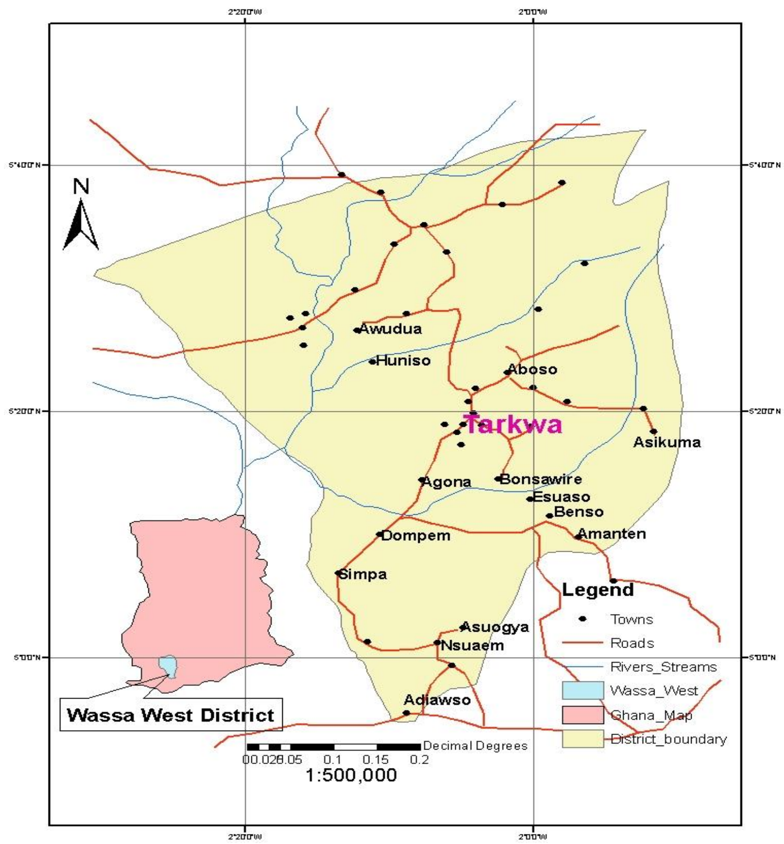

Figure 1: Map of Wassa West District showing location of Tarkwa (Issaka *et al*., 2010).

3. Materials and Methods

The materials and methods employed in this research are discussed in the following sections.

3.1 Materials

Control stations whose heights were reference to the geoid were identified and used as points of departure for the trigonometric leveling and also used as reference station for the GPS observations. Data obtained were downloaded using Prolink and Spectrum survey software from the total Station and GPS receivers respectively and analysed with Microsoft Excel.

3.2 Equipment

The field equipment includes four Sokkia Stratus single frequency GPS receivers, one Sokkia Set 3030R Total Station and accessories.

3.1.2 Software

Microsoft Excel was used for the statistical analysis and ArcGIS 9.3 for the plotting of the points and generation of contours.

3.2 Methods

A Trigonometric leveling survey was conducted using TSM CT1 and AFGO 10/1 as instrument and back stations respectively. DGPS survey technique was employed using TSM CT 1 as the reference station.

3.2.1 Sampling

A reconnaissance survey was conducted during which the points of departure used for the work were identified and the locations for the spot heights chosen and marked. The conditions of the instrument used were also ascertained by checking the battery strength. The receivers were also used to pick certain points and the information downloaded and processed to see if the receivers were functioning properly by comparing the coordinates obtained with the known coordinates of those points. The discrepancies were found to be minimal and therefore the receivers were observed to be in a good working condition. Also a point of departure test was conducted as a check on the stability of the base control point used, and was found to be good. Eighty-two sample points were identified within the study area; some within the valleys, plane and others on the ridges as showed on the contoured map (figure 5a and 5c).

3.2.2 Using the Trigonometric Levelling Technique

To enable effective determination of spot heights reference to the geoid, trigonometric leveling exercise was carried out to determine the coordinates of the sample points within the study area. Trigonometric leveling technique was chosen over spirit leveling technique due to the undulating nature of the terrain under consideration. A close link traverse was conducted over the sampled points to obtain the topogrophic positions using a Sokkia Set 3030R total station and a closing error of 0.002 m was obtained.

3.2.3 Using the GPS Receivers

A GPS receiver was mounted on a control point whose coordinates were known which served as the base station (TSM CT1) of the survey. The rest of the receivers (Roving receivers), were moved around the points whose coordinates are required using the leap frog technique. The time the receiver was switched on; the receiver's serial number and the name of the occupied pillar were recorded in the field book. The rover was switched off and the time recorded when a link between the rover and the base station was created. The recommended occupation time (30 to 40 minutes) was observed taking into consideration the distance between the base station and that of the rovers. This procedure was repeated until all the controls were occupied. The base receiver was switched off and the time recorded. In processing the data to obtain the coordinates of the points, the rovers were treated as autonomous to give the actual height reference to the ellipsoid. For comparison purposes, the coordinates of the reference station were used to compute the differential which was in turn used to adjust the GPS coordinates of the rover stations.

4 Geoid –Ellipsoid separation determination

The geoid also identified as the *mathematical figure of the earth* is an equipotential surface of the Earth's gravity field, which closely approximates mean sea level and is by definition perpendicular to the direction of the gravity vector at all points. Since the mass distribution of the Earth is not uniform and the direction of gravity changes accordingly, the resultant shape of the geoid is irregular ((Torge, 2001; Anon, 2009). The ellipsoid on the other hand is a mathematical surface obtained by rotating an ellipse about its semi-minor axis. The dimension and orientation of the ellipse are chosen "best fit" the ellipsoid to the geoid over a given area, which in this case is over the whole earth (Anon., 2009).

The reference surface for heights is traditionally taken as the mean sea level. The geoid is a surface of equal gravity potential which closely approximates mean sea level. There is increasing use of GPS receivers for surveying works, however, heights derived from GPS are relative to the GPS reference ellipsoid (WGS84). The separation between the geoid and an ellipsoid is known as the geoid-ellipsoid separation, or N value. To transform GPS heights to geoidal heights there is the need to determine geoid-ellipsoid separation (N). If the geoid is above the ellipsoid, N is positive.; and negative if the geoid is below the ellipsoid (figure 2).

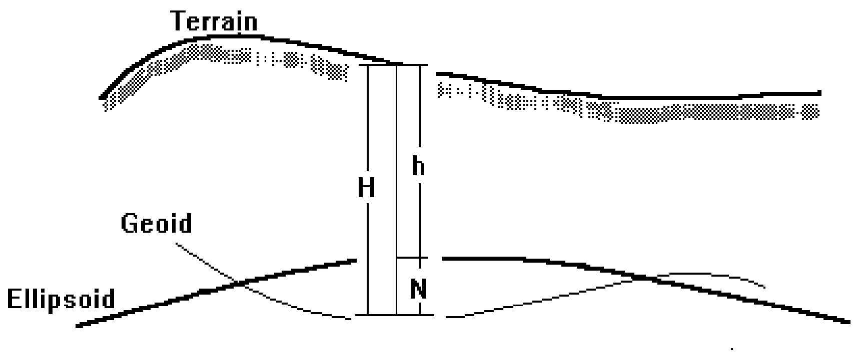

Figure 2 Geoid-Ellipsoid separation (Anon, 2009)

H = h – N

H = height with respect to the geoid (MSL

h = height with respect to the ellipsoid

N = Geoid-ellipsoid separation (N value)

The most common global geopotential model is the Earth Gravity Model 1996 (EGM96). EGM96 was published jointly by the United States National Aeronautical and Space Agency (NASA) and the National Imagery and Mapping Agency (NIMA) in 1998. The estimated accuracy of the EGM96 model is 1-2 metres at its maximum resolution of 56 kilometres. This means that gravity field changes that are smaller than this resolution will not be depicted by itand again; an error of this size may be introduced into heights that use the EGM96 in their calculation (Fiigure 3) (Anon, 2009c).

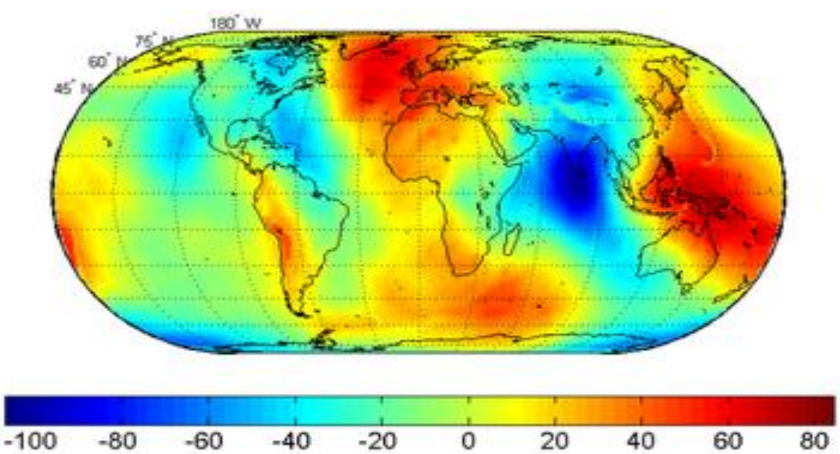

Figure 4 EGM96 Geoid Height in meters

5.0 Results

The results from the processing of the data using both the trigonometric leveling techniques which gave the orthometric height and the GPS receivers which gave the ellipsoidal height were obtained (Table 1). The geoid-ellipsoid separation was determined for each sample point and the most probable value (MPV) found to be 8.4 m. Again the volume between the two surfaces generated from the respective spot heights was computed from which the approximate average thickness of the volume generated gave the separation (N).

5.1 Analysis of Results

Linear regression analysis was applied to the scatter diagrams of the geoid and ellipsoidal heights to assess the relationship between the two heights. Figures 7, shows the relationship established among the variables – geoidal height and ellipsoidal height. The intercept on the vertical axis was determined to be 9.069 m which represents the separation value (N). Figure 7 is a plot of ellipsoidal verses the geodal heights.

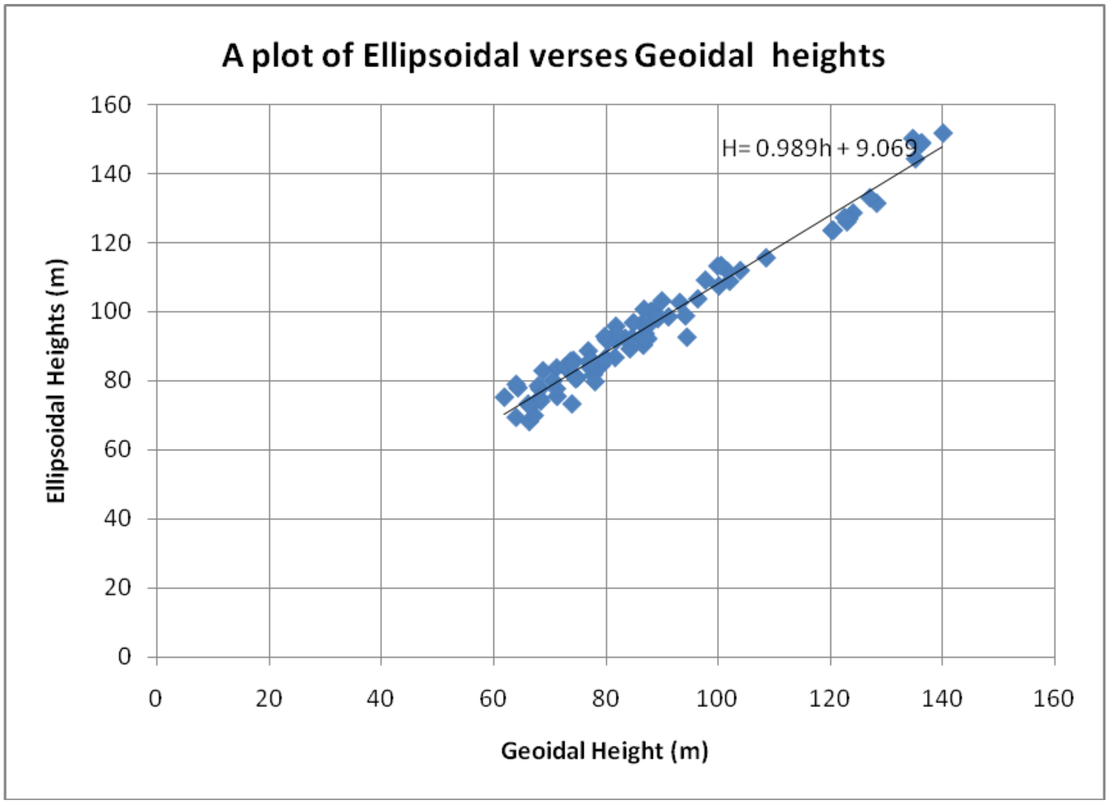

Figure 5 A plot of Ellipsoidal verses Geoidal heights

5.2 Contour and DTM generation for the two surfaces

From the coordinates (X, Y, Z) of the sample points of the study area, contour and DTM of the two surfaces were generated using Arcgis 9.3 and Surfer 7 softwares. The surface area and volume between the two surfaces were computed using surfer 7 software. From these the height between the surfaces was obtained to be 10.884 m.

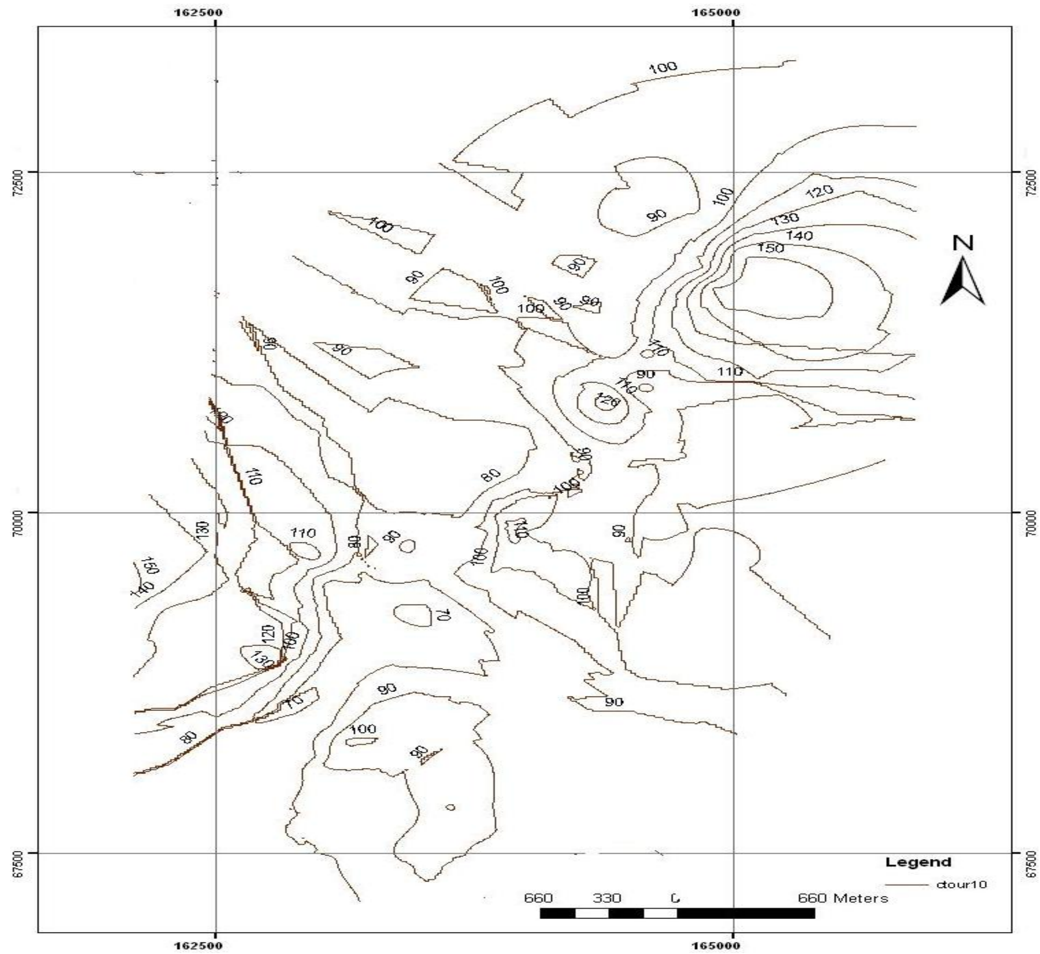

Figure 6a A contoured plan of the study area with respect to the Ellipsoid

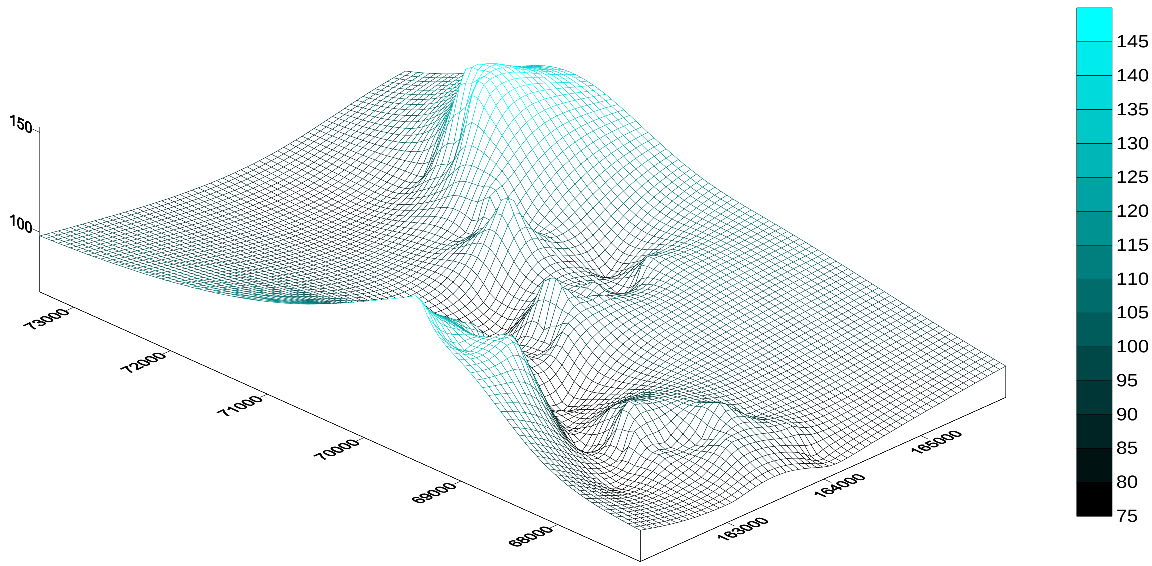

Figure 6b A DTM of the study area with respect to the Ellipsoid

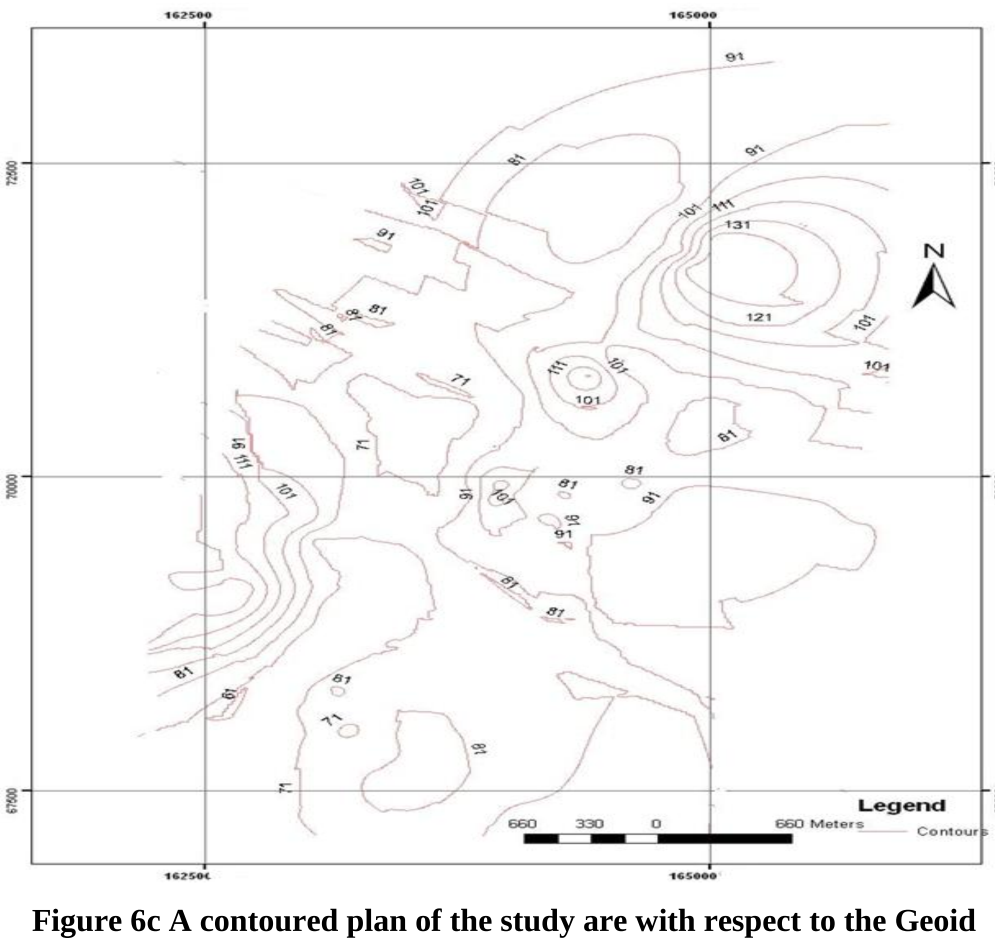

Figure 6c A contoured plan of the study are with respect to the Geoid

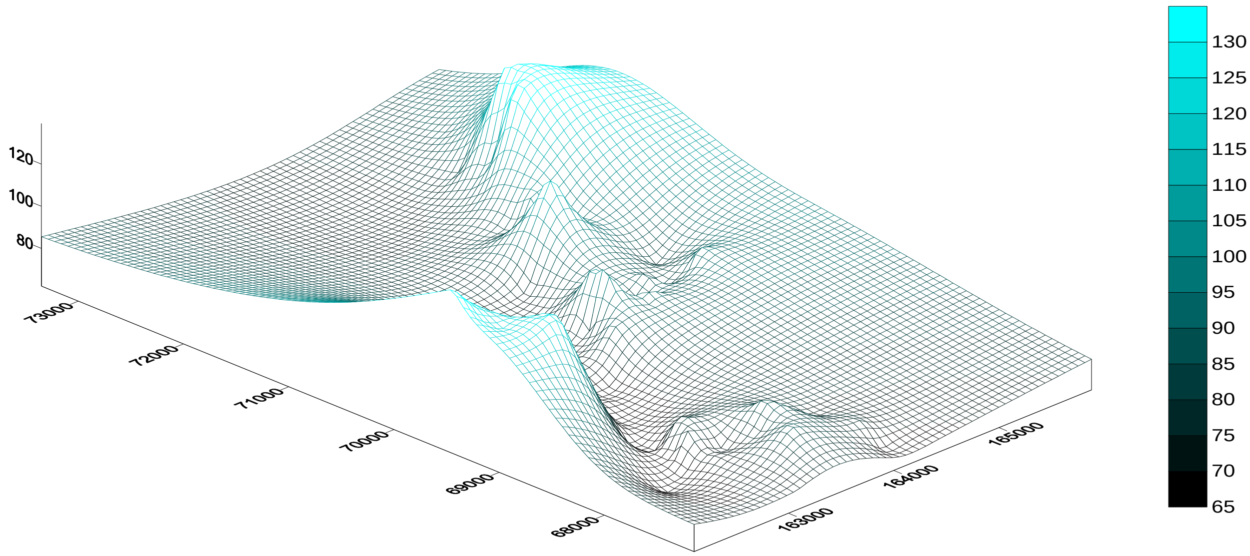

Figure 6d A DTM of the study are with respect to the Geoid

5 Discussions

From the analysis of the results obtained on the data collected, it was observed that there were some variations between the various N values for each sampled point; hence an average value was found (Table 1). Also, when the volume and surface area between the two surfaces were found and the height (N) computed, there were still some variations in the value of N.

The variations in the N values for each sampled point may be due to GPS observations this calls for more research in this direction to increase number of sample points and also to cover the whole region.

6. Conclusions and Recommendations

The application areas of GPS has proven it to be a very useful tool for the determination of geoid-ellipsoid separation when use with other classical geodetic survey techniques such as trigonometric leveling. The two techniques employed in the determination of the height reference to both the geoid and ellipsoid yielded acceptable results. Sensitivity analysis on selected controls gave an accuracy of 0.05 mm.

The correlations between the scatter plots of the various heights of the sample points are strong and the regression equations for the heights were linear.

Further research is therefore required to determine the separation value for the entire country.

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Appendix

Table 1 Coordinates and computation geoid-ellipsoid separation the of samples

N	E	Z_ELLIPSOID(H)	Z_GEOID (h)	N=H-h	Remarks
69484.508	163049.327	77.53	68.238	9.292	P 22
69178.08	162703.276	123.423	120.305	3.118	P 33
69007.646	162636.745	131.316	128.296	3.02	P 23
69808.581	163380.184	85.839	74.221	11.618	P 19
69936.598	164225.587	91.553	81.895	9.658	P 9
69818.5	163942.933	115.528	108.573	6.955	P 5
69812.147	163906.767	108.705	102.074	6.631	P 2
69317.151	162211.227	132.916	127.086	5.83	P 25
69676.933	164200.462	102.583	93.187	9.396	P 8
70229.503	164680.716	89.164	84.315	4.849	P 14
69581.692	163498.56	83.335	73.126	10.209	GCG WP 10 52
69969.836	163924.28	107.371	100.159	7.212	P 3
69849.578	163805.252	88.569	76.874	11.695	P 1
69112.737	162715.954	128.53	124.085	4.445	PBC
69888.023	163945.625	111.717	101.626	10.091	DMP 10
70272.493	164882.787	81.634	77.777	3.857	P 15
69952.351	164591.124	79.654	78.081	1.573	P 32
69937.096	164454.633	98.41	91.175	7.235	P 12
69377.096	162122.474	144.195	135.196	8.999	P 26
69752.028	163501.65	83.653	71.235	12.418	UMaT U4 08
69437.249	162118.874	151.6	140.117	11.483	P 27
69858.392	163993.945	98.69	94.204	4.486	P 7
69385.152	162101.687	148.46	136.111	12.349	WUC 102
69996.365	164607.821	86.635	81.663	4.972	P 31
69630.943	163759.628	96.452	86.41	10.042	P 16
69537.939	163718.96	93.523	87.04	6.483	P 17
69685.742	163733.017	84.322	79.185	5.137	P 18
69867.58	164786.296	90.206	86.714	3.492	P 13
69845.625	164828.079	103.622	96.389	7.233	P 11
69931.481	163985.03	111.836	103.979	7.857	P 4
68933.455	162549.843	126.824	123.298	3.526	P 29
69609.115	164114.724	100.059	88.527	11.532	P 6
69222.042	162336.077	123.486	120.542	2.944	WUC 101
69061.981	162628.25	125.907	122.971	2.936	P 30
69678.943	162979.04	113.161	99.955	13.206	DMP 12
69766.919	163286.771	79.897	70.629	9.268	P 20
69852.554	164063.736	96.732	84.941	11.791	SA 5
69844.81	164293.415	91.938	80.078	11.86	P 10

N	E	Z_ELLIPSOID(H)	Z_GEOID (h)	N=H-h	Remarks
70356.056	163905.887	73.201	73.976	-0.775	BP 1
71678.235	164642.722	103.047	90.006	13.041	BP 4
70808.474	164396.72	127.207	122.527	4.68	B 5
71650.399	164895.102	150.062	134.742	15.32	B 11
71799.86	164846.91	109.025	97.755	11.27	BP 5
71898.816	165030.651	148.836	136.286	12.55	B 13
71238.952	164464.677	92.192	83.482	8.71	BP 3
71878.646	164619.201	93.648	81.915	11.733	BP 6
71169.017	164570.568	113.249	100.553	12.696	B 9
73351.01	165881.26	96.917	88.174	8.743	GCG WP BOGO
72157.838	164895.945	92.395	84.642	7.753	B 15
70637.315	164025.311	80.786	74.927	5.859	BP 2
70917.799	164557.768	86.501	79.953	6.548	B 7
70104.447	163708.536	75.383	71.325	4.058	B 1
70567.266	164201.616	92.518	94.469	-1.951	B 3
72579.279	165186.911	99.981	88.017	11.964	B 17
68038.009	162889.752	77.858	64.336	13.522	C 7
68157.185	163026.37	91.393	80.33	11.063	C 6
68215.597	162970.257	78.396	67.968	10.428	C 5
68322.985	163135.854	101.783	82.217	19.566	C 4
68596.946	163237.59	89.457	69.071	20.386	C 3
69151.256	163297.374	74.833	68.217	6.616	C 2
69281.604	163392.685	68.068	66.375	1.693	AA WOR
69321.679	163506.143	69.826	67.255	2.571	C 1
68026.495	163749.548	92.746	79.811	12.935	T 9
67833.022	163623.684	100.613	86.82	13.793	T 8
67858.416	163408.983	85.966	77.439	8.527	T 7

N	E	Z_ELLIPSOID(H)	Z_GEOID (h)	N=H-h	Remarks
67873.255	163046.017	85.572	73.737	11.835	T 5
67969.934	162655.568	75.15	61.945	13.205	T 4
68444.988	163049.356	78.895	64.026	14.869	T 3
68634.741	162981.706	69.309	64.008	5.301	T 2
68959.705	163103.879	72.45	66.588	5.862	T 1
69396.583	163371.877	74.022	68.415	5.607	SGW 4 03 303A1
69541.693	163373.178	83.944	72.767	11.177	WUC 04 12
69614.762	163399.476	83.664	77.255	6.409	UMaT CP 1 08
67546.788	163403.55	92.077	87.481	4.596	C 12
67815.427	163411.228	84.658	77.393	7.265	C 11
67935.108	163553.962	97.892	89.275	8.617	C 10
68026.495	163749.548	85.686	79.811	5.875	C 9
67833.022	163623.684	91.978	86.82	5.158	C 8
67146.769	163076.319	77.607	71.205	6.402	T 14
67503.647	163120.623	80.461	74.621	5.84	T 13
67327.103	164078.488	73.16	66.158	7.002	T 12
67499.524	164053.709	85.671	77.117	8.554	T 11
67865.218	163944.731	84.969	74.06	10.909	T 10
			Average	8.404 m	