Biopolymer Drill-in Fluid Performance for Different Rheological Models using Statistical Characterisation*

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Abstract

Appropriate selection of rheological models is important for hydraulic calculations of pressure loss prediction and hole cleaning efficiency of drilling fluids. Power law, Bingham-Plastic and Herschel-Bulkley models are the conventional fluid models used in the oilfield. However, there are other models that have been proposed in literature which are under/or not utilized in the petroleum industry. The primary objective of this paper is to recommend a rheological model that best-fits the rheological behaviour of xanthan gum-based biopolymer drill-in fluids for hydraulic evaluations. Ten rheological models were evaluated in this study. These rheological models have been posed deterministically and due to the unrealistic nature have been replaced by statistical models, by adding an error (disturbance) term and making suitable assumptions about them. Rheological model parameters were estimated by least-square regression method. Models like Sisko and modified Sisko which are not conventional models in oil industry gave a good fit. Modified Sisko model which is a four parameter rheological model was selected as the best-fit model since it produced the least residual mean square of 0.61 $Ibf^2/100ft^4$. There is 95% certainty that the true best-fit curve lies within the confidence band of this function of interest.
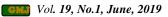
Keywords: Biopolymer, Least-Square Regression, Residual Mean Squares, Rheogram

1 Introduction

The use of rheological models to approximate the behaviour of non-Newtonian fluids is very paramount in the oil and gas industry especially during drilling, well completion, workover and acidizing. In drilling operations, mathematically designed rheological models are used to describe the viscous forces to develop frictional pressure loss equations. Accurate prediction of pressure losses help in the determination of bit optimization hydraulics, estimation of Equivalent Circulating Density (ECD) and drilling fluid compressibility. The benefits of a more accurate estimation of ECD is adequate for hole cleaning efficiency to enhance total drilling rate which in turn reduces total drilling cost. Prevention of circulation lost, maintenance of under-balanced drilling conditions and detection of potential kick are achieved if ECD is rightfully predicted (Bailey and Peden, 2000). Estimated model parameters help to perform other hydraulics calculations.

Power Law and Bingham Plastic models are widely used for hydraulics evaluation. They are assumed for standard America Petroleum Institute (API) hydraulics calculations. Herschel-Bulkley, Roberston-Stiff and Casson models have been accepted to some extent in the petroleum industry. These models and the corresponding hydraulic calculations do provide a way for fair estimates of hydraulics for conventional wells using simple drilling fluids (Guo and Hong, 2010). Power Law model predicts shear stress accurately at low shear rate (in the annulus) and Bingham Plastic model describes the characteristics of drilling fluid at high shear rate (in the drill pipe).

Biopolymer drill-in fluid is a complex fluid formulated with several compositions to desired properties for optimum performance particularly in unconventional wells. It is a water soluble 'rheology engineered' drilling fluid designed to optimize the performance of rotary drilling. It is a complex high molecular weight (MW) polymer with a strong bond between the chains of its molecules which is efficiently used in unconventional wells like onshore and offshore horizontal wells, coiled tubing drilling and slim holes. The elastic structures of biopolymers make them have a higher carrying capacity than the other polymers applied in the petroleum industry during drilling. Due to the complex nature of this type of fluid and its unusual behaviour, it is very prudent to use a more precise rheological model to characterize its behaviour over a full range of shear rate to achieve a proper hydraulics evaluation. Drill-in fluids are specially designed fluid system for drilling through the reservoir interval of a wellbore. They are basically formulated to drill the reservoir zone successfully, often a long horizontal drainhole, to minimize damage and optimize the production of the exposed zones and to enhance the well completion needed. It contains additives that can principally control filtration loss and facilitate optimum carrying capacity. Its composition may be brine with right aggregate size (salt crystals or calcium carbonates) and polymers (Anon, 2010). Polymers typically used as drill-in fluids are

xanthan gum, starch, cellulose and scleroglucans (Brian *et al*., 1997). Herschel-Bulkley model which is a three-parameter model is more likely to approximate the non-Newtonian behaviour of polymeric fluids (Hemphill *et al*., 1993). This paper focuses on ten rheological models proposed in various literatures and come out with a statistical criterion to select the most likely model to predict the rheological characteristics of xanthan gum-based biopolymer drill-in fluids.

2 Resources and Methods Used

The research was conducted by collection of Fann viscometer readings on 'rheology engineered' solid free xanthan gum-based biopolymer drill-in fluids used in coiled tubing drilling. The appropriate models were specified and statistical regression model was used. After data collection and model specification, the estimation of model parameters was done using Least-square regression approximation method. Matrix Laboratory (MATLAB) software code was developed to solve each non-linear model function using a quasi-Newton's numerical iterative approach. Results of the regression analysis were plotted and the residual sum of squares were analysed. Due to small sample size, residual mean squares were employed as a statistical tool to account for the error variance. The model with the minimum residual mean squares was selected. Confidence interval of the selected model function of fitted shear stress values were also estimated. Relevant graphs were plotted to make judicious engineering analysis and decision based on the results and literature knowledge.

2.1 Development and Application of Statistical Model on Data

2.1.1 Collection of Data

Rheological data of xanthan based biopolymer drill-in fluid from rotational viscometer readings were collected. Equations (1) and (2) were applied to convert the dial readings in degrees to shear stress (τ_i) in Ibf/100ft^2 and shear rate (γ_i) in second^{-1} respectively. Table 1 shows the experimented Fann viscometer readings, shear stress and shear rate data.

Table 1 Results of Rheological Data from Viscometer Reading

Speed (rpm)	Readings (°)	γ (sec^{-1})	τ (lbf/100ft^2)
600	54.0	1021.8	57.62
300	44.0	510.9	46.95
200	41.2	340.6	43.94
100	36.8	170.3	39.29
60	33.3	102.2	35.58
30	29.1	51.1	31.10
6	21.0	10.2	22.41
3	19.0	5.1	20.27

$$\tau = 1.067\theta \qquad (1)$$

and

$$\gamma = 1.703S \qquad (2)$$

where τ is in Ibf/100ft^2; θ is rotational viscometer dial readings in degrees; γ is shear rate in sec^{-1} and S is speed of rotation of outer cylinder of the viscometer in rpm.

2.1.2 Model Specification

Models that relate shear stress to set of shear rates were selected. These models are specified as a function of form f (γ_1, γ_2, ...γ_N) but still depend on unknown parameters (β_1, β_2, ……......β_q). The model function can be linear or non-linear. Ten popular rheological models were selected and analysed. For this research, apart from Bingham Plastic rheological model the rest of the model functions are nonlinear. A list of rheological models employed is shown in Table 2.

There is a functional relationship between the shear stress and shear rate in the models used. Therefore, the values of shear stress(τ) to be predicted by each model is a function of shear rate (γ) and q number of parameters ($\beta = \beta_1$, β_2, β_3, ……......β_q) to be estimated in each model. But practically, readings of data are accompanied by some amount of errors ($\acute{\epsilon}$) which might result from poor measurements and instrument error. These errors are assumed to be random constituting the discrepancies in the models approximation. These errors are added to the model function to cater for the failure of the model to fit the experimental data exactly. Hence, a general statistical regression model is formed as shown in Equation (3) to approximate the relationship between shear stress and shear rate.

$$\tau = f(\gamma, \beta) + \acute{\epsilon} \qquad (3)$$

where τ is shear stress in Ibf/100ft^2 and γ is shear rate in sec^{-1}; β is the value of model parameter and $\acute{\epsilon}$ is random error in lbf/100ft^2.

2.1.3 Choice of Fitting Method and Model Fitting

The next task is estimation of model parameters after collection of relevant data and defining the models to be used. Least-squares approximation method was used to performed regression analysis to estimate parameters in each model based on the given data sets. Least-square method was used due to the following assumptions made about the data and the regression model:

    (i)   The scatter follows a Gaussian (normal) distribution;

Table 2 Parameter Constraints and Initial Guess to Evaluate the Rheological Model Functions

Model Name	Model Equation	Parameter Constraints	Initial Guess
Bingham Plastic	$\tau = \tau_o + \mu_p \gamma$	$\mu_p > 0,\ \tau_o \geq 0$	$\mu_p = 0.02,\ \tau_o = 1$
Power Law	$\tau = K\gamma^n$	$K > 0,\ 0 < n < 1$	$K = 2,\ n = 0.4$
Herschel-Bulkley	$\tau = \tau_o + K\gamma^n$	$\tau_o \geq 0,\ K > 0,\ 0 < n < 1$	$\tau_o = 1,\ K = 2,\ n = 0.4$
Robertson-Stiff	$\tau = A(\gamma_o + \gamma)^B$	$A > 0,\ 0 < B < 1,\ \breve{Y}_o \geq 0$	$A = 2,\ B = 0.4,\ \breve{Y}_o = 1$
Modified Robertson-Stiff	$\tau = \tau_o + A(\gamma_o + \gamma)^B$	$\tau_o \geq 0,\ A > 0,\ 0 < B < 1,\ \breve{Y}_o \geq 0$	$\tau_o = 0,\ A = 2,\ B = 0.4,\ \breve{Y}_o = 1$
Prandtl-Eyring	$\tau = A\sinh^{-1}(\gamma/B)$	$A > 0,\ B > 0$	$A = 16,\ B = 30$
Modified Prandtl-Eyring	$\tau = \tau_o + A\sinh^{-1}(\gamma/B)$	$A > 0,\ \tau_o \geq 0,\ B > 0$	$\tau_o = 0,\ A = 10,\ B = 50$
Sisko	$\tau = a\gamma + b\gamma^c$	$a \geq 0,\ b \geq 0,\ 0 < c < 1$	$a = 0,\ b = 2,\ c = 0.4$
Modified Sisko	$\tau = \tau_o + a\gamma + b\gamma^c$	$a \geq 0,\ b \geq 0,\ 0 < c < 1,\ \tau_o \geq 0$	$\tau_o = 0,\ a = 0,\ b = 2,\ c = 0.4$
Casson	$\tau = (\sqrt{\tau_o} + \sqrt{\mu\gamma})^2$	$\mu > 0,\ \tau_o \geq 0$	$\mu = 1,\ \tau_o = 1$

(After Becker et al., 1991)

(ii) Errors are random errors that are independent and identically distributed with mean of zero and variance, σ^2.

In Least-square we look for a function (model) that minimizes the sum-of-squares of vertical distances (residuals) between the fitted model regression line and the observed data points. Considering N number of data points (τ_i, γ_i), least-square is expressed mathematically in Equation (4).

$$RSS(\beta) = \sum_{i=1}^{N} (\tau_i - f(\gamma_i, \beta))^2 = \varepsilon^2 \qquad (4)$$

where $RSS(\beta)$ is the residual sum of squares and β is the value(s) of model parameters that gives minimum RSS (also called least square estimators). β has to be determined so that $RSS(\beta)$ will be minimum. Therefore, for the sum of squares to be minimum,

$$\frac{\partial(RSS(\beta))}{\partial} = 0 \qquad (5)$$

Equations (4) and (5) were used to estimate model parameters and sum of squares (RSS) of each model. For non-linear models, the aforementioned equations were solved using iterative estimation algorithm. MATLAB code was developed to optimize the system of non-linear equation derived from each model equation by a quasi-Newton optimization method. Detailed procedure of how each model was applied is as follows:

(i) A relationship (statistical correlation) between the shear rate and shear stress data points were determined before each model function is fitted to Fann viscometer data points

(ii) Functions for Equation (5) for each model were created in MATLAB.

(iii) Newton iterative algorithm was created to solve each model function (Equation (4)) by calling each function defined in step (ii) above.

(iv) Appropriate initial values for each model parameters were chosen by looking at a graph of their model function behaviour and constraints set for each parameter. Parameters constraints were formed with the idea that shear stress are positive and increase with shear rate. Table 2 depicts the initial guess and constraints for the models.

(v) The algorithms developed were run to solve (converge) each model and relevant output results well tabulated and plotted.

(vi) Residual sum of squares and mean squares of the fitted models were calculated to assess for goodness-of-fit.

2.1.4 Model Comparison

Residual mean squares given in Equation (6) was employed as a statistical tool to account for the error variance because of small sample size;

$$RMS = \frac{RSS}{N - q} \qquad (6)$$

where RMS is residual mean squares or residual variance; RSS is residual sum of squares, N is number of data points; q is number of parameters in a model and $N - q = df$ = degree of freedom in a fitted model.

RMS was used as a performance measure of each model. The model with a minimum RMS was selected as most likely model to describe the behaviour of biopolymer based drill-in fluids.

2.1.5 Confidence Interval

Confidence interval of the selected model function of fitted shear stress values were also estimated. To solve any model function fitted to a measured viscometer data, it is dependent on estimation of rheological model parameter. However, these measured data are subjected to instrument measurement or reading error. It is therefore conceivable to quantify the degree of certainty attached to the fitted functions by calculating level of confidence interval. This is computed by statistical formula developed by Gallant in 1985 to approximate the true confidence interval [$100(1-\alpha)$ %] of non-linear function of concern. This method is applied as follows;

Let $h(\beta)$ be the nonlinear function of interest that is obtained using the rheological model parameters, β. Then, using the results of Gallant (1985), an approximate $100(1-\alpha)$ % confidence interval estimate of the true value of the nonlinear function is given by:

$$h(\beta) \pm t_{[(N-q)\propto]}\sqrt{\hat{H}\left(\hat{F}^T\hat{F}\right)^{-1}\hat{H}^T s^2} \qquad (7)$$

where

$$\hat{H} = \left(\frac{\partial[h(\beta)]}{\partial\beta_1} \quad \frac{\partial[h(\beta)]}{\partial\beta_2} \cdots \frac{\partial[h(\beta)]}{\partial\beta_q}\right) \qquad (8)$$

is the row vector of partial derivatives of $h(\beta)$ with respect to the rheological model parameters.

$$\hat{F} = \begin{bmatrix} \frac{\partial[f(\gamma_1;\beta)]}{\partial\beta_1} & \frac{\partial[f(\gamma_1;\beta)]}{\partial\beta_2} & \cdots & \frac{\partial[f(\gamma_1;\beta)]}{\partial\beta_q} \\ \frac{\partial[f(\gamma_2;\beta)]}{\partial\beta_1} & \frac{\partial[f(\gamma_2;\beta)]}{\partial\beta_2} & \cdots & \frac{\partial[f(\gamma_2;\beta)]}{\partial\beta_q} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial[f(\gamma_N;\beta)]}{\partial\beta_1} & \frac{\partial[f(\gamma_N;\beta)]}{\partial\beta_2} & \cdots & \frac{\partial[f(\gamma_N;\beta)]}{\partial\beta_q} \end{bmatrix} \quad (9)$$

F is the N x q matrix of partial derivative of $h(\beta)$ written in terms of the rheological model evaluated at β and N data points γ_1. s^2 is the estimated error variance given by the *RMS* value and $t_{[(N-q)\alpha]}$ is the t-distribution value corresponding to the significance level α.

It has $(N - q)$ degrees of freedom. The accuracy of the above approximations will, of course, increase with small sample size. For small data sets (as exist with fitted rheological models where typically eight, or fewer, samples are available) close-to-linear model behaviour is necessary to ensure that the above formulas are valid (Bailey and Peden, 2000).

3 Results and Discussions

3.1 Residual Sum-of-Squares and Mean Squares

The results obtained by the application of the general statistical regression model to each rheological model function using least-square approximation of function are discussed in this section. Total variability of model functions from the observed data is needed to make any plausible conclusions from the goodness-of-fit.

Based on the least-square regression analysis on the data, the parameters of the fitted models were calculated by minimising the sum-of-squares of the residuals in order to produce a good fit. Fig. 1 shows the comparison of the various fitted rheological models to the observed raw rheological data based on least-square regression analysis.

A summary of result from the least square regression approximation using MATLAB including the RMS values is shown in Table 3. It can be observed from Table 3 that Prantl-Eyring model has the highest RSS and RMS values which are 2041.48 Ibf²/100ft⁴ and 340.25 Ibf²/100ft⁴, respectively. Modified Sisko model has the lowest RSS and RMS values of 2.47 Ibf²/100ft⁴ and 0.61 Ibf²/100ft⁴, respectively. In this study RMS is used as the main criterion to measure the performance of fit to select the model which is able to describe the rheological behaviour of the biopolymer drill-in fluid over all realistic range of shear it is exposed to. This is because of the small data sets of eight that can be produced by the viscometer readings This criterion takes into consideration the varying number of parameters (degree of freedom) between models and produces an estimate of error variance.

In ranking the RMS results in Table 3 it can be seen that some models perform (fit) better than others because of their low RMS values relative to other models. Some of these models are Sisko, modified Sisko and Herschel-Bulkley. This is because of the flexibility of these models to adapt to the rheogram the biopolymer drill-in fluid will exhibit. Prandtl-Eyring mathematical model should not be used since it gave the poorest fit and hence will result in wrong hydraulics predictions.

Most of the conventional industrially accepted models particularly Bingham Plastic model are not the best to model the pseudoplastic behaviour of the data as compared to some of the models based on their respective RMS values. Modfied Sisko gave the best-fit because of its least RMS value followed by the Sisko model.
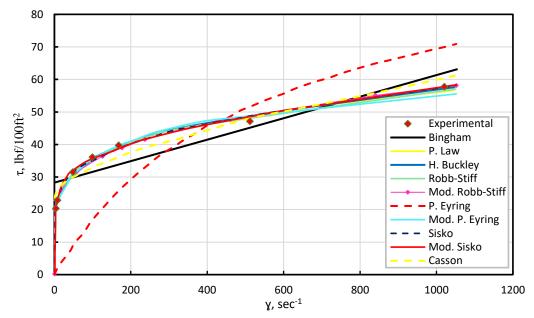
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Fig. 1 Comparison of Rheological Models

Table 3 Summary Result from Least Square Regression Approximation

Model Name	q	RSS (Ibf²/100ft⁴)	RMS (Ibf²/100ft⁴)	Estimated Model Parameters
Bingham Plastic	2	192.576	32.096	$\mu = 0.032$, $\tau_o = 28.08$
Power Law	2	7.747	1.291	K=14.19, n = 0.20
Herschel-Bulkley	3	5.938	1.188	$\tau_o = 7.47$, K = 8.56, n = 0.25
Roberston-Stiff	3	6.732	1.346	A =13.38, $\check{Y}_o = 2.38$, B = 0.21
Modified Roberston-Stiff	4	9.362	2.341	$\tau_o = 13.04$, K= 4.79, $\check{Y}_o = 0.60$, B = 0.32
Prandtl Eyring	2	2041.477	340.246	A = 30.67, B = 205.97
Modified Prandtl Eyring	3	15.556	3.111	$\tau_o = 19.55$, A = 8.73, B=34.89
Sisko	3	3.982	0.796	a = 0.006, b =15.33, c = 0.17
Modified Sisko	4	2.468	0.617	τ_o=0.489, a= 0.012, b = 501.35, c= 0.009
Casson	2	63.612	10.602	$\mu = 0.009$, $\tau_o = 22.71$

3.2 Confidence Interval Results

Modfied Sisko model is selected as a suitable model to describe the behaviour of biopolymer drill-in fluid because of the minimum RMS value. Once the suitability of the model is checked, it is possible to infer and create prediction intervals more reliably and hence to estimate shear stress with greater confidence. Within the range of experimental points, the prediction interval $100(1-\alpha)$% for a particular shear stress is estimated by Gallant's formula in Equation (7) through to (9). The 2-tailed t-value being taken at the required probability level, 0.05 and 4 degrees of freedom is 2.78. The confidence interval for the fitted modified Sisko is narrow enough as shown in Fig. 2.

This means we obtain the smallest uncertainty near the centroid of the Modified Sisko function plot and can be 95% sure that the true best-fit curve (which could only be known if you have an infinite number of data points) lies within the confidence band.

4 Conclusions and Recommendation

Statistical evaluation of the biopolymer drill-in rheological data using least-square regression statistical method has revealed that there are suitable rheological models to approximate the behaviour of this fluid other than the conventional industry Power law and Bingham plastic models. Conclusions drawn at end of this study are as follows:

(i) The most likely rheological model to characterise the behaviour of xanthan based drill-in fluid is the Modified Sisko model. This model gave the minimum error variance (residual mean square) and there is 95% certainty that the true best-fit curve lies within the confidence band.
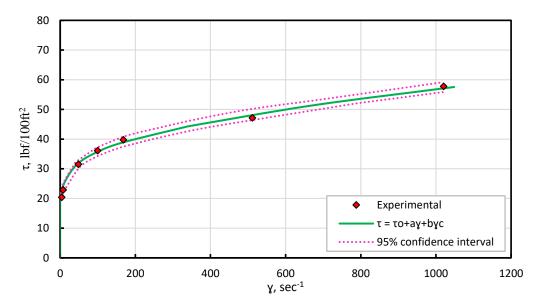
Fig. 2 Experimented Rheological Data, Fitted Modified Sisko Values and its Confidence Interval

(ii) Prandtl-Eyring mathematical model gave the poorest fit and should not be applied since it will result in wrong hydraulic predictions as far as this drill-in fluid is concerned.

The rheological properties of this xanthan based biopolymer drill-in fluids were measured at a nominal temperature of 120 °F. Using the parameter obtained from this rheological model at this temperature conditions might result in inaccurate hydraulic calculation especially when drilling offshore because drilling fluids experience high temperatures downhole and very cold temperatures in risers, while both locations are associated with high pressures. It is recommended that future work should be done on temperature and pressure effects on the rheological behaviour of this xanthan based biopolymer drill-in fluid in order to select the appropriate model.

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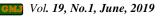
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