Alternative Methods of Determining Translation Parameters for Geocentric Translation Model Applications*

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Abstract

Several studies have been reported in geodetic sciences on the application of geocentric translation model (GTM) for coordinate transformation. The most commonly used method for estimating GTM parameters has been the simple averaging approach popularly known as the arithmetic mean. However, to the best of our knowledge, little or no alternative techniques have been fully explored for estimating GTM parameters. Therefore, the present study is focused on testing the capability and applicability of alternative mean estimation methods of median, geometric mean, harmonic mean and quadratic mean; by comparing with the arithmetic mean. In view of these developments, the methods were applied in Ghana’s geodetic reference network to transform Global Navigation Satellite System acquired data to the two local datums (Accra and Leigon) used in Ghana. The suitability of the mean estimation methods for coordinate transformation between local datums was also carried out. In determining the GTM parameters, the Vaníček and Steeves concept (VSC) was also applied for the first time as part of the GTM coordinate transformation process. The performance of the mean estimation approaches were assessed based on standard deviation, root mean square error, mean absolute error, minimum and maximum horizontal errors and horizontal positional errors. On the basis of the results, it was clearly revealed that the strength of a mean estimation method depends on whether it is applied for: global to local coordinate transformation; only local coordinate transformation, and the type of reference ellipsoids used in the transformation process. This study will therefore serve as a contribution to the dynamics in the implementation of GTM for surveying and mapping purposes in Ghana.

Keywords: Geocentric Translation Model, Vanicek and Steeves Concept, Coordinate Transformation

1 Introduction

Coordinate positions are significant in all aspects of applications within the Earth Science disciplines for spatial locations of object on, beneath the Earth surface or the outer space. Presently, the general method of acquiring coordinates information is predominantly through the use of Global Navigation Satellite System (GNSS) such as Global Positioning System (GPS) based on the World Geodetic System 1984 (WGS84) geocentric datum. In order to localize these datasets obtained from the GPS necessitates the need to carry out coordinate transformation. Transformation of coordinates from one reference frame to another is a common problem in spatial data processing. This transformation creates the opportunity to integrate data from different reference frames into a single datum (Mandich, 2014) and is recognised as a step for utilising some of the potential of GNSS technology.

Over the years, coordinate transformation has been chiefly dominated by the conformal transformation methods of Bursa-Wolf (Bursa, 1962; Wolf, 1963) and Molodensky-Badekas (Molodensky *et al.*, 1962; Badekas, 1969) model. Other conventional techniques such as Abridged Molodensky, multiple regression and polynomial have also been comprehensively studied (Applebaum, 1982; NIMA, 2000; Newsome and Harvey, 2003; Soycan and Soycan, 2008; Kutoglu, 2009). Yet, few researchers have investigated into Geocentric Translation model (GTM) (Deakin *et al.*, 1994; Featherstone, 1997; Newsome and Harvey, 2003; Tierra *et al.*, 2008; Solomon, 2013). The reason could perhaps be related to the low accuracy produced by the GTM as compared to the other coordinate transformation based models. This lower accuracy could be attributed to the assumption made in the GTM that the coordinate axes of the two datums are parallel and no scale factor and rotations exist in both systems (Deakin, 1994; Featherstone, 1997). Moreover, the inability of the GTM to model out heterogeneity in data relating different reference frames effectively contributes to its low transformation accuracy.

Nevertheless, GTM could be applied in situations where higher accuracy is not paramount (Featherstone, 1997; Newsome and Harvey, 2003). For instance, in field surveying works such as recce, the GTM parameters could be applied just by using a calculator which overcomes the complexity of the similarity formulas; in GIS data collection for geodatabase generation; and small scale topographic mapping surveys. Also, it could be used to transform thematic type data such as vegetation, soil type and geology where the accuracy is not critical. Generally, the translation parameters applied in the GTM to transform coordinates between different datums are primarily obtained from similarity models such as Abridged

Molodensky, Bursa-Wolf, and Molodensky-Badekas (NIMA, 2000; Featherstone, 1997). Interestingly, one requires good mathematical knowledge and programming skills to understand and apply these similarity methods. In addition, Featherstone (1997) contended that using the similarity based models could produce different translation parameters and thus could lead to inconsistent results when GTM is applied. Alternatively, the arithmetic mean has been widely adopted as a technique that could represent the characteristics of the datasets more appropriately in the estimation of GTM parameters (Deakin, 1994; Newsome and Harvey, 2003; Solomon, 2013).

On the other hand, upon careful review of existing studies, little or no alternative techniques have been investigated and tested for estimating translation parameters in GTM application. This study applied the mean estimation techniques of arithmetic mean (AM), median (MD), harmonic mean (HM), geometric mean (GM) and quadratic mean (QM). These estimation approaches were used to determine the GTM translation values for the transformation between WGS84 and the two local geodetic datums (Accra and Leigon) used for surveying and mapping purposes in Ghana. It is important to state that the authors adopted the Vaníček and Steeves concept (VSC) (Vaníček and Steeves, 1996) in the coordinate transformation process. This was necessary because it was argued by Vanicek and Steeves (1996) that the local ellipsoidal height is assumed to be determined to a lower accuracy not deserving for 3D coordinate transformation and thus, unwarranted distortions are introduced into the local geodetic network. This often renders the ability of a transformation model to produce satisfactory results. Vaníček and Steeves (1996) further suggested as a prerequisite that the height component in both global and local datums should not be used in the coordinate transformation process. This global assumption has been further echoed in Featherstone and Vaníček (1999). Therefore, the Ghana local geodetic datums established based on astro-geodetic observations make it suitable to apply the Vaníček and Steeves concept since there is no local ellipsoidal height.

From this reasoning, the present authors were therefore motivated to apply the VSC with the primary objective of reducing the distortions especially caused by the geodetic heights (local) in the transformed coordinates produced by GTM. In addition, suggest the approach as a case of application for developing countries like Ghana where the non-geocentric datum is still utilised for their surveying and mapping activities. To achieve these, the present study for the first time applied, compared and discussed alternative mean estimation methods (AM, MD, HM, GM and QM) by using VSC in GTM application in Ghana. The suitability of the mean estimation methods for transformation between Accra and Leigon datum was also carried out.

The mean absolute error, root mean square error and standard deviation are the dimensioned error statistic used as performance indices. In addition, the horizontal positional error, mean horizontal position error, minimum and maximum horizontal residuals were also utilized. This study will therefore contribute to the implementation of the GTM for low level accuracy surveying and GIS related works carried out in Ghana geodetic reference network.

2 Resources and Methods Used

2.1 Resources and Study Area

The research was carried out in Ghana located at the Western part of Africa. Ghana shares boarders with three countries which include Togo to the East, Ivory Coast to the West, and Burkina Faso to the North. To the South is the Gulf of Guinea. The country lies between latitudes 4^{o} 30' and 11^{o} N and longitude 1^{o} E and 3^{o} W and covers a total land area of 239,460 sq. km. The topography is generally of low plains with divided plateau in the South-Central area and scattered areas of high relief (Baabereyir, 2009). Two national horizontal geodetic datums namely, the Accra and Leigon datum have been the only available datums used in Ghana. These two datums are non-geocentric and were established based on astro-geodetic observations. The reference surface for the Accra datum 1929 is the War Office 1926 ellipsoid suggested by the British War Office with semi-major axis a = 6378299.99899832 m, semi minor axis b = 6356751.68824042 m, flattening f = 1/296. The Leigon datum 1977 has the Clark 1880 (modified) ellipsoid as its reference surface having a semi-major axis a = 6378249.145 m, semi minor axis b = 6356514.870 m, and flattening f = 1/293.465006079115 (Ayer, 2008; Ayer and Fosu, 2008).

The necessity for geospatial professionals in Ghana to adopt GNSS technology for surveying and mapping lead to the Land Administration Project (LAP) sponsored by the World Bank. GPS observations were made on existing historical triangulation points at the mid Southern part of Ghana (Fig. 1) to obtain satellite positions of the controls in the two astro-geodetic datums (Kotzev, 2013). In effect, common point coordinates between global datum and the local datums (Accra and Leigon) were obtained. It should be noted that the positions in the global datum were obtained through differential processing of the GPS receiver acquired coordinates with the International GNSS service (IGS) stations. The GPS obtained satellite coordinates was defined in the International Terrestrial Reference Frame 2005 (ITRF2005) specified at epoch 2007.39 (Kotzev, 2013). The mid Southern regions (Fig. 1) are the first phase of the nationwide renewal of the local datums referred to as the golden triangle. The second phase covering the North most part of the country is yet to be completed. This study applied secondary data acquired from the Ghana Survey and Mapping Division of Lands Commission in its on-going LAP. Existing Projected grid coordinates in the Accra datum and Leigon datum were also provided to help evaluate the performance of the mean estimation methods. Tables 1, 2 and 3 show the cartesian coordinates of common points used to perform the transformation in this study.

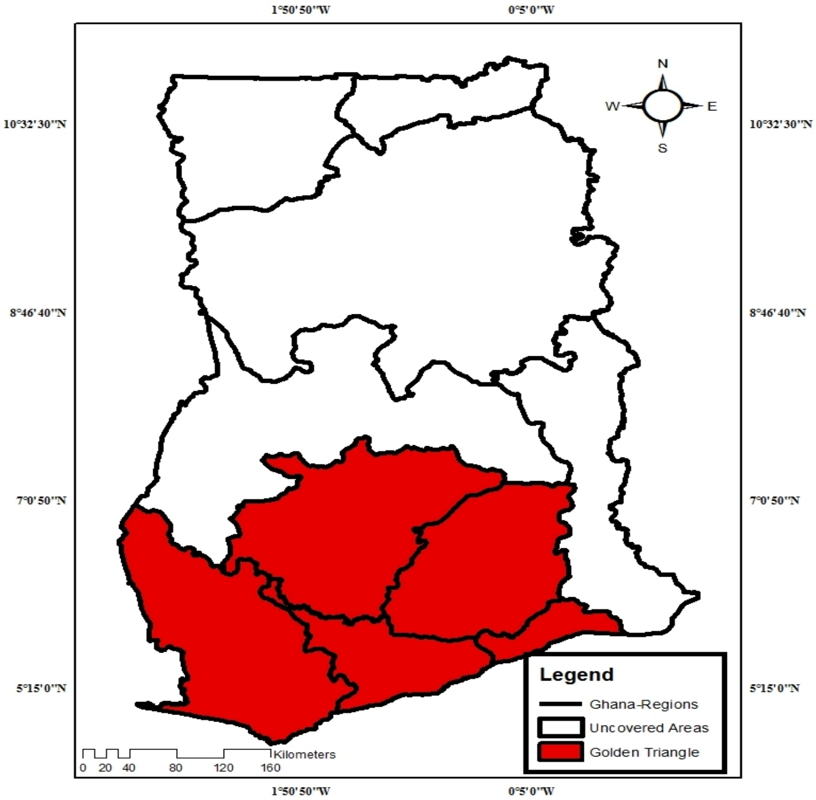

Fig. 1 Study Area (Ghana)

Table 1 Data for transforming from WGS4 to War Office 1926 system using GTM

PT ID	War X	War Y	War Z	WGS X	WGS Y	WGS Z
1	6349409.327	-46970.670	602528.309	6349216.526	-46937.595	602850.422
2	6347511.181	-61996.956	620767.330	6347317.555	-61963.994	621089.479
3	6347456.143	-166364.103	601974.614	6347263.655	-166331.080	602299.601
4	6350895.804	-81439.118	583003.237	6350704.167	-81406.532	583324.885
5	6344311.354	-13541.146	655229.232	6344115.678	-13508.014	655552.515
6	6338161.167	-114348.252	702846.772	6337963.682	-114315.111	703169.740
7	6341401.872	-142407.686	667850.435	6341206.186	-142375.250	668173.703
8	6343764.185	-80827.204	655678.421	6343568.942	-80794.501	656000.509
9	6336087.817	-84696.251	725228.756	6335889.215	-84663.620	725550.613
10	6341585.798	-82924.611	676032.731	6341389.492	-82892.051	676355.510
11	6335400.386	-128828.430	724740.654	6335201.992	-128794.655	725062.881
12	6334220.039	-212939.599	715227.300	6334022.452	-212905.291	715549.999
13	6325317.473	-180075.852	797697.362	6325116.025	-180042.328	798017.357
14	6328359.648	-222871.878	762205.871	6328160.051	-222837.357	762526.598
15	6329201.162	-159721.648	770957.863	6329000.768	-159688.632	771278.987
16	6330226.210	-192706.232	754944.210	6330026.793	-192671.938	755265.209
17	6334967.992	-156168.638	723153.737	6334769.765	-156134.602	723476.351
18	6335189.568	-187480.113	713775.381	6334991.985	-187446.158	714097.595
19	6341604.179	-217730.715	645392.842	6341409.929	-217696.318	645716.644

Table 2 Data for transforming from WGS4 to Clark 1880 (modified) system using GTM

PT ID	CLARK X	CLARK Y	CLARK Z	WGS X	WGS Y	WGS Z
1	6325268.660	-180072.729	797655.269	6325116.025	-180042.331	798017.351
2	6350847.002	-81439.573	582961.521	6350704.166	-81406.490	583324.897
3	6347406.889	-166363.862	601937.718	6347263.652	-166331.081	602299.632
4	6343715.387	-80827.249	655636.799	6343568.941	-80794.477	656000.521
5	6330177.324	-192703.545	754903.176	6330026.765	-192672.157	755265.385
6	6329152.289	-159719.587	770916.339	6329000.769	-159688.630	771278.978
7	6335351.567	-128826.648	724699.085	6335201.987	-128794.677	725062.927
8	6346218.801	-7249.474	636194.924	6346072.860	-7215.682	636559.414
9	6342637.266	-5528.302	670747.513	6342489.559	-5561.330	671112.093
10	6344262.493	-13541.030	655188.256	6344115.677	-13507.953	655552.524
11	6334919.064	-156166.678	723113.204	6334769.767	-156134.579	723476.335
12	6336039.052	-84696.007	725186.476	6335889.219	-84663.454	725550.591
13	6341555.100	-217728.482	645354.496	6341409.940	-217696.122	645716.602
14	6349360.552	-46970.978	602486.466	6349216.525	-46937.591	602850.430
15	6347462.388	-61997.472	620725.659	6347317.555	-61964.085	621089.463
16	6335140.581	-187477.989	713735.529	6334991.985	-187446.135	714097.600
17	6341536.957	-82924.572	675991.442	6341389.490	-82892.093	676355.526
18	6338112.326	-114347.458	702805.411	6337963.685	-114315.182	703169.702
19	6341352.924	-142407.335	667810.194	6341206.206	-142374.803	668173.607
20	6328310.769	-222868.253	762165.012	6328160.052	-222837.320	762526.601

Table 3 Data for transforming between War Office 1926 and Clark 1880 (modified) system using GTM

PT ID	War X	WAR Y	WAR Z	CLARK X	CLARK Y	CLARK Z
1	6349409.327	-46970.670	602528.309	6349360.552	-46970.978	602486.466
2	6347511.181	-61996.956	620767.330	6347462.388	-61997.472	620725.659
3	6347456.143	-166364.103	601974.614	6347406.889	-166363.862	601937.718
4	6350895.804	-81439.118	583003.237	6350847.002	-81439.573	582961.521
5	6344311.354	-13541.146	655229.232	6344262.493	-13541.030	655188.256
6	6338161.167	-114348.252	702846.772	6338112.326	-114347.458	702805.411
7	6341401.872	-142407.686	667850.435	6341352.924	-142407.335	667810.194
8	6343764.185	-80827.204	655678.421	6343715.387	-80827.249	655636.799
9	6336087.817	-84696.251	725228.756	6336039.052	-84696.007	725186.476
10	6341585.798	-82924.611	676032.731	6341536.957	-82924.572	675991.442
11	6335400.386	-128828.430	724740.654	6335351.567	-128826.648	724699.085
12	6334220.039	-212939.599	715227.300	6334171.046	-212937.008	715187.635
13	6325317.473	-180075.852	797697.362	6325268.660	-180072.729	797655.269
14	6328359.648	-222871.878	762205.871	6328310.769	-222868.253	762165.012
15	6329201.162	-159721.648	770957.863	6329152.289	-159719.587	770916.339
16	6330226.210	-192706.232	754944.210	6330177.324	-192703.545	754903.176
17	6334967.992	-156168.638	723153.737	6334919.064	-156166.678	723113.204
18	6335189.568	-187480.113	713775.381	6335140.581	-187477.989	713735.529
19	6341604.179	-217730.715	645392.842	6341555.100	-217728.482	645354.496

2.2 Methods Used

2.2.1 Geocentric Translation Model

The Geocentric translation model (GTM) considers only the displacement vectors between the origins of two reference systems. The displacement vectors estimated and applied in GTM is usually between two different datums. These displacement parameters represent the shifts in origin between two geodetic datums. In the GTM, translation parameters are estimated in (X, Y, Z) cartesian coordinates by assuming a centric value of (0, 0, 0) for one datum and defining the other datum centre by the translation vectors $(\Delta X, \Delta Y, \Delta Z)$. Mathematically, the GTM could be defined by Equation (1) (Featherstone, 1997) as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{global} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{local} \tag{1}$$

where $(X, Y, Z)_{global}$ is the cartesian coordinates based on the WGS84 and $(X, Y, Z)_{local}$ is the cartesian coordinates based on the local datum (Accra and Leigon). As indicated earlier, mean estimation techniques of AM, QM, HM, GM and MD were respectively applied to estimate the unknown translation parameters $(\Delta X, \Delta Y, \Delta Z)$. However, before the implementation of these estimation approaches the VSC was first applied in the transformation process. Detailed description about the VSC and mean estimation methods is given in Sections 2.2.2 and 2.2.3 respectively.

2.2.2 Vaníček and Steeves Concept (VSC)

An essential step in coordinate transformation between different datums demands the conversion of curvilinear geodetic coordinates into its corresponding cartesian coordinates. To achieve this, the standard forward transformation equation expressed by Equation (2) (Heiskanen and Moritz, 1967) is mostly used.

$$\begin{aligned} X &= (N+h)\cos\varphi\cos\lambda \\ Y &= (N+h)\cos\varphi\sin\lambda \\ Z &= [N(1-e^2)+h]\sin\varphi \end{aligned} \tag{2}$$

where φ, λ and h is the geodetic latitude, geodetic longitude and ellipsoidal height respectively. N is the radius of curvature in the prime vertical defined by Equation (3) as

$$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}} \tag{3}$$

where a and b are the semi-major axis and semi-minor axis of the geodetic ellipsoid and e is the first eccentricity as defined in Equation (4).

$$e = \frac{\sqrt{a^2-b^2}}{a} \tag{4}$$

In Vaníček and Steeves (1996), the local geodetic reference network heights is assumed to be less accurate than the horizontal positions. Hence, in order to apply the VSC correctly, requires a modification of the standard forward transformation equation (Equation (2)) by eliminating h. The modified version of Equation (2) is given by Equation (5) as:

$$\begin{aligned} X &= N\cos\varphi\cos\lambda \\ Y &= N\cos\varphi\sin\lambda \quad . \\ Z &= N(1-e^2)\sin\varphi \end{aligned} \tag{5}$$

This study used Equation (5) to convert all the geodetic coordinates denoted as $(\varphi,\lambda)_{WGS84}$, $(\varphi,\lambda)_{Accra}$ and $(\varphi,\lambda)_{Leigon}$ into cartesian coordinates. The transformed cartesian coordinates for the WGS84, Accra and Leigon datums using Equation (5) are designated in this study as $(X, Y, Z)_{WGS84}$, (X_A, Y_A, Z_A) and (X_L, Y_L, Z_L) respectively. On the use of this concept (VSC), the real case when local geodetic reference network heights are under consideration (Featherstone and Vaníček, 1999) was implemented in the present study.

2.2.3 Translation Parameter Estimation and Coordinate Transformation

Step 1: The geodetic coordinates of $(\varphi,\lambda)_{WGS84}$, $(\varphi,\lambda)_{Accra}$ and $(\varphi,\lambda)_{Leigon}$ were first converted into their respective (X, Y, Z) cartesian coordinates using Equation. (5) as elaborated in Section 2.2.2.

Step 2: The degree of shift $(\Delta X, \Delta Y, \Delta Z)$ of the common points between Accra datum (X_A, Y_A, Z_A) and global datum $(X, Y, Z)_{WGS84}$ were estimated. This was done by subtracting each $(X, Y, Z)_{WGS84}$ from (X_A, Y_A, Z_A) using Equation (6) given as:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} . \tag{6}$$

The objective here is to transform the global (X, Y, Z)$_{WGS84}$ into the Accra datum so that its applicability in the local geodetic network could be realised. In the case of (X, Y, Z)$_{WGS84}$ to (X_L,Y_L, Z_L), Equation (7) was used to determine their origin shifts. Here, (X_L, Y_L, Z_L) represents the Leigon datum coordinates.

$$\begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix}=\begin{bmatrix}X_L\\ Y_L\\ Z_L\end{bmatrix}-\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{WGS84} \qquad (7)$$

Similarly, Equations (8) and (9) were respectively applied to calculate the translation values between Leigon and Accra datum.

$$\begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix}=\begin{bmatrix}X_A\\ Y_A\\ Z_A\end{bmatrix}-\begin{bmatrix}X_L\\ Y_L\\ Z_L\end{bmatrix} \quad \ldots.. \qquad (8)$$

$$\begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix}=\begin{bmatrix}X_L\\ Y_L\\ Z_L\end{bmatrix}-\begin{bmatrix}X_A\\ Y_A\\ Z_A\end{bmatrix} \qquad (9)$$

Step 3: Here, the mean estimation methods (AM, QM, HM, GM, MD) were applied to the shifts values estimated in Step 2. Their respective mathematical representations (Gleb *et al.,* 2009) are given by Equations (10) to (15).

Arithmetic Mean (AM)

As stated earlier, AM is the most commonly used type of averaging for estimating GTM parameters. In order to apply Equation (10), the set of ΔX, ΔY and ΔZ calculated values for each common points were added. The summation value for the set of ΔX, ΔY and ΔZ was then divided by the number of observations (*n*) to get the AM parameters denoted as (X_{AM}, Y_{AM}, Z_{AM}).

$$X_{AM}=\frac{1}{n}\sum_{i=1}^{n}\Delta X_i$$

$$Y_{AM}=\frac{1}{n}\sum_{i=1}^{n}\Delta Y_i \qquad (10)$$

$$Z_{AM}=\frac{1}{n}\sum_{i=1}^{n}\Delta Z_i$$

Geometric Mean (GM)

The GM translation parameters were estimated using Equation (11). Here, the product of the set of ΔX, ΔY and ΔZ values determined in Step 2 was calculated. To get the GM parameters (X_{GM}, Y_{GM}, Z_{GM}), the *n*th root was found for the product value of ΔX, ΔY and ΔZ set.

$$X_{GM}=\left(\prod_{i=1}^{n}\Delta X_i\right)^{\frac{1}{n}}$$

$$Y_{GM}=\left(\prod_{i=1}^{n}\Delta Y_i\right)^{\frac{1}{n}} \qquad (11)$$

$$Z_{GM}=\left(\prod_{i=1}^{n}\Delta Z_i\right)^{\frac{1}{n}}$$

Harmonic Mean (HM)

Equation (12) was used to compute the HM parameters (X_{HM}, Y_{HM}, Z_{HM}). In this approach, the reciprocal of the set of ΔX, ΔY and ΔZ computed values (Step 2) for each common point was carried out. This was then followed by dividing *n* by the sum of the reciprocal values.

$$X_{HM}=\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\Delta X_i}\right)^{-1}$$

$$Y_{HM}=\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\Delta Y_i}\right)^{-1} \qquad (12)$$

$$Z_{HM}=\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\Delta Z_i}\right)^{-1}$$

Quadratic Mean (QM)

The estimated ΔX, ΔY and ΔZ set values for each common point coordinate was squared. The average of the squared values was then estimated. Now, a square root of the averaged value was performed to get the quadratic mean translation parameters (X_{QM}, Y_{QM}, Z_{QM}).

$$X_{QM}=\left(\frac{1}{n}\sum_{i=1}^{n}\Delta X_i^2\right)^{\frac{1}{2}}$$

$$Y_{QM}=\left(\frac{1}{n}\sum_{i=1}^{n}\Delta Y_i^2\right)^{\frac{1}{2}} \qquad (13)$$

$$Z_{QM}=\left(\frac{1}{n}\sum_{i=1}^{n}\Delta Z_i^2\right)^{\frac{1}{2}}$$

Median (MD)

ΔX, ΔY and ΔZ (Step 2) of *n* points were each arranged in ascending order into three ranges such that

$$\Delta X = [\Delta X_1, \Delta X_2, ..., \Delta X_n],$$
$$\Delta Y = [\Delta Y_1, \Delta Y_2, ..., \Delta Y_n] \text{ and}$$
$$\Delta Z = [\Delta Z_1, \Delta Z_2, ..., \Delta Z_n].$$

The middle most number of the coordinate distribution for (X, Y, Z) in the three datums (WGS84, Accra and Leigon) was determined using Equations (14) and (15).

For even numbered *n*;

$$X_{MD} = \frac{\Delta X_k + \Delta X_{k+1}}{2}$$
$$Y_{MD} = \frac{\Delta Y_k + \Delta Y_{k+1}}{2} \quad \text{, where } k = \frac{n}{2} \qquad (14)$$
$$Z_{MD} = \frac{\Delta Z_k + \Delta Z_{k+1}}{2}$$

For odd numbered *n*;

$$X_{MD} = \Delta X_k$$
$$Y_{MD} = \Delta Y_k \quad \text{, where } k = \frac{n+1}{2} \qquad (15)$$
$$Z_{MD} = \Delta Z_k$$

Hence, the k^{th} number position of the coordinates in each datum was selected as the MD translation parameter (X_{MD}, Y_{MD}, Z_{MD}).

Step 4: The determined translation parameter values at Step 3 were then utilized to transform coordinates between WGS84 and the local datums (Accra and Leigon). Coordinate transformation was also performed between Accra and Leigon datum. The respective GTM used is given by Equation (16) where (ΔX_{tr}, ΔY_{tr}, ΔZ_{tr}) represent the translation parameters to be applied in the GTM. In order to transform $(X, Y, Z)_{WGS84}$ into Accra datum, (ΔX_{tr}, ΔY_{tr}, ΔZ_{tr}) (Equation (16)) was added to the $(X, Y, Z)_{WGS84}$. Similar procedure was adopted for the rest of the coordinate transformation process carried out between WGS84, Accra datum and Leigon datum respectively.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{Accra} = \begin{bmatrix} \Delta X_{tr} \\ \Delta Y_{tr} \\ \Delta Z_{tr} \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} . \qquad (16)$$

2.2.4. Assessment of Model Quality

In order to quantify the accuracy and reliability of the transformed coordinates by AM, QM, HM, GM and MD, mean absolute error (MAE), root mean square error (RMSE) and standard deviation (SD) were used. In addition, the horizontal position error (HD), mean horizontal position error (MHD), minimum and maximum horizontal errors were applied. The performance indicators (PI) are expressed in Equations (17) to (23).

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|t_i - o_i| \qquad (17)$$

$$SD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e-\bar{e})^2} \qquad (18)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(t_i - o_i)^2} \qquad (19)$$

$$HD = \sqrt{\Delta E^2 + \Delta N^2} \qquad (20)$$

$$MHD = \frac{1}{n}\sum_{i=1}^{n} HE_i \qquad (21)$$

$$Max\ Error = \max(t_i - o_i)_{i=1}^{n} \qquad (22)$$

$$Min\ Error = \min(t_i - o_i)_{i=1}^{n} \qquad (23)$$

Here *n* is the total number of coordinate points, *t* is the existing projected grid coordinates, and *o* is the projected grid coordinates results obtained by the various mean estimation procedures. ΔE and ΔN is the residual between the existing projected coordinates and the results produced by the mean estimation methods in Eastings and Northings respectively. Also, *e* represents the error, estimated as the difference between the measured and new projected grid coordinates while $\bar{e}$ is the mean of the error values.

3 Results and Discussion

The initial results obtained from each mean estimation technique were in (X, Y, Z) cartesian coordinates. However, in Ghana, the projected grid coordinate system is used for surveying and mapping purposes. Therefore, the obtained results were converted from cartesian coordinates into geodetic coordinates using Bowring inverse equation stated in Bowring (1976). These estimated geodetic coordinates were then projected onto the Transverse Mercator 1^0 NW using the relationship stated in Dzidefo (2011) to obtain the new projected grid coordinates. The obtained projected grid coordinates for the Accra and Leigon datum were then compared with the known projected coordinates for the test analysis. The test results for the various mean estimation techniques are presented in the following sections.

3.1 Coordinate transformation from WGS84 to Accra datum

The translation parameters produced by each mean estimation technique are shown in Table 4. The distribution of coordinate positions in the Accra datum is presented in Fig. 2.

Table 4 Derived Transformation Parameters from WGS84 to Accra Datum

Method	Parameter		
	ΔX (m)	**ΔY (m)**	**ΔZ (m)**
AM	196.653	-33.372	-322.292
MD	197.485	-33.133	-322.214
GM	48.891	0.680	40.787
HM	196.617	33.359	322.288
QM	196.674	33.379	322.294

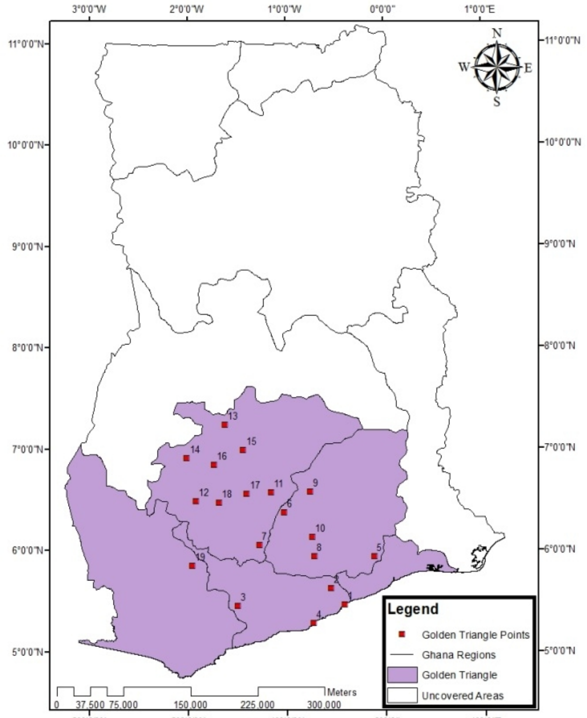

Fig. 2 Points Distribution in the War Office 1926 Reference Frame

Tables 5 and 6 present the statistical results obtained by each of the mean estimation techniques for Eastings and Northings.

Table 5 Model Performance Assessment for Eastings (WGS84 to Accra Datum)

PI	Method (m)				
	AM	**MD**	**GM**	**HM**	**QM**
MAE	0.588	0.561	66.984	66.922	66.998
RMSE	0.666	0.718	66.995	66.931	67.010
MHD	1.0572	1.0728	644.235	520.1752	644.236
Min	-1.108	-1.376	-69.098	-68.824	-69.113
Max	0.914	0.656	-68.824	-65.665	-65.488
SD	0.488	0.535	-69.113	1.272	1.374

Table 6 Model performance assessment for Northings (WGS84 to Accra datum)

PI	Method (m)				
	AM	**MD**	**GM**	**HM**	**QM**
MAE	0.779	0.779	640.742	515.851	640.742
RMSE	0.949	0.952	640.744	515.853	640.743
MHD	1.057	1.073	644.235	520.175	644.236
Min	-2.287	-2.285	-643.921	-518.848	-643.922
Max	1.6748	1.7016	-637.796	-513.153	-637.796
SD	0.488	0.5347	1.373	1.272	1.374

From Tables 5 and 6, it is clear that GM, HM and QM performed poorly in all the statistical analytical techniques considered. Hence, the discussion here will be centred on the AM and MD approaches. The MAE and RMSE results (Tables 5 and 6) of less than 1 m attained by AM and MD indicate the quality of their resulting transformed coordinate values. That is, the MAE signifies the average magnitude of how close the projected grid coordinates produced by AM and MD deviates from the existing data in absolute terms. The RMSE, on the other hand, measure the error rate of the AM and MD by given how much their results differ from the existing projected grid coordinates. The maximum and minimum errors (Tables 5 and 6), on the other hand, depict the highest and lowest range of error that could be produced when AM and MD are used within the study area. These maximum and minimum error values clearly demonstrate the level of uncertainty produced by the GTM model using AM and MD, respectively. A careful study of Tables 5 and 6 revealed that AM and MD methods achieved the least MHD of approximately 1.1 m while higher values were produced by GM, HM and QM methods. These MHD values reveal the extent by which AM and MD deviate averagely in terms of horizontal positional accuracy. It should be noted that the SD values (Tables 5 and 6) was computed from the HE results. These SDs show the precision of the transformed coordinates furnished by AM and MD.

Fig. 3 illustrates the horizontal positional variations for the results obtained for AM and MD.

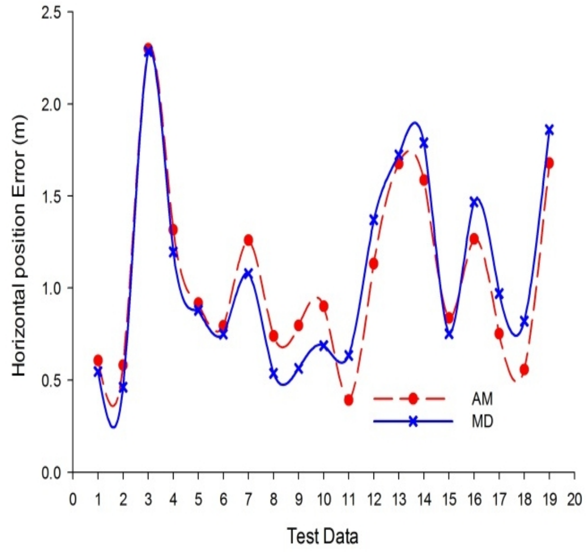

Fig. 3 Horizontal positional displacements (WGS84 to Accra datum)

With reference to Fig. 3, it is clearly demonstrated that, by virtue of the nineteen test data points applied, about 52.63 % of the MD transformed coordinates were in better agreement to the existing projected grid coordinates than the AM where only 47.37 % was realised. Nonetheless, both AM and MD obtained identical MHE values of approximately 1.1 m even though the HE for each pair of coordinates revealed slight variation. This means that the coordinate differences exhibited in the HE results by AM and MD do not have any significant impact. In line with this, it could be stated that both AM and MD could produce closely related results and thus, are better techniques to be used in GTM for transforming WGS84 coordinate to Accra datum.

3.2 Coordinate Transformation from WGS84 to Leigon Datum

Table 7 gives the translation parameters for the respective mean estimation methods. Fig. 4 shows the points distribution in the Leigon datum.

Table 7 Computed Transformation Parameters for the Direction from WGS84 to Leigon Datum

Method	Parameter		
	ΔX (m)	ΔY (m)	ΔZ (m)
AM	147.628	-29.052	-363.284
MD	147.587	-32.420	-363.568
GM	147.604	32.343	363.283
HM	147.579	32.331	363.282
QM	151.489	33.208	372.723

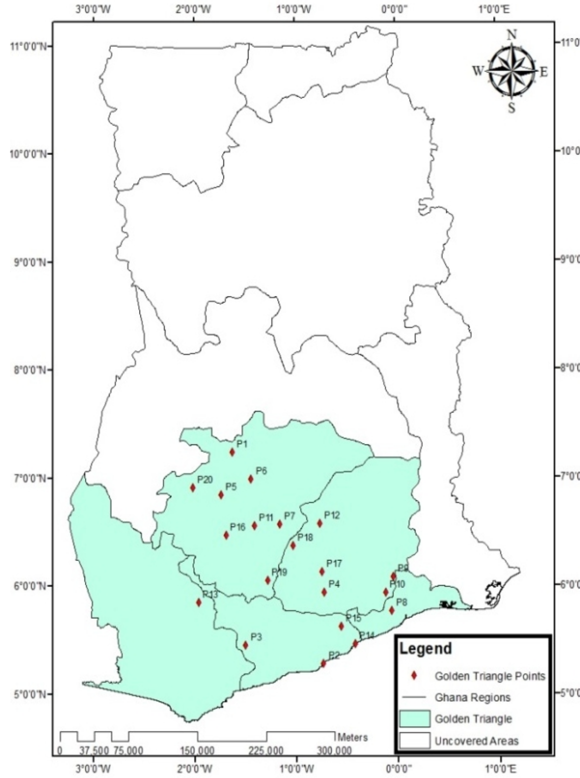

Fig. 4 Coordinates Distribution in Clark 1880 (modified) Reference Frame

Transforming WGS84 coordinates to the Leigon datum; it was observed that GM, HM and QM performed poorly (Tables 8 and 9). Hence, the discussion here will be made on the AM and MD methods.

Table 8 Model Assessment Results for Easting Coordinates (WGS84 to Leigon Datum)

PI	Method (m)				
	AM	MD	GM	HM	QM
MAE	3.240	0.733	64.793	64.780	65.734
RMSE	3.372	0.946	64.795	64.783	65.737
MHD	3.365	1.177	725.103	725.10	734.115
Min	-4.728	-1.361	-65.967	-65.954	-66.980
Max	-1.188	2.179	-63.545	-63.532	-64.532
SD	0.938	0.641	2315.383	2315.386	2313.306

Table 9 Model Assessment Results for Northing Coordinates (WGS84 to Leigon Datum)

PI	Method (m)				
	AM	MD	GM	HM	QM
MAE	0.741	0.750	722.202	722.204	731.166
RMSE	0.884	0.939	722.203	722.205	731.167
MHD	3.365	1.177	725.103	725.104	734.115
Min	-1.119	-0.836	-723.950	-723.952	-732.952
Max	1.774	2.058	-720.037	-720.039	-728.942
SD	0.938	0.641	2315.383	2315.386	2313.306

With regard to Tables 8 and 9, it was noticed that the AM achieved a minimum Easting and Northing error of approximately -4.73 m and -1.12 m. On the other hand, approximately -1.9 m and 1.8 m were gotten as the maximum Easting and Northing error. On the contrary, the MD produced more satisfactory results with approximately -1.36 m and -0.84 m as the minimum error in Easting and Northing respectively. A maximum coordinate shift of approximately 2.18 m (Easting) and 2.06 m (Northing) (Tables 8 and 9) were obtained by MD. These minimum and maximum values suggest the extent of the error variations that could be generated when AM and MD are applied within the study area. Hence, providing an insight into the practical strength of AM and MD, respectively.

The quantitative results for MAE and RMSE (Tables 8 and 9) also show the dominance of MD over the AM technique. In conformance with the results (Tables 8 and 9), it could be seen that the MD had the least SD compared to the AM. This was further corroborated by Fig. 5 which clearly displays the horizontal positional accuracies of the transformed coordinates given by MD and AM. Therefore, transformation from WGS 84 to Leigon datum, the MD is the appropriate approach to be used. This assertion has been made based on the analyses of the results presented in Tables 8 and 9 where a transformation accuracy of 0.641 m was produced by MD and 0.938 m achieved by AM.

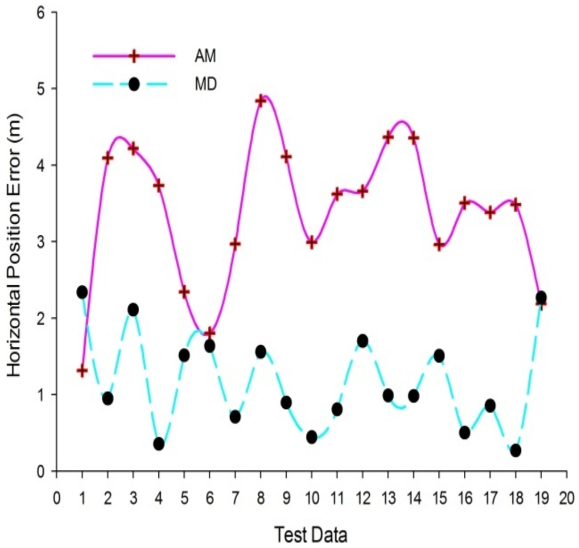

Fig. 5 Horizontal Positional Displacements (WGS84 to Leigon Datum)

3.3 Coordinate Transformation from Leigon Datum to Accra Datum

The translation parameters determined for the various mean estimation techniques are presented in Table 10.

Table 10 Computed Transformation Parameters from Legion to Accra Datum

Method	Parameter		
	ΔX (m)	ΔY (m)	ΔZ (m)
AM	48.891	-1.192	40.809
MD	48.861	-0.795	41.290
GM	48.891	0.680	40.787
HM	48.891	0.237	40.764
QM	48.892	1.763	40.830

Here, it was uncovered that the GM, HM, and QM methods that performed poorly in the transformation from WGS84 to the local datums (Accra and Leigon) gave satisfactory results when transforming coordinates from Leigon to Accra datum. Comparatively, AM and MD performed better than GM, HM and QM (Tables 11 and 12).

Table 11 Model Assessment Results for Easting Coordinates (Leigon to Accra Datum)

PI	Method (m)				
	AM	**MD**	**GM**	**HM**	**QM**
MAE	1.208	1.190	1.879	1.496	2.962
RMSE	1.302	1.364	2.286	1.939	3.236
MHD	1.682	1.671	2.336	2.040	3.248
Min	-2.432	-2.829	-4.302	-3.860	-5.385
Max	1.705	1.308	-0.166	0.276	-1.249
SD	0.8	1.046	1.262	1.189	1.324

Table 12 Model Assessment Results for Northing coordinates (Leigon to Accra Datum)

PI	Method (m)				
	AM	**MD**	**GM**	**HM**	**QM**
MAE	0.982	0.911	0.988	0.997	0.972
RMSE	1.319	1.404	1.319	1.320	1.319
MHD	1.682	1.671	2.336	2.040	3.248
Min	-3.928	-4.410	-3.909	-3.886	-3.954
Max	1.477	0.995	1.495	1.518	1.450
SD	0.8	1.046	1.262	1.189	1.324

With respect to the results in Tables 11 and 12, it could be stated that AM and MD gave better representative values that are in better agreement with existing dataset than GM, HM and QM, respectively. Fig. 6 intuitively confirms this assertion.

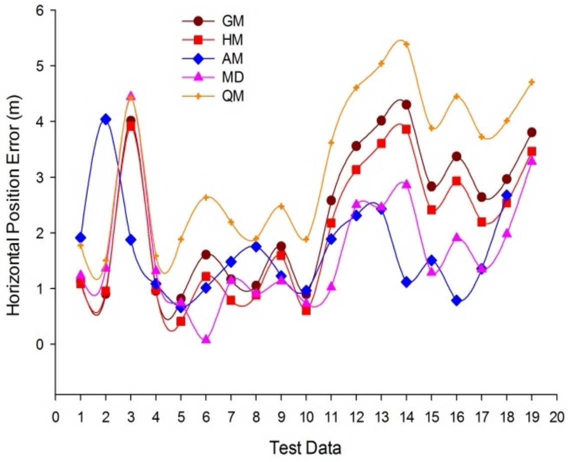

Fig. 6 Horizontal Positional Displacements (Leigon to Accra Datum)

3.4 Coordinate Transformation from Accra to Leigon Datum

The GTM is highly dependent on the direction of the transformation process. Consequently, there is the need to determine transformation parameters from Accra to Leigon datum (Table 13).

Table 13 Transformation Parameters in the Direction from Accra to Leigon Datum

Method	Parameter		
	ΔX (m)	ΔY (m)	ΔZ (m)
AM	-48.891	1.192	-40.809
MD	-48.861	0.795	-41.290
GM	48.891	0.680	40.787
HM	48.891	0.237	40.764
QM	48.892	1.763	40.830

An interesting observation made is that, GM, HM and QM performed poorly in this regard. The authors gathered after careful study of the transformation process that, there was quite a large margin of deviation in GM, HM and QM estimated (X, Y, Z) coordinates compared to AM and MD (X, Y, Z) values. Hence, contributing to such a higher deviation exhibited (Tables 14 and 15) between the existing and projected grid coordinates estimated for GM, HM and QM. Only the AM and MD produced favourable results (Tables 14 and 15).

Table 14 Model Assessment Results for Easting Coordinates (Accra to Leigon Datum)

PI	Method (m)				
	AM	MD	GM	HM	QM
MAE	1.205	1.186	1.562	1.122	2.645
RMSE	1.299	1.362	1.699	1.304	2.728
MHD	1.681	1.671	70.473	70.441	70.550
Min	-1.694	-1.297	-3.052	-2.610	-4.135
Max	2.447	2.844	-0.421	0.021	-1.504
SD	1.334	1.334	0.687	0.687	0.687

Table 15 Model Assessment Results for Northing Coordinates (Accra to Leigon Datum)

PI	Method (m)				
	AM	MD	GM	HM	QM
MAE	0.985	0.913	70.452	70.429	70.497
RMSE	1.319	1.404	70.467	70.443	70.512
MHD	1.680	1.671	70.473	70.441	70.550
Min	1.477	-0.996	-73.147	-73.124	-73.192
Max	3.927	4.409	-67.992	-67.969	-68.037
SD	1.355	1.356	1.464	1.464	1.464

Visual inspection of Fig. 7 shows that the AM positional accuracies were slightly better than MD. Nevertheless, both AM and MD could produce comparable results. However, making reference to Fig. 7 and the SD values, it is fair to state that AM had a little advantage over MD. Thus, AM is the most appropriate to be used to transform coordinate from Accra to the Leigon datum.

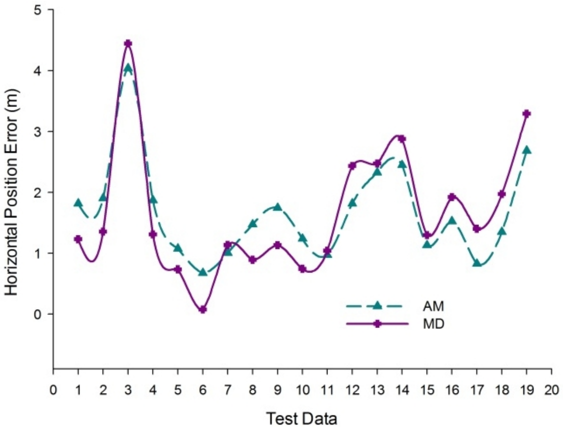

Fig. 7 Horizontal Positional Displacement (Accra to Leigon Datum)

4 Conclusions

The geocentric translation model is implemented in most Geographic Information System, handheld GPS receivers and surveying softwares for geospatial purposes. The results from this study have clearly shown that to implement geocentric translation model correctly and accurately, require the testing of all possible mean estimation methods. This will help the field practitioner to select the most appropriate method suitable for the area of survey.

The evidence from the dynamics in the results obtained by the various mean estimation methods confirmed the aforementioned assertion. That is, taken into consideration the mean horizontal positional error; approximately 1.06 m and 1.07 m were achieved by the arithmetic mean and median for WGS84 to Accra datum transformation. From WGS84 to Leigon datum, approximately 3.36 m and 1.18 m were gotten by arithmetic mean and median respectively. Corresponding values of approximately 1.68 m and 1.67 m were produced by the arithmetic mean and median when transforming coordinates based on the Leigon datum to Accra datum and vice versa. It was clearly noticed from the overall analysis of the results obtained in this study that, only the arithmetic mean and median could successfully produce satisfactory transformed coordinates results compared with geometric mean, harmonic mean and quadratic mean, respectively.

Moreover, this study has shown that for smaller areas like Ghana (study area) the arithmetic mean and median results could be used for less accuracy demanding survey and mapping works like recce, GIS data collection for geodatabase generation, small scale topographic mapping, and transforming thematic type data such as vegetation, soil type and geology.

Furthermore, the authors recommend that the Vaníček and Steeves (1996) concept could be adopted as part of the coordinate transformation process in developing countries with similar astro-geodetic datums like Ghana. This will further prevent the need to estimate the local ellipsoidal height in order to get 3D geodetic data for the local network when performing global to local datum transformation and vice versa. This is important because almost all the local geodetic networks do not have ellipsoidal heights and most researchers resort to the Abridged Molodensky approach which is computational intensive and difficult getting convergence for the ellipsoidal height correction values.

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