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A PhD THESIS ENTITLED

AN EXTENDED NONLINEAR ORDINARY DIFFERENTIAL EQUATION MODEL FOR UNEMPLOYMENT DYNAMICS ON GHANA'S LABOUR MARKET

BY

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THESIS SUPERVISORS

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DECLARATION

I declare that this thesis is my own work. It is being submitted for the award of Doctor of Philosophy in Mathematics in the University of Mines and Technology (UMaT), Tarkwa. It has not been submitted for any degree or examination in any other University.

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(Signature of Candidate)

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ABSTRACT

Globally, unemployment constitutes a major socio-economic concern and Ghana cannot escape its threats. The absence of jobs and its creation as always hunted by individuals who legitimately fall within the labour force actively searching for jobs worsens the menace rapidly in a burgeoning economy. Therefore, this study analysed unemployment dynamics on Ghana's labour market with three economic sectors using nonlinear ordinary differential equations model. Seven dynamic variables from the three sectors were introduced and used to develop the nonlinear ordinary differential equations model. Stability analysis of the system of differential equations was carried out to establish the characteristics of the system subject to time. The Routh-Hurwitz stability criterion was used to establish both the local and global asymptotic stability of the equilibrium point. SageMath and MatLab softwares were employed for computations, qualitative analyses, programming, simulation and graphing. Perturbation analysis of some parameters was performed to confirm their impacts on unemployment, employment and newly created vacancies at equilibrium. The analysis of the model confirmed the rates of movement of unemployed persons joining the employed in agriculture, industry and services sectors as inverse correlates of unemployment across all three sectors. The rate at which new vacancies are created in favour of the unemployed persons is an inverse determinant of the number of unemployed persons across all three economic sectors. The number of employments relates directly with the rate at which new vacancies are created in favour of the already employed persons. Sensitivity analysis was performed to measure the degree of responsiveness of unemployment and employment to variations of the model's parameters across the three sectors. The results of the sensitivity analysis confirmed that unemployment and newly created vacancies in all the three sectors is highly sensitive to variations in the rates of transitions of individuals from the unemployment class to the employment class in agriculture, industry and services sectors. Thus, unemployment and newly created vacancies are associated with higher elasticities than employment with respect to variations in these parameters across the three sectors. The implication is that a unit change in Ghana's economic growth rate will cause the rate of movement of unemployment to employment to increase with high positive margins. In this study, the incorporation of the three economic sectors into the model, the inclusion of the rate at which new vacancies are created in favour of the employed persons among others are relevant contributions to knowledge.

DEDICATION

To the Lord God Almighty, I dedicate this Work.

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CHAPTER 1

INTRODUCTION

1.1 Background to the Study

One of the most important socio-economic problems in every nation is unemployment (Pathan and Bhathawala, 2015). This phenomenon affects most developing countries in distinct ways and Ghana as such, lies within such criteria. The situation quickly worsens in a rising economy due to the lack of employment and the emergence of new ones as a result of people in the labour market actively searching for jobs. As it is known, Africa, mainly has a problem of unemployment; this is due to lack of means to the multiplication of sustainable jobs and also to equip the labour force properly (Kappel, 2021). Added to this, Africa faces a lack of access to productive and decent work opportunities. The state of unemployment is a global phenomenon where each year, working age forces mostly from the African continent leave their mother countries in search for greener pastures abroad (Anon., 2019b). Job seekers use available means of getting employed whilst the system provides fewer opportunities for them (Anon., 2019b). This phenomenon affects most developing countries in distinct ways and Ghana as such, lies within such criteria. Job search for the Ghanaian labour force over the years has become a national dilemma where the majority of the working age population struggle to secure permanent employment (Anon., 2016b). Unemployment and migration have become a global phenomenon. Since a larger section of the population now depends on income from work, it is essential to create jobs for those entering the labour market and improve the quality of existing ones (Anon., 2019a). Several governments have implemented employment policies that are intended to provide employment and improve the quality of employment, especially for those in precarious employment situations, in order to support the poor and unemployed in taking benefit of the opportunities to be productively employed and contribute their fair share to the goal of national development (Anon., 2019a). However, the COVID 19 pandemic brought about a very unique problem of job losses, redundancy among others thus, many job seekers are finding ways to go around it whilst maintaining their career ambitions on top (Anon., 2021b). Many job seekers who had high hopes for finding respectable employment and economic progress in the middle of the pandemic are now growing disillusioned.

With the continued economic turmoil, the ability to respond to the unemployment issue and the long-term effects it entails cannot be compromised (Brand, 2015). After completion of training (formal and informal), many working age populaces find it difficult securing employment (Anon., 2017b). On the contrary, reasons ranging from the labour skill required, capital available to start one's business, increasing number of graduates emanating from the various tertiary institutions and etc. have been assigned to contribute to the unemployment uncertainties (Anon., 2017b). Acquisition of knowledge through education, whether formal/informal, is an important facet of development and job creation. Within Ghana's population, the distribution of educational level to employment allocation cannot be undermined. However, there are 57.2% of persons with a basic level of education dominating the labour force while 25.2% of the working population are without education (Anon., 2014). Enrolment into the tertiary institution has experienced progress with the introduction of new programs, addition of institutions (both private and public) and population growth; it is obvious employment will also be affected. Accordingly, in 2017, a study by the Institute of Statistics, Social and Economic Research (ISSER) reported that only 10% of graduates from the tertiary institution are able to find jobs upon completion of their national service whilst huge numbers remain unemployed for a number of years, it might take a number of years before those unemployed even secure jobs (Anon., 2017b). The basis of ISSER's report underlined some of the causes of unemployment as poor attitude towards work, limited room for graduates to be absorbed by the industries, a lack of employable skills, and a shortage of finance capital for entrepreneurship (Anon., 2017b). New graduates do not have the component information technology (IT) skills in Africa to venture into job openings with the youth unemployment on the rise (Anon., 2019b).

Due to Ghana's labour force growth, there is a higher unemployment rate, which if not tackled well will affect the development of the state and its stability. Economic vectors, a wide range of demographic groups and other social attributes contribute to changes in the unemployment rate (Barnichon and Mesters, 2018). However, in the African continent, an estimated 77% of its population are younger than 35 years, which also are part of its workforce (Anon., 2019b). The continent's youthful population presents a powerful accelerator for economic growth and innovation whilst some countries face an ageing population subsequently affecting demand for skilled and qualified labour (Anon., 2003). There are a number of graduates who are disproportionately unemployed with the absence of jobs (Anon., 2019b). In light of the nation's expanding population and alarmingly high rates of unemployment, the Ghanaian government has acknowledged the need to give youth development priority (Anon., 2018). Though there have been some plans and policies to curb this developmental menace, unemployment issues continue to remain a serious social economic crisis.

The development of other significant technologies and innovations, population increase, the expansion of infrastructure, agriculture, and changes in economic performance and indicators have all had an impact on employment dynamics in the countries of sub-Saharan Africa (Bloom and Freeman, 1986). To address the issue of unemployment, successive governments in Ghana have laid down plans and policies to create employment opportunities as population dynamics escalate (Anon., 2016c). Again, sluggish employment responses to growth have been caused by the economy's limited sectoral job prospects (Aryeetey and Baah-Boateng, 2015). Every country has a specific agenda toward employment gains within the labour market whilst thriving on to implementing its set of policies which Ghana is not an exception. The United Nations Sustainable Development objectives eight, a worldwide call to action component of the 17 global objectives, were created in response to the need to set a global target on reducing the unemployment gap. These goals included the need to attain decent work and economic growth by 2030 (Anon., 2016e). Inclusion of equitable economic growth as part of the core component in sustaining development creating greater opportunity globally (Anon., 2018).

The strength of Ghana's economy depends on sector indicators for employment development and job creation through its workforce. Agriculture, industry, and services are the three sectors that make up Ghana's economy (Anon., 2015a). However, most of its economically active persons in the labour force are engaged in agriculture and services sectors (Anon., 2016a). In Ghana, there is a large disparity between the labour force and the jobs that are available, making employment and unemployment a severe economic crisis that must be addressed before it turns around (Anon., 2015a). The Ghanaian government in 2018 established the Nation Builders Corps programme (Anon., 2019a) which is an intervention system. Consequently, the need to examine the sector's robustness when tested will provide sustainable jobs for the labour force. This can be achieved by using appropriate techniques in finding out the optimal performing sector for employment and creating vacancies to be filled with job seekers. The Interventional system is for short term graduate employment into various public services as a means of controlling the rising menace of graduate unemployment. Other sectors and agencies have also been established to provide short term jobs to the informal populace adjusting the increasing unemployment levels (Anon., 2016c). As a result of the associated economic threats, this thesis intends to delve deeply into this phenomenon in a burgeoning country constrained with limited financial resources.

1.2 Statement of the Problem

In view of the fact that unemployment is a global socio-economic problem, there have been collaborative attempts by researchers, local and international organisations, among others in pursuit of reducing the unemployment canker if not to eradicate it. Goal 8 of the 17 United Nations goals was to foster comprehensive and sustainable economic progress, full and productive employment, and decent work for all by 2030. This goal was agreed upon by 193 countries in 2015 (Anon., 2016). Owing to this, there is the need for countries to work towards achieving their set targets. Albeit this global effort, there are a number of qualified job seekers and graduates who are disproportionately unemployed worldwide (Anon., 2019b). The threats of unemployment is evidently clear and intense in the region (Kazeem *et al.,* 2018). In Ghana, unemployment is consistently affecting the country's Gross Domestic product and it has become a serious socio-economic problem where both skilled and unskilled labour struggle to secure jobs, whilst a part of those already employed keep losing employment due to the deteriorating economic standings of specific sectors (Anon., 2017b). In Ghana, there is a significant disparity between the skilled and unskilled labour force and the jobs that are available, which makes unemployment a serious economic challenge that requires to be tackled (Anon., 2017b). This point is buttressed by the result of a study conducted in 2017 by the Institute of Statistics, Social and Economic Research (ISSER), which uncovered that it could take up to 10 years for many graduates to find employment (Anon., 2017b) after completing their national service.

The desire to control unemployment has given rise to multifaceted quality-driven global research with emphasis on model development. Gains have been made in the mathematical literature, with much focus on the application of system dynamic modelling techniques ((Misra and Singh, 2011; Kazeem *et al.*, 2018; Harding and Neamtu, 2016; Galindro and Torres, 2017) to study and model unemployment dynamics for effective control.

Notwithstanding this attempt, most of the applied techniques under the system dynamic modelling approach to study and control unemployment do not adequately reflect real-world dynamics of unemployment. To justify this claim, it is important to reflect on the broad generalisation of these unemployment models which neglect the crucial role of the economic sectors, thereby failing to reflect the sector's contribution to economic growth. In addition, not only do they fail to reflect the employment elasticities of economic indicators that play critical role in reducing unemployment, they do not also reflect other key determinants of employment opportunities.

Nonlinear ordinary differential equation (ODE) models have been applied to examine unemployment controls. This is significant as it allows a system to be analysed in a more precise way using theories and algorithms (Schichi, 2020). In Ghana, nonlinear ODE models have not been explored for unemployment control; also, the creation of new vacancies in the three main economic sectors (Anon., 2021b), has escaped the attention of researchers and policy makers.

ODEs were used by Misra and Singh (2011) to model unemployment while taking into account three variables, inclusively, the proportions of unemployed, temporarily employed, and regularly employed people. The differential equations' stability theory was the main focus of the model. Although, the results confirmed that the overall number of temporary vacancies is a direct correlate of the employment opportunities for the number of people who are temporarily employed, but an inverse determinant of the number of people without jobs; the model does not take into account additional factors affecting new government or private sector employment opportunities.

Misra and Singh (2013) applied ODE to analyse unemployment with a key objective to control it. Three dynamic variables were considered in the modelling process; the population of the: unemployed, employed, and new vacancies created based on the concepts of Misra and Singh (2011), and Nikolopoulos and Tzanetis (2003). The proposed model in this study did not consider the new avenues within the economic sectors which must be produced within the allowed time frame to make the unemployment controllable.

In a study, Addor *et al*. (2015) applied a system of linear ODEs to evaluate the robustness of the Job-finding, Job Loss (JFJL) model in modelling the unemployment and employment rates in Ghana. In this study, it was demonstrated that by varying the assumption of constant labour force to maintain its realistic characteristic, the long run equilibrium of the labour market remained unchanged. Although, predictions with the derived models confirmed the robustness of the JFJL model in predicting employment and unemployment in Ghana's labour market, the model did not integrate the main economic sectors of the Ghanaian economy together with the creation of vacancies across these sectors. Hence, the model does not reflect the true dynamics of employment and unemployment in Ghana.

Further, Pathan and Bhathawala (2015) applied a nonlinear ODE modelling method to model the effect of self-employment on the unemployment rate. The designed model concentrated on the following time-dependent variables: unemployed population, employed population and quantity of new vacancies created by the intervention of the government and non-government sector. The study did not present exhaustive representation of the unemployment dynamics since the role of vacancy creation was not evaluated with regard to the employed across all the economic sectors. To address this gab, Pathan and Bhathawala (2016) applied ODE to discuss unemployment considering the situation of job competition between native unemployed and new migration. Four dynamic variables; number of unemployed persons, number of new immigrants, the number of employed persons and newly created vacancies, were introduced. However, no attempt was made to establish employment elasticities for both native and immigrants.

Pathan and Bhathawala (2017a) applied ODE to nonlinear system dynamic models for unemployment control. This study analysed the state of job competition amid indigenous unemployed and fresh migrant workers. The work detected the impact of activeness in generating fresh vacancies provided by government and the non-government sectors with and without delay together with the efforts of indigenous unemployed and fresh migrant workers to gain self-employment. In another study relating to action-taking with regard to unemployment without delay, Pathan and Bhathawala (2017b) applied nonlinear ODE to model and study unemployment. The parameters considered include the unemployed population, employed population, quantity of present jobs in the market and the units of new vacancies created. The model established one non-negative point of equilibrium of the system in a test to confirm the stability. It emerged that unemployment can be reduced by improving more newly created vacancies and with higher rates of self-employment.

Pathan and Bhathawala (2017c) proposed a dynamic mathematical model to study and analyse unemployment with delay. Four variables for unemployment, namely: the employed population, quantity of current jobs in the market, units of generated new vacancies and the role of self-employment were considered in developing the model. The model only described the behaviour of the four variables; however, the authors did a generalised analysis on unemployment and vacancies without accounting for the role of sectoral contribution.

Most of the literature reviewed revealed that existing research works modelled unemployment in general without connecting it with the economic sector dynamics. This is a gap which needs to be filled. The argument here is that the proposed models do not epitomize the dynamic role sectoral contribution of the Ghanaian economy in promoting employment and reducing unemployment. Although they can provide eminent direction in modelling general unemployment, they do not come close to reflecting the economic sector specific opportunities with the labour force. Also, they do not relate the employed class to vacancy creation within the economic sectors and its role in reducing unemployment. Therefore, it is inadequate to apply them to model the unemployment dynamics of the labour market given the changing phases of unemployment crises in this recent unemployment epidemic.

This thesis aims to tackle these economic gaps in a burgeoning country under the restraints of inadequate financial resources to improve upon the previous works and existing models. These existing gaps constitute the key drive behind this research title, "An Extended Nonlinear Ordinary Differential Equation Model for Unemployment Dynamics on Ghana's Labour Market".

1.3 Objectives of the Thesis

This research seeks to:

i. Develop a nonlinear differential equation model to study the unemployment dynamics on the three main economic sectors of Ghana's labour market;
ii. Perform the stability analysis of the model; and
iii. Simulate the model to ascertain the effect of unemployment, employment and newly created vacancy on the three major economic sectors.

1.4 Methods Used for the Research

The methods used include:

i. Systems of nonlinear ordinary differential equations (ODEs) to develop the model
ii. Proof by Contradiction Method to establish the positivity of the system
iii. Linearisation technique using the Jacobian matrix in determining the local stability of the equilibrium points
iv. Routh's, and Routh-Hurwitz methods to confirm the global stability of the non-negative equilibrium points
v. Euler method in MATLAB for the simulation of the model.

1.5 Facilities Employed

The facilities employed in the execution of the thesis include:

i. Personal computer
ii. Both University of Mines and Technology (UMaT) and the University of Cape Coast Libraries
iii. Internet
iv. SageMath and Mathlab software

1.6 Organization of the Thesis

The thesis is organised into six chapters. Chapter 1 highlights the introduction of the thesis, including the background, statement of the problem, objectives, methods used, facilities employed and the organisation of the thesis. Chapter 2 is the literature review, which consists of a review of relevant studies on mathematical modelling of unemployment, youth unemployment globally, history of modern unemployment, employment and unemployment, forms of work, types of employment and Ghana's economic sectors. Chapter 3 deals with the theoretical background and discussion of some fundamental concepts used in this research. Chapter 4 consist of the development of the nonlinear unemployment dynamics model. It presents an improved or a new version of a nonlinear model for the unemployment model in Ghana and its underpinning assumptions. In Chapter 5, simulations, sensitivity analysis were performed and the results were discussed. Finally, Chapter 6 summarises the results, presents the contribution to knowledge, outlines the recommendations and conclusion.

CHAPTER 2

LITERATURE REVIEW

2.1 Background

In general, the literature review covers the following thematic areas: review of related literature on unemployment control using nonlinear ODE's, unemployment and employment in Ghana's economic sectors. Finally, the literature examines a very crucial concept in breaking down of unemployment and employment into economic sectors as agriculture, industry and services to understand how unemployment, employment and vacancy creation works at the various sector levels, which, to the best of my knowledge and as far as the literature is concerned, has not yet received any attention.

2.2 Review of Related Literature

The extensive unemployment situation in South Africa requires an in-depth examination, hence, Kingdon and Knight (2004) studied the unemployment situation in South Africa. The study focused on two key inquiries related to unemployment in the country. First, it investigates why the unemployed in South Africa don't typically engage in informal sector activities, a trend often observed in other developing nations. Second, it delves into the reasons behind the unemployed individuals' limited transition into wage-based employment. The findings of the study cast doubt on the notion that the unemployed deliberately opt for joblessness. In fact, the unemployed population experiences significantly worse conditions and lower life satisfaction compared to what they would experience if they were informally employed. Various obstacles that hinder entry into the informal sector contribute to the prevalence of open unemployment. The study makes the argument that the frequently reported high reservation wages (in comparison to projected wages) should not be mistaken for a lack of interest in working when examining the possibility that the unemployed have excessively high salary expectations. Albeit the study investigated into the behaviour of unemployment within the formal and informal sector, it was unclear how the unemployed reacts to specific economic sectors within the South African economy. This amounts to sector aggregations which misplaces the homogenous nature of labour force interaction which is sector specific. This makes it difficult to understand the comprehensive dynamics of unemployment relative to the rate of transitions of the unemployed across the different sectors of the economy for paid jobs.

However, the idea of transition of the unemployed into wage-based employment or the notion of the reluctance of individual unemployed to engage in the formal sector conveys the concept the existence of vacancies within the sectors. As a result, a useful insight was gained which guided this thesis.

Kingdon *et al.* (2006) studied labour markets and Sub-Saharan Africa's adaptability, salaries, and incomes. The article offered an overview of the performance of African labour market during the 1990s. The contention is that the inability of these markets to generate well-paying jobs has led to an oversupply of labour, manifesting as either open unemployment or a growing self-employment segment. Lack of "flexibility" in the labour market, which maintains formal sector wages higher than their equilibrium level and restrains job growth, is one reason for this result that might be put up. The flexibility of the labour market was broken down into three aspects. The first is the direction of actual wage trends over time, the second is the likelihood of earnings to adjust in reaction to unemployment, and the third is the severity of compensation disparities amongst various industries or businesses of various sizes. Recent studies reveal that actual salaries in Africa in the 1990s may have shown greater flexibility in the negative direction than originally thought and a surprisingly strong reactivity to unemployment rates. However, there are still large salary gaps between businesses in the legitimate and unofficial sectors. This third component of "inflexibility" can shed light on a factor that unites the various African economies: the widening income disparity between workers at larger companies and those working elsewhere. Similarities exist between those working in Ghana's thriving self-employment sector and job seekers in South Africa, both of which have significantly lower earnings prospects than those hired by larger businesses. Though the oversupply of labour, which induced to unemployment, was alluded to the inability of the unemployment market to produce jobs that pay high, the specific economic sectors of the economy were not explicitly captured in the studies, which still amounts to sector aggregation. This makes it difficult to relate the constrained vacancy creation resulting from unrealistic higher equilibrium sector wages to specific economic sectors such as agriculture, industry, and the service sector. Also, the study does not reflect the structural time-dependent transitional relationships between the generation of well-paid jobs, supply of labour, unemployment, vacancy creation sector incomes, thereby concealing the realistic dynamic representation of these variables. Thus, the study does not represent the complete dynamic variation of the labour market.

Misra and Singh (2011) present a valuable contribution to the understanding of unemployment dynamics by providing a mathematical model and analysing its properties using stability theory and numerical simulations. According to the data, there may be a link between a rise in regular job openings and a drop in temporary and unemployed jobs, while increasing the number of temporary vacancies can increase temporary employment and decrease unemployment, under certain conditions. The paper also highlights the importance of considering the rate of transfer from temporary to regular employment when trying to reduce unemployment. Specifically, the transfer rate should only be increased under a given condition, otherwise it may lead to further decline in unemployment. Overall, the paper provides useful insights into the dynamics of unemployment and emphasises the need for timely and appropriate policy measures to prevent unemployment from increasing rapidly and becoming harder to tackle. However, as with any mathematical model, there are limitations and assumptions that may not fully capture the complexity of real-world phenomena. Specifically, the key sectors of the economy in which vacancies are created through economic growth were not captured in the study, thereby reducing the complete dynamic representation of unemployment and employment in the labour market. Therefore, this current research will focus on improving the model by incorporating additional dynamic variables such as the key sectors of the Ghanaian economy as well as vacancy creation within these sectors.

During recent years, after the emergence of the a world financial emergency and the challenges associated with EU sovereign debts, there has been an upsurge in the unemployment rate and a decline in output growth. Notably, certain authors, including Smets, Wouters, and Galì *et al.* (2012), have incorporated the unemployment rate as an observable variable within Dynamic Stochastic General Equilibrium (DSGE) New Keynesian models. In the context of these models, elements such as proportional taxation, a trajectory of public debt, and productive public expenditure are introduced by Rollin (2015). The main goal of their research is to evaluate the ramifications of fiscal policy and to capture the significance of tax rates. Furthermore, with the help of search costs, this study tries to examine the effects of labour market flaws.. Taxation introduces distortions to the framework, and the introduction of a shock related to public expenditure yields positive effects, particularly on the unemployment rate and output. On the other hand, the absence of search costs exclusively influences demand shocks. The study presented the correlates of the unemployment rate and output growth, but contentiously, the interactions of unemployment and output growth was not linked to any specific key economic sector due to failure of the study to capture the contribution of sector expansion to output growth and unemployment reduction. Physical policies such as tax rates, public debts, productive public expenditure, are sector specific shocks which may place limitations on vacancy creation and induce labour market imperfections, therefore, excluding the sectoral contributions in the study does not present a complete realistic variational impact on employment, unemployment and labour force in general.

Wolf-Power (2013) explored the predictors of employment evolution and unemployment in the US central cities. The study examined employment expansion and unemployment during the period from 1990 to 2010 across a selection of cities, with a focus on practical strategies available to city governments for alleviating these issues. The study specifically examines the effects of educational attainment (both initial levels and changes over time) and public capital investment on job creation and changes in unemployment rates within 83 central American cities. A possible influence on improved job growth and decreased unemployment is suggested by patterns of educational achievement modifications through time. The implication drawn is that efforts aimed at attracting and retaining college-educated professionals, along with investments targeted at enhancing college attainment among current residents, could potentially lead to a decrease in joblessness and an improvement in societal well-being. The overall relationship between capital spending and the health of the labour market is not strong, despite some data suggesting that public capital expenditures helped create jobs and lower unemployment rates during the 1990s. Intergovernmental spending had an interesting effect on unemployment rates in 2009 and 2010 as a result of the American Recovery and Reinvestment Act (ARRA). It's significant to note that the analysis's two dependent variables, employment growth and unemployment rates, have only weak correlations, which emphasises the moderating effects of migration and labour force participation on the conversion of job creation into actual employment growth for those in the unemployed demographic. Although the predictors explored are useful in explaining the correlates of unemployment, it gives little or no idea with regard to job-creation within the main economic sectors since these sectoral variables were not considered. That notwithstanding, the study gives a reasonable enlightenment on the correlates of job-creation which can be translated into vacancies within the main sectors of the economy. This helps to understand the validate the dynamic behaviour of unemployment in this thesis.

Baah-Boateng (2013) conducted a study to explore the determinants of unemployment in Ghana. The research provides evidence indicating that, despite economic growth, employment expansion in Ghana lags behind due to the rapid expansion of sectors with low job creation potential in contrast to sluggish growth in sectors with high labour absorption capacity. Additionally, an analysis using a probit regression model across different sections demonstrates that demand-related factors significantly impact unemployment, revealing the limited job-generating effects of economic growth. Empirical examination further confirms that young individuals and urban residents are more susceptible combining education and gender to address unemployment being explanatory factors in certain cases. The concept of a reservation wage is also found to increasingly contribute to unemployment. In response, the study proposes policy recommendations that focus on stimulating investment in agriculture and manufacturing, as these sectors exhibit higher employment elasticity in relation to output. Addressing the elevated unemployment rates among youth and those with secondary education, calls for targeted interventions such as providing funding for entrepreneurial training and start-up capital to encourage young school graduates to become job "creators" rather than job "seekers." Additionally, a potential reduction in jobseekers' expectations regarding their reservation wage could potentially aid in alleviating unemployment challenges in Ghana. Although the study related unemployment to sector expansion in the event of low job creation, the application of a probit regression model limits the study in terms of the structural linkages among the sectors which does not reflect the gross flow of unemployed or unemployed persons across the various sectors. Regardless of this drawback, the study provides some degree of insights in respect of the effects of vacancy creation within the economic sectors on job creation, employment and unemployment.

A nonlinear delay mathematical model was proposed (Misra and Singh, 2013) to address the problem of unemployment and the job crisis. The model incorporates delay to account for the time-consuming process of creating new job opportunities. The analysis of the model showed that creating new jobs proportional to the number of unemployed individuals can lead to a decrease in unemployment. Additionally, the paper discovered that the stable equilibrium of the model may become unstable as delay increases beyond a critical value. The paper highlights the importance of creating new job opportunities in accordance with the number of unemployed individuals to effectively control unemployment. The findings also suggested that delay in creating new vacancies can destabilise the system, making the situation harder to control. The government must produce new jobs in proportion to the number of jobless individuals in the system, taking into consideration the influx of unemployed people, in order to solve the unemployment crisis. But, the process of creating new vacancies takes time, and given the population's ongoing growth, it's possible that the vacancies that are eventually generated will be different. As a result, new opportunities must be opened up as soon as possible because a longer wait time causes the system to become unstable. The study provides a given level of insights into the dynamic behaviour of unemployment/employment in the labour market. Contentiously, by neglecting the role of sectoral contribution to economic growth which drives new job opportunities or vacancy creation, the dynamic behaviour of new job openings as presented in the model lacks a complete dynamic representation. Thus, the study does not represent a complete real-world dynamic of unemployment in the labour market. Integrating the dynamic behaviour of sectoral variables in connection with vacancy creation within these sectors constitute the major drive behind this thesis.

Addor *et al*. (2015) applied a system of linear ODEs to evaluate the robustness of the Job-finding, Job Loss (JFJL) model in modelling the unemployment and employment rates in Ghana. The main focus was to assess the transition of the labour force in and out of employment within the Ghanaian context. In this study, it was demonstrated that by varying the assumption of constant labour force to maintain its realistic characteristic, the long run equilibrium of the labour market remained unchanged. The models for employment and unemployment were derived using the method of variation of parameters. By using data from the ministry of Employment and Labour Relations, the results were obtained via codes in Microsoft Excel. The predictions with the derived models for employment and unemployment confirmed the robustness of the JFJL model in predicting employment and unemployment in Ghana’s labour market. That notwithstanding, the study did not integrate the main economic sectors of the Ghanaian economy together with the creation of vacancies across these sectors into the model. In addition, formulating the model using a linear system of ODE cannot be a sufficient representation of a real-world transition between employment and unemployment. Also, the absence of the key economic sectors makes it difficult to establish a complete validation of the variation in the job-finding and job loss rates since they are influenced by economic growth due to expansion in the various sectors of the economy. Hence, the model does not reflect the complete dynamics of employment and unemployment in Ghana. However, the study provides a foundation based upon which other studies including this research can receive insightful direction. This insufficient representation of the dynamic behaviour of the labour market will be enhanced by integrating the key economic sectors of Ghana.

Pathan and Bhathawala (2015) presents a non-linear mathematical model for unemployment that takes into accounts the effect of self-employment. The model considers three dynamic variables: the number of unemployed persons, the number of employed persons, and the number of newly created vacancies by the government and private sector. The authors analysed the stability of the equilibrium point of the model and verified it with numerical data. They find that increasing the rate of newly created vacancies can lead to a decrease in unemployment, and that self-employment can also help to control unemployment. The study concludes that efforts by the government and private sector to create new vacancies, as well as individual efforts towards self-employment, can help to address the problem of unemployment.

Harding and Neamtu (2016) presented a dynamic model for reducing unemployment that takes into account the effects of ongoing migration. The model takes into account five variables at any given time: population of the unemployed, the population the employed, the population of new immigrants, the units of total jobs in the market, and the quantity of vacancies provided by government. It is based on a differential-typified nonlinear system with dispersed time delay. The paper analyses the unique positive equilibrium point in the presence of dispersed time delays to confirm the stability. It is found that without delay, the model is locally asymptotically stable about the equilibrium point subject to specific circumstances of the parameters. However, when delay is introduced, a Hopf bifurcation is observed for Dirac kernels, and the stable equilibrium assumes the state of instability as delay surpasses given critical values. The effect of migration on policy interventions is studied and it is observed that taking migration into account results in a more significant reduction in unemployment levels. Overall, the paper provides a comprehensive model for reducing unemployment that takes into account the effects of migration and time delays. The findings have important implications for policymakers who seek to address unemployment in the context of ongoing migration. Nonetheless, the dynamic role of vacancy creation cannot be completely confirmed in the absence of the key sectors of the economy since the expansions in these key sectors propel economic growth which drives vacancy creation. Hence, the study does not represent a complete realistic dynamic variation of employment/unemployment in the labour market. This concern will be addressed in this present study by integrating the key economic sectors into the unemployment/employment model to enhance the dynamic variation of the same in the labour market.

A nonlinear unemployment mathematical model is proposed and examined by Pathan and Bhathawala (2016) utilising four time-dependent variables: the unemployed population, new immigrant population, employed population, quantity of new jobs generated. The equilibrium point was discovered to be locally asymptotically stable. According to the study, the rate of unemployment decreases as the rate of transfer from unemployment to the unemployment class increases. Additionally, it was found that if the number of people working for themselves increases and if the number of new immigrants who are unemployed rises, unemployment will also rise. It was argued that in order to control unemployment, the government and private sector should create new jobs in proportion to the population of unemployed natives in addition to the population of unemployed immigrants. To minimise unemployment to some extent, new vacancies should be generated in proportion to both the unemployed and migration. The model also implies that as the rate of self-employment rises, so does the unemployment rate for both native and immigrant unemployed people.

Pathan and Bhathawala (2017a) presented a mathematics-based model to study unemployment variations and control. The employed population, population of unemployed individuals, the self-employed population, and the quantity of new vacancies provided by the public and private sectors are the model's four dynamic variables. To examine the state of stability of the point of equilibrium, in addition to the influence of different variables on unemployment rates, the authors conducted a numerical study. According to the outcomes of their numerical simulation, a rise in the quantity of new jobs provided by the public and private sectors causes the unemployment rate to fall. Additionally, they discover that a rise in the percentage of unemployed people who work for themselves causes the unemployment rate to fall. The authors also investigate how immigration affects unemployment rates and discover that a rise in the proportion of newly unemployed immigrants causes a rise in the general unemployment rate. The authors conclude that their mathematical model can be useful for policymakers in designing and implementing effective strategies for controlling unemployment. It was suggested that policymakers should focus on creating new vacancies and promoting self-employment among the unemployed individuals to bring down the rate of unemployment.

Pathan and Bhathawala (2017b) used four dynamic variables—the population of the jobless, the quantity of jobs currently available on the market, the employed population, and the quantity of new vacancies created in a nonlinear ODE-based model to study the unemployment dynamic. The research demonstrated that the model is locally asymptotically stable around the points of equilibria. Through mathematical manipulation, it was determined that reducing the jobless class, requires higher rate of transfer of the unemployment to employment class. As a result, both the public and private sectors should work to increase the number of new job openings. According to the findings, if a person's current employment fluctuates, then their self-employment rate should be higher in order to lower unemployment. To reduce unemployment, it is important to increase the number of unemployed people who become employed and, if current work is unstable, the number of people who are self-employed. The proximity of the jobless person to time suggests that self-employment rates should be high to reduce unemployment.

Another mathematical model was studied by Pathan and Bhathawala (2017c). ODE is used in unemployment dynamics modelling to examine the steadiness of the equilibrium point in the presence and absence of delay. The model took into account a number of time-dependent variables: the jobless population, the quantity of jobs presently available on the market, the population of the employed, and new openings created. The paper discovered that without any restrictions, the point of equilibrium is locally asymptotically steady without delay. This indicates that by raising the rate of self-employment and creating more openings, the unemployment rate can be reduced. Furthermore, the variation of jobs presently in the market can also have a positive or negative effect on unemployment, contingent on its upward and downward variations. However, in the presence of delay, the steady state of the point of equilibrium is subject to certain conditions, then the equilibrium point is stable for small delays and unstable for large delays. This suggests that controlling unemployment is more difficult in the presence of delay, and that reducing delays in the creation of new vacancies could be an effective strategy for reducing unemployment. Their simulations showed that high rates of self-employment can lead to a limited reduction in unemployment, and that significant effort from both and private and government sectors is necessary to generate fresh vacancies and reduce unemployment.

The dynamic variables used in the model in all the study of unemployment conducted by Pathan and Bhathawala (2015; 2016; 2017a; 2017b; 2017c) make the model less representative of the complete dynamic behaviour of unemployment in the labour market as a fact is reiterated that increasing the variables without integrating the dynamics of the sectoral variables does not in any way cure the problem of incomplete dynamic representation . For instance, other dynamic variables such as the key economic sectors within which vacancy occurs due economic growth emanating from physical expansion within these sectors are needed to establish a comprehensive conclusion on the dynamic variation of vacancy creation. The thesis therefore seeks to address the issue of unrealistic representation of unemployment/employment in the labour market of Ghana.

Galindro and Torres (2017) introduced a straightforward mathematical framework to describe unemployment. Regardless of its simplicity, it was established that the model was associated with greater realism and utility compared to recent models in the existing literature. The assertion was supported by a case study involving actual data from Portugal. Furthermore, the authors formulated and addressed an optimal control problem, which results were used to discuss various policy options to eliminate unemployment. Although the model presented a more realistic unemployment situation of the Portuguese economy, the role of economic sectoral contribution in reducing unemployment was missing, making it difficult to accurately reflect on the role of vacancy creation. Arguably, vacancies are created within sectors, and given the heterogenous nature of vacancy across the various sectors, the role of vacancies across the different sectors is aggregated due to inexplicit representation of the sectors. This aggregation confers homogeneity on vacancy across the myriad key economic sectors leading to unrealistic representation of unemployment. Nonetheless, the study offered a significant direction in respect of the dynamic behaviour of the various variables around the equilibrium point, which partly provided a basis to validating the dynamic behaviour of the variables employed in this thesis.

Misra *et al.* (2017) researched into a nonlinear mathematical model for studying and analysing the role of skill advancement in controlling unemployment. Three distinct classes were examined in the modelling process: jobless, self/temporary employed, and regularly employed. A model was developed to evaluate the impact of skill development avenues on unemployment control by taking into account four time-dependent variables: unemployed population, temporary/self-employed people, reliable employed people, and skill development avenues for unemployed people. It was assumed that at any given time, the avenues for skill development programmes are constant. Also, persons enter the state of the unemployed at a constant rate and transits from this state to others for employment.

The rate at which the unemployed migrate is considered proportionate to their population. Also, it was held that population of the employed will exit their employment to be unemployed. It was considered that some academic institutions build skills among youngsters to help them find work; nevertheless, the opportunities for skills development midst the unemployed grow with an increase in the population the unemployed. It is assumed that the transition rate of the unemployed to the self/temporarily employed class grows as the opportunity to develop skills expand. The system model was examined using differential equation theory of stability analysis. It was discovered that the proposed model only has one point of equilibrium whose stability was locally asymptotic without any condition and stable globally subject to specific conditions. The study demonstrated that when the potential of developing skills channels grows, the unemployed population reduces while the temporary/self-employed population surges. It was also revealed that developing skill amidst the unemployed is essential for reducing the unemployed population, and that these pathways need to be developed in proportion to the unemployed population. Arguably, inherent in the assumptions depicts vacancy creation, whose absence from the model does not reflect the complete transition of the unemployed to the employed group as this transition is engineered by vacancy creation through incapacitation, retirement, migration and death. Also, increasing the number of dynamic variables without the variable key sectors of the economy cannot be a cure to incomplete dynamic representation. However, the modelling analytic techniques provided a useful guide in formulating and analysing the seven-state model in this current research.

Morén and Elias Wändal (2018) conducted a global study on trends and determinants for the years spanning the period 2000 – 2017. The study undertook a global assessment by calculating the employment elasticity of economic growth for 168 countries. The term "employment elasticity" denotes the percentage variation in employment induced by a 1% variation in GDP, thus indicating the level of labour-intensive growth. To examine patterns among diverse demographic categories, the elasticity is computed for the entire population of each country, as well as its subgroups including adults, youth, females, males, female youth, male youth, female adults, and male adults. These outcomes are subsequently scrutinized on national, regional, and global scales, with contrasts made between burgeoning and advanced nations. Moreover, an econometric model is employed to identify potential factors influencing the employment elasticity metric. The findings exhibit considerable divergence among countries, with the highest and lowest recorded elasticities being 2.61 and -0.32, respectively. At a regional level, the most employment-intensive growth is observed in the Caribbean, Central America, and Southern Europe. Developing nations display higher elasticity in comparison to developed ones, while gender differences are more pronounced in developed countries. Across most regions, the greatest elasticity measure is found among female adults, followed by adults. Factors like labour force growth, the proportion of total employment in the service and industry sectors, foreign direct investment (FDI), and trade exhibit an influence on the employment elasticity measure, at least for specific demographic subsets. The study gives some insightful ideas on employment elasticities which guided in the computations and validation of the elasticities in this study, that notwithstanding, it failed to reflect the key drivers of economic growth since the little was said about the main contributing sectors of the economy of the countries involved.

A mathematical model was formulated and analysed by Kazeem *et al*. (2018) to establish threshold parameter to control unemployment in the society. Recognizing that understanding one's adversary prior to conflict yields a 50 percent chance of success, this study aimed to shed light on the intricate dynamics of unemployment, considering retirement and potential control strategies. The research objectives encompassed formulating a mathematical model grounded in deterministic modeling and mathematical epidemiology, followed by in-depth model analysis. This analysis encompassed a numerical semi-analytical approach to validate the accuracy of analytical solutions. The model exhibited mathematical coherence and biological significance. Two equilibrium points were identified. A decisive threshold for recruitment from the pool of unemployment, crucial for triumphing over unemployment, was also derived. Effectively managing this threshold emerged as a critical aspect in combatting the societal issue of unemployment. The Variational Iterative Method was applied as a numerical semi-analytical approach to offer solution to the model. The resultant approximated results provided realistic and comprehensive insights that aligned with the analytical outcomes and validated the model's assumptions. The study concluded by emphasizing three important notions: the collective responsibility of all individuals to combat this societal challenge, the plea for government and policymakers to formulate supportive policies encouraging population growth, and the necessity for retirees to plan for life post-service, as excessive reliance on pension schemes can lead to hardships due to corruption within the system. The study gives some reasonable level of comprehensiveness into the dynamic behaviour of the employed and unemployed in the labour market as far as certain scope of this thesis comes to play; however, a lot of insight is missing as long as the sectoral contribution of the economy, which drives economic growth, is concerned. Albeit vacancy was introduced as a dynamic variable, its contribution cannot be completely understood without the its main driver, which is economic growth induced by physical sectoral expansions. This makes the study to lack the exhaustive dynamic variation of unemployment/employment in the labour market.

Al-Maalwi *et al. (*2018) applied a system of nonlinear mathematical model to study unemployment. Their study simulates unemployment in poor countries using a nonlinear ODE-based model, considering three variables: unemployment, employees, and available vacancies. They relied on the absence of government resources as a potential correlate of unemployment. The unemployed population, the population of the employed, and the quantity of open positions were the three dynamic variables in the study's basic mathematical model. The theory of stability analysis of nonlinear systems was used to analyse the model. The two points of equilibria were obtained: an employment-free and positive points of equilibria. The analysis method is adapted from mathematical epidemiology to compute the basic reproductive number and investigate its effect on stability state of the equilibria. The first equilibrium is known as the employment-free equilibrium point. Its presence, which is dependent on the basic reproductive number R_0, is regarded as a menace to society. It was discovered that it is both LAS and GAS (locally and globally asymptotically stable) subject to the condition that $R_0 < 1$. The second stationary point is fascinating and is known as positive equilibrium. It was evidenced that it was both LAS and GAS when $R_0 > 1$. Moreover, it was revealed that the model exhibits Transcritical Bifurcation at $R_0 = 1$. Numerical experimentations were performed with regard to the positive point of stationarity were to validate the theoretical assertions. The numerical simulations significantly validated the GAS behaviour of the model about its positive point stationarity. Further roles of changing several of such elements as the rate of: employment, diminution, and the fundamental reproductive number in arresting the unemployment phenomenon was numerically examined. The findings imply that, in order to successfully address the unemployment issue, it is necessary for the government to establish and generate plenty job possibilities, thus boosting the basic reproductive number by accelerating upward variation in the employment rate and decelerating its rate of reduction. Although vacancy creation was one of the dynamic variables, the absence of the key economic sectors makes if difficult to understand the impact of growth due to sectoral expansion on the creation of vacancies. Irrespective of the fact that only three dynamic variables make the model less representative of the employment and unemployment dynamic in the labour market, the results are useful for the purpose of analysis, numerical simulation and validation of the results in this thesis.

A mathematical model was formulated to analyse labour force evolution (El Fadily and Kaddar, 2019). The study introduced a delayed differential system as a representation of the evolution of the labour force, encompassing both occupied labour force and unemployed individuals. The mathematical examination of our model primarily centers on the behavior of the labour force in the vicinity of a positive equilibrium state. We further investigate the presence of a series of periodic solutions emerging from this positive equilibrium, employing the Hopf bifurcation theorem. As a concluding step, we conducted numerical simulations to visually depict the outcomes of our theoretical findings. The dynamics of unemployment and employment represented using a two-state nonlinear system of ODE and the stability analysis are useful to comprehend the transitional dynamics of labour force in and out of employment. However, the model provided the basic two-state employment/unemployment models which does not exhaust the complete dynamic characteristics of labour force in the labour market. As a result, the study cannot not relate the variational characteristic of labour force (both those in active employment and those out of employment) to economic growth which can be boosted by the physical expansion of the economic sectors. Hence the explicit role of economic sectoral contribution in job or vacancy creation cannot be explained.

Guerrini *et al*. (2020) explored a model of economic growth featuring a dispersed time-delay function for investment, wherein the time-delay indicator corresponds to the average time delay represented by a gamma distribution. Employing the linear chain trick technique, the delay differential equation system was transformed into an equivalent set of ODEs. Given the consideration of both weak and strong kernels, the system was simplified into a three-dimensional ODE system for weak kernels and a four-dimensional ODE system for strong kernels. The potential occurrence of Hopf bifurcation by focusing on two crucial indicators: the time-delay indicator and the growth rate parameter were investigated. The analysis presents sufficient conditions regarding the existence and stability of a limit cycle solution during the Hopf bifurcation concerning the time-delay parameter. Through numerical exploration involving the Dana and Malgrange investment function, the study revealed the emergence of two Hopf bifurcations in relation to the rate of growth parameter. Remarkably, in this scenario, the presence of stable long-period cycles within the economic system was identified. Interestingly, the admissible range of values for the rate of growth parameter was divided into three distinct intervals based on the time-delay and adjustment speed parameters. The first interval was characterized by a stable focus, followed by the limit cycle, and eventually leading back to a stable solution marked by two Hopf bifurcations. Such behaviour is particularly pronounced within a specific intermediate range of the admissible rate of growth parameter values. What raises the contention in this study rests in the fact that economic growth is driven by sectoral expansion of the economy, however, the roles of specific key economic sectors were ignored in the study. Although products and capital stocks were modelled as correlates of investment, a determinant of growth, they cannot be related to any specific sector. This suggests that the key economic sectors such as agriculture, industry and service were considered in aggregation. The relevance of this study under review to this thesis is that it models economic growth which is a key driver of vacancy creation, which opens up job or employment opportunities in the various sectors of the economy to reduce unemployment. Hence, some of the variables play significant role in the validation process of this thesis.

Kaslik *et al.* (2021) proposed a mathematical model comprising five dimensions, aimed at analyzing the dynamics of the labour market. This model focuses on variables such as unemployment, migration, fixed-term contractors, full-time employment, and the count of available job openings. The model takes into account distributed time delay in the rate of change of available vacancies, which is influenced by past levels of regular employment. The model is presented in non-dimensional form, and the existence of equilibrium points is examined. The article also establishes the positivity and boundedness of solutions and provides global asymptotic stability analyses for both the equilibrium point representing absence of employment and the positive equilibrium point. The theoretical findings are supported by numerical simulations. Regardless of the fact that the study extensively introduced five dynamic variables, the dynamic incompleteness emanates from the absence of important dynamic variables, namely the main economic sectors of the economy, thereby making it difficult to explicitly and exhaustively confirm the role of vacancy creation which sterns from economic growth due to sectoral expansion of the economy. However, the nonlinear and non-dimensional ODEs presented constituted effective modelling and analytical tool, which guided in the model formulation, analysis, interpretation of the dynamic equilibrium, and the validation of the results in this present study.

Al-Sheikh *et al.* (2021) in their study, suggested and formulated a non-linear mathematical model of the unemployment problem to analyse unemployment with the effect of limited jobs, taking into account three major variables: the number of unemployed people, the number of employed people, and the number available vacancies. They investigated the impact of government assistance in lowering the unemployment rate, assuming that the rate of job creation by the government is proportionate to the number of unemployed persons due to limited financial and economic resources. The unemployment model has just one positive equilibrium points, which was both LAS and GAS. In the model, the restraint of producing fresh jobs at an unvaried rate when the unemployed solution exceeds a certain level has an unsatisfactory influence on the unemployment problem. The findings indicate that the problem will worsen by the growth in the number of unemployed people annually and will turn into a disaster threatening the society. To prevent an increase in the number of unemployed people, the maximum ability to create more jobs must be higher than or equal to the equilibrium point of unemployment. The point of contention in this review is that only three dynamic variables cannot accurately represent the dynamics of unemployment in the labour market. In addition, the need for government role in reducing unemployment is initiated at the various key economic sectors; hence, neglecting these key economic sectors makes it uneasy to establish the dynamic behaviour of vacancy within these sectors. This makes it difficult to formulate sector-specific policies to effectively address the issue of unemployment. That notwithstanding, the findings and analysis provided great assistance in understanding important aspects of the unemployment dynamics which became useful in executing this current research.

Petaratip and Niamsup (2021) studied and analysed an unemployment model using differential equations with time delay, taking into account the role of government in promoting job creation. In this study, Munoli and Gani (2015) unemployment model was improved with new assumptions. Their model takes into account three variables: the number of unemployed, employed, and vacancies. It was assumed that workers are fired and may become unemployed, migrate to other countries, or become business owners under some additional assumptions. Also, vacancies occur as a result of retirement, and that some job roles are retained and not terminated. Furthermore, it was assumed that the newest openings are caused by government policies and the government leverages previously obtained unemployment data to determine new openings. New vacancies are formed at a rate proportional to the number of unemployed people, and there is a time delay in establishing new vacancies. The unemployment model was presented using delay differential equations. The model system's dynamic behaviour was evaluated and a stability study was performed. Conditions for the absence of delay-induced instability and the local asymptotic stability of positive equilibrium points were derived. It was demonstrated that the model contains three feasible positive equilibrium points. Following that, existence of the local and global stability of all positive equilibrium points were confirmed. The requirements in the absence of delay-induced instability were established. The investigation has revealed that the phenomenon of Hopf-bifurcation cannot exist under the conditions specified in these hypotheses. It was revealed that when the rate of establishing new vacancies or the employment rate increases, so does the number of unemployed people. Their observation indicates that a government's approach of creating new openings and speeding up the matching process between unemployed people and entrepreneurs has an influence on the number of unemployed people. Although, government intervention is useful in job openings or vacancy creation, it will be difficult to comprehend how government can initiate varying policies within the different sectors as theses sector variables were not covered in the study. This can be explained by the fact that vacancy is dynamically variant in each sector and it correlates positively with economic growth originating from sectoral expansion. Hence, the study does not represent the complete dynamic behaviour of unemployment/employment in the labour market. That albeit, the inclusion of vacancies, the modelling approach, the computation of equilibria and stability analysis provided a useful direction in validating the results in this thesis.

El Fadily and Kaddar (2022) using delay differential equations, they investigate the problem of unemployment in a study titled "a delayed model of unemployment with a general nonlinear recruitment rate". The goal was to assess the effect of the delay and recruitment function on the stability of the equilibrium. A general type nonlinear function was represented. The existence and uniqueness of the endemic equilibrium, as well as the global stability of the disease-free equilibrium were demonstrated using the Lyapunov-LaSalle invariance concept. First, they show that positive equilibrium exists and is unique. It was discovered in the study that the time lag between the generation of vacancies and their occupation had no effect on the stability of the unemployment model. Albeit the fact that the concepts of existence and uniqueness of the equilibrium point as well as the computation of the same gave insightful directions that can be appreciated in this thesis, the absence of the key sectors of the economy makes the study lacks the complete dynamic variation of the unemployment/employment dynamic in the labour market.

El Fadily and Kaddar (2023) studied the global asymptotic stability of a nonlinear unemployment model to establish the global asymptotic stability of an unemployment model employing geometric technique. The matching procedure between vacancies and unemployed people causes the non-linearity. As a result, the employment rate is a general nonlinear function, including the bilinear form reported in prior scientific research. The basic assumption was that matching between vacancies increases as the number of unemployed increases, it cancels if there are no unemployed people. Conditions were set to ensure the existence and uniqueness of a positive equilibrium. El The application of the geometric technique assured global asymptotic stability without the requirement for an additional condition. The existence and stability of positive equilibrium of the system were determined. Using Lyapunov's approach it was demonstrated that if $R_0 \leq 1$, the unemployment-free equilibrium P_0 is globally asymptotically stable, implying that the chomage cannot persist in the population and that the situation is under control. Furthermore, if $R_0 > 1$, the employment-free equilibrium becomes unstable, and the model exhibits a positive equilibrium. This result indicates that, if unemployment exists initially, it will persist at the unique positive equilibrium level. It is however eminent to note that the dynamic behavior of vacancy is sector-specific and it is also generated by economic growth which correlates positively with physical expansion of the various sectors of the economy; hence, its absence in an unemployment/employment model amount to hiding important real-world dynamic representation. However, the qualitative analysis and the computation of the equilibrium point provided some appreciable level of guide in analyzing and validating the results in this thesis.

In summary, these existing models have made great attempt to model the unemployment dynamics; however, these dynamics do not reflect the contribution of the key economic sectors in promoting and sustaining employment while reducing unemployment through the creation of vacancies, which in general, need to be addressed. This gap will be filled via an ODE-based compartmental modelling approach by integrating the role of the key economic sectors of Ghana.

2.3 Unemployment Overview

Africa is a big continent with great diversity, home to over 54 different countries (Anon., 2011). Making generalisation about the type and pattern of growth and development in Africa is obviously problematic (Anon., 2011). The root of Africa's unemployment issues is their failure to diversify their economies and promote meaningful work for a fast growing labour population, notwithstanding the critical need for it for both human development and economic prosperity (Anon., 2011). The number of unemployed persons in Africa in 2019 was over 34 million, 12.2 million of whom were young adults between the ages of 15 and 24 (Anon., 2020). In comparison to 2010, there were 6.4 million more young people without work, or around 1.5 million young people. As evidenced by the continent's regional rate of unemployment of 6.8%, which was considerably greater than the global mean of 5.0%, the International Labour Organisation claims that unemployment is a key labour market concern in Africa (Anon., 2020). Over 34 million people in Africa were unemployed in 2019. They numbered 12.2 million young people within the ages of 15 and 24 (Anon., 2020).

As a result, there is increased enthusiasm from African countries to develop sectoral employment programs with integrated policy frameworks designed to create jobs and have more equitable economic routes. Sustainable development in Africa depends on effective structural reforms (Anon., 2011). The creation of wage employment must be the main objective of any long-standing plan to improve working and residing circumstances for the labour force, even while self-employment contributes an important part in safeguarding the livelihood of a large number of employees in the near term. According to one argument, employment targeting and sectoral targeting are interrelated (Anon., 2011). Therefore, in order to decide which sectors to prioritize for policy interventions, the next generation of African development strategies must consider creating jobs and labour force size as important considerations. It is crucial to keep in mind that every country's economy identifies significant sectors as those that produce a portion of value added, present job possibilities, have strong connections to the national economy, and have large multiplier impacts (Anon., 2011). In order to ease the process of productive transformation and the building up of domestic capabilities, it is important to emphasize the crucial role that labour-intensive public infrastructure investments, human development, and skill-building initiatives play. Even though there has been a minor decline in agriculture's employment share from 61.5% in 2000-2006 to 59% in 2007-2009, it still provides a significant portion of the workforce in Sub-Saharan African (SSA) nations' income (Anon., 2011). In the past, the agricultural industry has been severely neglected. Africa is the only continent where food output per person has decreased over the past 30 years (Anon., 2011). In contrast to agriculture, external aid has favored health and education (Anon., 2011). The employment proportion going to industry has increased marginally from 9.4% to 10.6% and the employment share going to services has increased from 29.2% to 30.4% as proof that some workers have moved into these sectors (Anon., 2011). Agriculture provides substantially less employment in North Africa than other sectors, and its importance has gradually decreased over the past decade, from 28.7% in 2000–2006 to 28.0% in 2007–2009 (Anon., 2011). In contrast to SSA, the employment share of industry has increased significantly, while that of service has slightly decreased (Anon., 2011). According to the World Bank's enterprise survey, 46% of private sector enterprises in the Sub-Saharan Africa region view a lack of access to financing as a major barrier to their ability to do business and create jobs (Anon., 2011). This is the highest percentage in the world (Anon., 2011).

North Africa had the unpleasant distinction of having the highest unemployment rate in the world in 2000, registering 14.1%, partly due to the extremely high youth unemployment rate of close to 30% (Anon., 2011). In Sub-Saharan Africa, the unemployment rate in 2000 was 8.4% (Anon., 2011). The continent of Africa as a whole stood out in the early 2000s for having one of the highest employment growth rates in the world, at 3% and above (Anon., 2011).This slowed to below three percent in the years 2007 through 2009, suggesting once more the effects of the global recession of 2009 on employment chances (Anon., 2011).

The availability and timeliness of employment data for Africa are particularly poor (Fox and Pimhidzai, 2013). Low-income African nations face issues with the very definitions of labour force participation, employment, and unemployment used in emerging economies (Fields, 2012). Despite these economic changes, the poor have been able to invest in expanding their own chances for self-employment. The private sector makes greater investments to establish operations in a developing nation, causing the creation of jobs and the payment of of taxes that can be utilized to create roads, schools, and social services (Fields, 2012). However, a consistent pattern may be seen in the data that is currently accessible.

The stark contrast between very little formal employment and the labour markets in Africa is noticeable. Instead of open unemployment, there is widespread underemployment in the informal urban and agricultural sectors. Sub-Saharan Africa's labour force participation rates do not differ much from those of other developing nations; they are somewhat lower than those of Latin America and South Asia, but surprisingly greater for women, and lower than those of East Asia for males (Golub and Hayat, 2015). Unusually, with 77 percent of the labour force employed, the African labour market is dominated by informal (Golub and Hayat, 2015). Unusually, with 77 percent of the labour force employed, the African labour market is dominated by informal work (De Vreyer and Roubaud, 2013). Agricultural labour, non-wage employment, and part-time wage employment are all examples of informal employment in this context (Golub and Hayat, 2015). According to this definition of informal work, at least 80% and frequently 90% to 95% of all employment in SSA's low-income nations falls under this category. Benin, Burkina Faso, Cameroon, Mali, Nigeria, Rwanda, and Tanzania are among the low-income SSA nations where government employment is higher than that of the formal private sector, whereas Ethiopia, Ghana, Malawi, Mali, Senegal, Uganda, and Zambia are among those where the converse is true (Golub and Hayat, 2015). However, formal employment in the commercial and public sectors accounts for less than 10% of total employment in each of these nations, and is sometimes lower than 5% (Golub and Hayat, 2015). Although still significant at 38% in both Botswana and South Africa, informal employment is smaller in middle-income SSA nations than it is in higher-income countries. Egypt, a representative of North Africa, is an intermediate case, with 61 percent of its workforce working in the informal sector and 30 percent working in the state sector (Golub and Hayat, 2015).

In low-income SSA, unemployment rates are typically very low and frequently far below those of industrialized nations. For instance, they are 0.7% in Benin, 2% in Uganda, 2.3% in Burkina Faso, and 2.6% in Madagascar (Golub and Hayat, 2015). Ethiopia, Senegal, and Zambia are a few of the exceptions (Golub and Hayat, 2015). South Africa in particular, where unemployment has become pervasive and a topic of considerable concern, is one of the middle income SSA countries where unemployment tends to be greater (Kingdon and Knight, 2004). Additionally, in contrast to wealthy nations, documented unemployment rates in Africa climb with education levels, with university graduates typically experiencing the highest rates (Anon., 2012). While most young people in developing nations are employed, this is not the case in wealthier nations

(Anon., 2012). In poorer countries most young people are working, in the better-off countries there are more unemployed people than workers (Anon., 2012).

Unemployment is just not an option for the uneducated and underprivileged who seek security in subsistence farming and the urban informal sector (Fields, 2012). Given the difficulty in separating unemployment from leaving the labour force, the accuracy of the unemployment figures is in doubt. However, it is evident that Africa has an issue with employment as opposed to unemployment (Fields, 2012). After gaining their independence, nearly all African nations implemented highly interventionist import substitution industrialization (ISI) strategies that emphasized the expansion of the public sector and the defense of homegrown sectors (Golub and Hayat, 2015). Economic crises that were widespread in the 1980s prompted structural adjustment strategies that involved shrinking the public sector and reducing the protection of formal industries that competed with imports. Between the late 1970s and the mid-1990s, public employment decreased both in absolute terms and as a percentage of the labour force (Golub and Hayat, 2015). Initial changes to the structural programs also resulted in decreases in employment in the private sector due to the failure of ineffective import substitution companies and underwhelming non-traditional export growth. The evolution of employment in the private sector is not well studied, but it is obvious that wage employment growth has not been sufficient to significantly reduce underemployment. In the 1990s and early 2000s, there were changes with regard to Tanzania, Uganda, and Nigeria's sectoral employment patterns (Kingdon *et al*, 2006). The proportion of the labour force engaged in wage work, which is strongly tied to employment in the formal sector, fell over this period in all three nations, mostly as a result of large declines in the employment of the public sector, which includes state-owned firms (Kingdon *et al.*, 2006). Since the wage employment in the private sector expanded at a pace that was either too slow or, in some countries, even kept up with that of the public sector, it has continued to make up a very tiny percentage of the work force. Self- and family employment—which is primarily urban—rose considerably as a share of the labour force. Employment at a wage is significantly lower for women than it is for males. For instance, in Uganda in 1999–2000, only 6% of women had paid employment, compared to 16% of men (Kingdon *et al.*, 2006). Open unemployment is extremely low in the three countries and did not significantly vary across the time periods displayed (Kingdon *et al.*, 2006).

The portion of the working-age population in a country who actively participates in the labour market by either working or looking for work is known as the labour force participation rate (LFPR) (Anon., 2019a). It gives a hint as to the proportional size of the labour pool available for the creation of goods and services. LFPR is one of the helpful metrics used to track development progress toward national and global goals like the Sustainable Development Goals (SDGs) (Anon., 2019a). The total labour force is calculated as a percentage of the population that is 15 years of age and older, or otherwise referred to as LFPR (Anon., 2019a). According to the International Labour Organization's (ILO) definition of the labour force, it is composed of all working-age people who are both employed and unemployed (Anon., 2017a). In another jurisdiction, an individual is classified as unemployed in the labour force if the person has not been working in any activity within a defined time period, has no attachment to work and is vacant for work (Anon., 2014). Another concept on labour force classification includes persons who are employed or unemployed excluding non-job seekers (Anon., 2016d).

The level of unemployment for females aged 15 years and above within the labour force in Ghana is 5.5% higher than males 4.8% (Anon., 2014). Due to limited jobs available and rural urban migration, urban unemployment is 6.5% higher than rural unemployment, on average which is 3.9% (Anon., 2014). On the account of the labour force survey conducted in 2015, Ghana's unemployment rate is 11.9% with females recording the highest percentage of 12.5% and for males 11.1% (Anon., 2016d).

Anon (2015a), report on national employment highlights gender's economic demarcation with a greater number of males dominating the workforce in all the three main sectors of the economy accounting for 60.3% to 39.7% females. The classification of these sectors and its sub sectors under the agriculture sector comprises "crops, livestock and poultry, forestry and logging, fishing and aquaculture" (Anon., 2015a). Industry subsectors are "manufacturing, mining and quarrying, electricity and gas, water supply, sewerage, waste management and construction" (Anon., 2015a). Services comprises of "wholesale and retail trade, transportation and storage, accommodation and food, information and communication, financial and insurance, real estate, professional, scientific and technical, administrative and support service activities, public administration and defence, education, human health and social work, arts, entertainment and recreation, and other services" (Anon., 2015a). Male dominates largely in the agriculture sector in Ghana with 74.4%, 64.3% in the industrial sector and 59.0% in the service sector as compared to the corresponding female percentages (Anon., 2015a).

The distribution in persons engaged by sex and region indications showed that for establishment in the agriculture sector, the largest percentage of men was seen in the Greater Accra region. (31.9%) and females (35.7%) (Anon., 2015a). Eastern region followed by males (15.4%) and females (11.6%) having Upper East region recording the least number of individuals with males (0.6%) and females (0.5%) in the agriculture sector (Anon., 2015a). The industry sector saw the Greater Accra region recording the highest number of engagements for both males (42.9%) and females (36.3%) (Anon, 2015a). The Ashanti region followed recording males (15.5%) and females (14.3%) with the Upper West region recording the lowest sex male (1.6%) and female (2.0%) engagement for the sector (Anon., 2015a). The services sector for the Greater Accra region recorded the highest engagement with male dominance of 38.7% and female dominance of 35.8% (Anon., 2015a). Again, the Ashanti region recorded male dominance of 15.4% and female 17.5%. The region with the lowest gender engagement was the Upper West with males of 1.8% and females 1.7% (Anon., 2015a). Employment allocation is of paramount importance in the national distribution of job opportunities for all.

Kapsos (2005) research on employment elasticity and growth in the Asian and Pacific regions, recounted how East Asia and South-East Asia growth increased due to increase in their industry. The average annual growth rate in East Asia recorded 12.8 percent, 5.9 percent in South Asia, whilst South-East Asia had 5.4 percent in the region's industrial sector (Kapsos, 2005). The services sector showed growth of 8.8 percent in East Asia, South Asia recorded 6.9 percent with South- East Asia recording 4.6 percent (Kapsos, 2005). Productivity growth was the main engine behind the annual growth rate in the South-East Asia's agricultural sector within the years 1991-2003. The region of East Asia reported the fastest rate of yearly increase in the agricultural sector with 3.7 percent, South Asia's recorded 2.9 per cent whilst South East Asia had the least growth of 2.1 percent (Kapsos, 2005). South-East Asia's growth in both the industry and services sectors was a result of employment growth.

For the poor and uneducated, who seek solace in subsistence agriculture and the urban informal sector, unemployment is simply not a choice (Fields, 2012). Given the difficulty in distinguishing between employment and leaving the labour market, the accuracy of the unemployment figures is in doubt. Creating meaningful jobs continues to be a top priority in sub-Saharan Africa. Nonetheless, the behaviour of unemployment is explained directly by the fluctuations of employment rate (Rollin, 2015). Employment on the other side is a human rights concern as well as an economic one (Wray, 2009).

2.4 History on Modern Unemployment

Computers have replaced several occupations in recent years, including those of bookkeepers, cashiers, operators, and others (Bresnahan, 1999). Manyika *et al.*, (2013) claim that the development of technologies has caused a change in the course of events. In contrast, industrial robots are now performing physically demanding, hazardous, or filthy tasks like welding, spray painting, manufacturing, and aviation (Manyika *et al.*, 2013). (Manyika *et al.*, 2013). It is not surprising that new technology makes some forms of human labour obsolete or less profitable and raises demand for new skills (Manyika *et al.*, 2013). During the start of the Industrial Revolution, when the machine looms marginalized domestic weaving while generating jobs for mill workers, this situation has repeatedly occurred (Manyika *et al.*, 2013). Though, the potential impact of modern technological technologies on the nature of labour is startling (Manyika *et al.*, 2013). Computerized knowledge-based tools could entirely replace some professions, but they could also increase the abilities of many sorts of workers and assist create top-line improvements via innovations and better decision-making (Manyika *et al.*, 2013). More manual jobs might be automated as a result of advanced robotics, especially those in the services sector, where automation has so far had less of an influence. Employers, business executives, and policymakers will need to figure out how to take advantage of these technologies while developing fresh, inventive work practices and enhancing labour skill sets (Manyika *et al.*, 2013). Examining the likelihood that susceptible jobs will be computerized is necessary to understand how technological advancements will affect unemployment.

The debate over both technical and industrial unemployment among economies has recently been more heated due to the weak labour market performance in advanced economies. While there is still disagreement on what causes the unemployment rates to remain so high, some academics have suggested that the rise in the number of unemployed people recently may be related to computer-controlled technology (Brynjolfsson and McAfee, 2014).

England's late 18th century saw the beginning of the industrial revolution (Anon., 2021d). The industrial revolution had a significant impact on the textile business. In numerous fields, increasing steam power introduced new machines for mass production. Cities with lots of manufacturers saw the construction of factories and a concentration of the labour force (Anon., 2021d). Because they were less expensive than adults, even youngsters were employed. During the industrial revolution, some societal issues emerged that had never before. New social issues included the polarization of wealth, city sanitary issues, labour exploitation, and unemployment. The industrial revolution made a clear distinction between those who employ people, or employers, and those who are employed, or employees. Agriculture was also revolutionized by the industrial revolution (Anon., 2021d). Agriculture's requirement for labour has significantly decreased, and the additional labour has moved into urban areas (Anon., 2021d). The industrial era ushered in the Industrial Society (Park, 2008). Over the course of two centuries, the industrial revolution transformed society from an agricultural to an industrial one. In the late 18th and early 19th centuries, the industrial revolution extended throughout all of Europe. In the 20th century, it spread to Asia and other continents (Park, 2008).

Since the industrial revolution began in the United Kingdom, it was the first nation to become industrialized (Park, 2008). The economy and society of the United Kingdom underwent significant upheaval during the industrial revolution. According to Park (2008), rivers and forests served as the primary sources of energy prior to the industrial revolution. The newly constructed industrial districts might cluster in the urban areas thanks to the usage of coal as fuel (Anon., 2021d).

During the industrial revolution, the textile sector experienced significant growth because steam engines enabled the mass manufacture of textile products, which were formerly made by hand at home. People flocked to the city in search of work during this era of employment when factories required enormous amounts of labour. In those days, unemployment was not a significant problem. After several decades of the industrial revolution, there was an excess of labour and an overcrowding of metropolitan areas (Park, 2008). Beginning in 1855, the United Kingdom kept statistics on unemployment. Unemployment rates ranged from 1.8% to 7.3% between 1855 and 1896. Unemployment did not reach a high or low point abruptly; rather, the transition was gradual (Park, 2008). The percentage decreased further, from 3.7% in 1870 to 1.6% in 1871, from 5.9% in 1869 to 3.7% in 1870. The number remained close to 1% for the next 4 years. After another four years, starting in 1875, the proportion increased from 2.2% to 6.2%. In 1879, it reached 10.7% and then decreased by half to 5.2% the following year (Park, 2008). The second industrial revolution started in the UK at the beginning of the 1870s and continued there until the start of World War 1 (Park, 2008). Chemical, electrical, petroleum, and steel industries all arose during this second stage of the industrial revolution. Similar to how it was at the start of the industrial revolution, the rise of these industries created a huge demand for labour. This provides an explanation for the abrupt decline in the unemployment rate in 1870 (Park, 2008).

The Long Depression is known to have lasted from 1873 to 1896. The price of raw materials decreased, the monopolistic companies controlled the majority of the industries, and the sudden development of new technologies (primarily as a result of the second industrial revolution) encouraged an overproduction of goods that could not satisfy consumer demand (Park, 2008). Although the Long Depression was not as severe as the Great Depression, the theory of effective demand can nonetheless account for the rise in the unemployment rate (Park, 2008). Up until 1896, the employment rate was around 6–7%, but the rate fell from 5.8% in 1895 to 3.3% in 1896 and stayed around 3% for a period of ten years (Park, 2008).

The United Kingdom's unemployment was impacted by two major wars in the 20th century. As Europe's tension rose in the lead-up to war, the industrial structure changed. And in 1908–1909, this resulted in transient frictional unemployment. The United Kingdom's industrial sector entered a state of war during World War I. Since both the military and industry needed more labour than previously, there was not enough manpower (Park, 2008). In order to evaluate this, we start by applying a cutting-edge methodology to theorize the economic sectoral impact on employment creation.

2.5 Youth Unemployment Trends Globally

The likelihood of young, economically engaged people being unemployed is still significantly higher than it is for the whole population. Despite making up only 17% or less of the labour force in places like the Arab States, the northern part of Africa, and Southern Asia, constitute greater than 40% of the job seeking population overall in these areas (Anon., 2017b).

Youth make up about 10% of the labour force overall and about 20% of the unemployed in Northern, Southern, and Western Europe. In spite of advances in Eastern Asia and Northern America, the global ratio of youth to adult unemployment is expected to be 3.0 percent in 2017 (Anon., 2017b). This ratio has not changed from 10 years ago. However, in half of the regions examined, adolescent unemployment rates have increased during the previous ten years more quickly than adult rates. Southern, Southeastern, and the Pacific regions had the most significant growth in the ratio (Anon., 2017b). Youth rates of unemployment in Southeast Asia and the Pacific are more than five times greater than adult rates. The ratios in the Arab States and Southern Asia, which are 4.5 percent and 3.9 percent, respectively, in 2017, aren't far behind (Anon., 2017b). Northern Africa is expected to continue to have the second-highest youth unemployment rate of any region, at 28.8 percent in 2017 (Anon., 2017b). While the rate in the region is anticipated to modestly fall in 2018, the rate of youth unemployment in the region's large countries is still forecast to be much higher than 30%. For the second year in a row, the young unemployment rate in sub-Saharan Africa is predicted to increase, approaching 11.1 percent in 2017 (Anon., 2017b). This signals the end of the encouraging trends seen between 2012 and 2015, when the youth unemployment rate decreased by about 1 percentage point to reach 10.7 percent in the latter year (Anon., 2017b), with projections for a further increase in 2018 (Anon., 2017b). It is also anticipated that South Africa's youth labour market will continue to deteriorate within the area.

In developed nations, many young individuals go through prolonged unemployment, which may impede their ability to develop their skills, become employable in the future, and increase their earning potential (Anon., 2017b). For instance, nearly 18 percent of young people who were unemployed in the Organization for Economic Co-operation and Development (OECD) countries in 2016 had been out of work for a year or more. In the first quarter of 2017, this percentage was close to 29% among young people without jobs in the 28 nations that make up the European Union (EU-28), and it was close to 33% in the euro area (Anon., 2017b). Long periods of unemployment run the risk of causing young people to become disengaged from the labour market, particularly among those looking for their first job. However, 2.8 million youth in OECD nations are marginally attached to the labour market in 2017; they are neither employed nor actively looking for employment, but they are willing to work and ready to accept a job (Anon., 2017b).

Contrarily, it is predicted that the young unemployment rate in Northern America will fall to 10.4% in 2017, from 10.6% in 2016, the lowest level since 2000. The region's young unemployment rate is predicted to increase to 11.1 percent in 2018 (Anon., 2017b), despite the economy's generally favourable outlook. The Arab States will continue to have the highest youth unemployment rates in the world in 2017, but at 30.0 percent, it will be somewhat lower than in 2016. Even while some encouraging trends are anticipated for 2018, slower economic expansion in some oil-exporting nations, particularly Saudi Arabia, will help keep the rate of youth unemployment high (Anon., 2017b). The chances for youth employment in the remainder of the region will probably continue to suffer from geopolitical instability. The young unemployment rate in Eastern Asia is predicted to stay at 10.4% in 2017 and slightly increase to 10.5 percent in 2018 (Anon., 2017b). The little increase from 2016 is mostly due to the slowing Chinese economy, which is only somewhat offset by forecasts for declining youth unemployment rates in Japan and the Republic of Korea. However, the number of young people without jobs is expected to fall, from 10.9 million in 2016 to 10.2 million in 2018 (Anon., 2017b), as more young people in the region choose to continue their studies rather than enter the workforce. In Southern Asia, the rate of youth unemployment is anticipated to be steady at 10.9 percent in 2017 and 2018 (Anon., 2017b). This is due to the fact that the area's largest economy, India, will see rapid economic development that will be partially offset by slightly worsening labour market conditions elsewhere in the region. The issue of youth unemployment in Southern Asia will continue to be severe in absolute terms. In 2017, nearly 14 million economically capable youth will be jobless, which amounts to nearly 20 percent of total young unemployment worldwide (Anon., 2017b). The rate of youth unemployment is anticipated to rise by the second-highest amount, from 11.7 percent in 2016 to 12.0 percent in 2017 and 12.2 percent in 2018, in South-Eastern Asia and the Pacific (Anon., 2017b). The youth unemployment rate in Indonesia will remain high despite modest progress in 2017, while it is anticipated to climb in Malaysia and the Philippines. More than any other region in the world, Eastern Europe is predicted to have a decrease in the youth unemployment rate in both 2017 and 2018. According to Anon (2017b), the rate of youth unemployment is predicted to decline from 17.0 percent in 2016 to 15.2 percent in 2017 and 14.2 percent by 2018. The main drivers of the region's improvement are the Russian Federation's recovering economy and labour market, but rates are also falling in the Czech Republic, Hungary, and Poland. In contrast, it is anticipated that youth unemployment in Central and West Asia will increase in 2017 to 17.5 percent, up more than 0.5 percentage points from

2016 (Anon., 2017b). The rising youth unemployment rate in Turkey will play a significant role in this. Conditions of the youth labour markets in Northern, Southern, and Western Europe seem to be slowly but surely improving. According to projections, the region's young unemployment rate will decrease by a full percentage point in 2017 to 18.2 percent (Anon., 2017b). Since reaching a record high of 23.3% in 2013, this will mark the regional young unemployment rate's fourth consecutive fall (Anon., 2017b). Positive changes in a few high-unemployment nations, notably France, Italy, and Spain, account for a large portion of the predicted drop in the regional data. In 2018, the rate of youth unemployment is anticipated to decline to 17.8 percent, though to a lower extent (Anon., 2017b). In certain nations, youth unemployment rates are perennially high and represent systemic problems.

2.6 Youth Employment in Africa

With 226 million youth (aged 15–24), or nearly 20% of the world's youth population, in 2015, Africa is the continent with the highest youth population (Anon., 2015d) and with 70% of sub-Saharan Africa under the age of 30 (Anon., 2022). Kappel (2021), the main issue with employment in Africa is young employment. It is easy to identify the situation for young people from the trend in overall unemployment.

Sub-Saharan Africa's employment situation is getting worse. It seems improbable that just Africa's economic expansion will generate the required number of jobs (Kappel, 2021).

Africa's young population is expanding quickly and is projected to increase by 42% by 2030, but the youth population in other parts of the world has stagnated in size (Anon., 2015d). 30 million young people are predicted to enter the African work market each year by 2030 (Barlet and D'Aiglepierre, 2017). Youth in Africa faces significant economic challenges. Young individuals in Africa struggle with underemployment and a lack of opportunity (Anon., 2022b). Africa faces a series of challenges when it comes to youth employment. Young people are entering the labour field at an alarmingly rapid rate across Africa, adding to the already pressing need for adequate work for them. Africa is going through a significant demographic revolution that, for better or worse, will significantly alter its employment market (Turolla, 2022). Only 3 million formal jobs are produced each year, according to the World Bank, leaving 12 million young people looking for employment (Turolla, 2022). The 'young employment' crisis in Africa, according to Turolla (2022), is basically a ‘missing jobs' crisis.

Most African countries have a serious job crisis due to their highly youthful population structure and a quickly growing labour pool (Anon., 2019b). Under these conditions, it is essential that the economy develops productive jobs and means of subsistence at a breakneck pace. For instance, it is predicted that in Mali, the number of people joining the labour market for the first time was 171,800 in 2005 and will reach a peak of 447,800 in 2005 before beginning to fall (Anon., 2007). In Madagascar, there were a projected 286,200 new workers in 2005, and by 2035, that number will rise to 473,400 annually before starting to fall (Anon., 2007). Due to the scarcity of employment prospects in the formal sector, both men and women are increasingly turning to the informal economy and are, for the most part, locked in informal employment. Due to the amount and type of youth employment there, Sub-Saharan Africa constantly exhibits the second largest youth employment-to-population ratio, meriting special attention. The number of young people working climbed by a notable 33.3 percent only in Sub-Saharan Africa between 1998 and 2008 (Anon., 2007). Unfortunately, this does not signal a good thing for the area and serves as a warning that the rise in young people's employment is not always a good thing. The issue is how to determine when a bad trend causes young employment to increase and when it does not. To address the first question, it is crucial to recognise that increasing youth employment rates and job-to-population ratios are not indicators of success when they show that more young people must work for a living in an environment marked by persistent poverty and a lack of social assistance (Islam, 2005). At 9.9 percent in 2009 and an anticipated 9.8 percent in 2010, North Africa's unemployment rate remained among the highest in the world (Islam, 2009). The exceptionally high rate of youth unemployment is the main cause of the high total rate. When it comes to population growth, North Africa ranks third, with a working-age population that increased by 27.8% between 2000 and 2010. However, GDP growth rates of 4 to 5 percent are insufficient to generate productive and dignified jobs for this expanding population, as evidenced by the extremely low rates of increase in labour productivity and the enormous number of unemployed individuals (particularly women and young people who only represent the tip of the iceberg). Many of the employment that is currently available are, in fact, of poor quality, underpaid, unstable, and unrepresentative of workers or basic labour standards (Anon., 2011). The large percentage of unofficial employment and the continuance of working poverty serve as indicators of this. It is impossible to ignore the root reasons of the region's persistently severe lack of adequate employment opportunities or the challenging circumstances that women and young people in particular face. Current population expansion is undoubtedly a burden, as economic growth does not generate enough high-quality jobs. Additionally, there is a widening gap between the skills that employers need and the abilities that young people can supply (Anon., 2011). This is evident, for instance, in the high and occasionally rising unemployment rates among African university graduates. As many young people lament, they would be willing to work if they knew where to get a job, it is difficult to match the supply and demand of employment. Furthermore, what the labour market offers is considerably different from what young people anticipate when they search for work: traditional government jobs, which most young people aspire to, are becoming harder to find. Because of the dearth of youth entrepreneurship and the lack of incentives for young entrepreneurs, private sector positions are frequently viewed as being excessively demanding, and self-employment continues to be seen as impossibility. Despite the fact that population growth is predicted to reduce in the next few years, this will result in a lost generation of young people who are jobless (Anon., 2011). It will be more challenging for them to acquire employment later on the longer they go without one. They run the risk of losing both skills and drive as a result. This is the reason the area needs to put a lot of emphasis on this youthful generation. It is encouraging to see that all of the nations in the region have prioritised youth employment challenges in their national development goals. National action plans for youth employment have been created in various nations, and the creation of regional action plans is seen as a potential strategy to address youth employment difficulties in other nations (such as Egypt, Sudan, and Tunisia) (Anon., 2011). Enhancing skills, particularly technical, IT, and language abilities (as in Rwanda's successful scenario) should be a key component of the plan to solve these difficulties (Anon., 2011). The youth employment plan may also include encouragement of self-employment as well as job growth in contemporary economic sectors (such as eco-tourism, IT, and green occupations). More positive views among young people and employers can be fostered with the use of strengthened social dialogue institutions. Youth employment is now regarded as a vital element of PRSs since it is a significant issue for both development and peace building in the wake of the "Arab Spring" and the growing demands of youth for access to decent jobs (Anon., 2011). All member states reaffirmed their commitment to stepping up efforts to eradicate youth and women's unemployment and underemployment in Africa and to create a youth employment pact at the most recent

African Union Heads of State and Government conference in Malabo in June 2011 (Anon., 2011). However, achieving full, productive, and freely chosen employment for young people requires long-term, coordinated, and comprehensive work across a variety of economic and social policies. The integration of national policies and activities promoting youth employment into overall macroeconomic and sectoral policies maximises their effectiveness. The need for long-term action on a variety of economic and social issues to provide young people with good and productive work is becoming more and more apparent to policymakers. This calls for an integrated and cogent strategy that combines macroeconomic and microeconomic intervention, focuses on labour supply and demand, and addresses both the quantitative and qualitative components of employment (Islam and Anwar, 2011). Considering how closely tied youth employment is to overall employment, programmes aimed at enhancing young people's employment chances perform best when they are a part of larger initiatives to boost aggregate demand. To guarantee that economic growth results in quality employment, employment must be at the centre of economic and social policies. This necessitates a thorough framework that addresses young employment via an integrated strategy for employment growth (Islam and Verick, 2011). The integration of national employment-related policies and programmes into general macroeconomic and development strategies is a prerequisite, but it is not sufficient. The employment possibilities for young people vary depending on a number of variables, including gender, age, ethnic background, socioeconomic class, household size, and amount of education and training. Hence, it is essential to add specialised measures to the wider employment and other economic and social policies in order to combat the unique difficulties that many young people face while joining or continuing in the labour market. Numerous nations have made efforts to address the issue of youth unemployment during the past three decades. Despite these initiatives, there has been little improvement in young people's chances of finding full-time, productive jobs of their choosing. The impact of these activities has been restricted for a number of reasons. First, the majority of them focused on individual projects, many of which were time and space constrained and had a limited reach, as opposed to developing policies and strategies that would encourage youth employment. Furthermore, these interventions were prioritised largely at random, and the priority was frequently tied to the business cycle because young people are typically the last to benefit from economic prosperity and the first to be affected by recession (Anon., 2011). Third, the emphasis was mostly on pre-employment initiatives, with little attention devoted to the subpar working circumstances of many young people, who are disproportionately employed in informal sectors or in temporary positions. Finally, many schemes that either focused on job creation or on improving young people's employability failed to address several facets of the youth employment dilemma (Anon., 2011). As a result, many African nations are taking the necessary steps to lower the high rates of youth unemployment that are currently present, and Africa is now offering many lessons and best practices from youth labour market initiatives that have demonstrated positive and promising effects on their employment and financial outcomes (Anon., 2011). By making it easier for young people to access vocational training, fostering the growth of a competitive job training sector, and providing them with efficient active labour market programmes, they hope to address the technical skill gap that exists among young people (Anon., 2011). These initiatives also aim to help young people who are unemployed or who live in remote areas where the market does not reach. Additionally, it might entail the implementation of workplace training programmes, the development or improvement of apprenticeship systems, the promotion of subsidised training initiatives that offer financial incentives to employers for in-service training, and the provision of entrepreneurship education to young people looking for self-employment opportunities (Anon., 2011). They concentrate on the demands of the industrial sector and offer instruction in the appropriate needed abilities. In response to demand on the labour market, Tanzania is reviewing its TVET strategy (Anon., 2011). A system for apprenticeship training is being developed by employers in the hospitality sector. Programmes for developing, evaluating, and certifying the abilities of those working in the unofficial economy have been started. The non-formal and formal vocational training systems' training curricula were examined to ensure they followed a competency-based training strategy. In terms of skill development, South Africa has made significant strides (Anon., 2011). To address the systemic issues that have been discovered, the National Skills Development Strategy III has been introduced and the Skills Development Act has been modified. A new quality council for trades and occupations has been established to better focus on skills training (Anon., 2011). The emphasis continues to be on young men and women who are not in training or employment, as well as young women and men who are not in school or have a job (Anon., 2011). Meanwhile, encouraging young people to start their own businesses is increasingly recognised as a crucial strategy for generating employment and ensuring that nations can take advantage of the enormous socioeconomic potential of their youthful populations (Anon., 2011). However, initiatives to encourage young people to pursue entrepreneurship as a career path have to be carefully thought out and long-term in nature.

This is because entrepreneurship is both demanding and inherently risky, especially for those who are still in their formative years and are generally lacking in both life experience and material resources (Anon., 2011). This is demonstrated by the fact that a large number of young people struggle in the informal sector and are sometimes forced into self-employment out of necessity rather than choice (Anon., 2011). Programmes for the growth of the private sector and small businesses are not new to Africa. Instead, over the past ten years, bi-lateral and multilateral institutions have increased their funding for such activities (Anon., 2011). The emphasis has instead been on general business development services that are pertinent for numerous sub sectors rather than comprehensive and integrated youth-specific entrepreneurship development activities. Although this strategy is pertinent for company development as a whole, it can leave young people in a difficult position where they may succeed in beginning a firm (how to start) but not knowing which industry to enter (where to go) (Anon., 2011). Young entrepreneurs who graduate from these start-up curricula frequently cluster into a small number of well-known businesses as a result, where margins are thin and competition is strong (Anon., 2011). Based on the aforementioned findings, interventions must focus on the very specific difficulties that young people encounter in the enabling environment for youth entrepreneurship, at the level of service providers and financial providers, as well as at the individual level, in order to ensure long-term and sustainable impact on youth entrepreneurship in Africa (Anon., 2011).

Africa's youth face a variety of difficulties, from economies that developed but were unable to generate enough jobs before the global financial and economic crisis to sluggish post-crisis growth that was partially caused by unfavourable weather and low commodity prices (Anon., 2021b). Africa is the only region where the youth bulge will keep expanding in the near future, offering both a chance to benefit from the demographic dividend and a potential time bomb that could threaten social cohesion and cause massive migration in search of opportunities if the right policies are not put in place to capture the dividend (Anon., 2021b). In 2019, little over one in five young people (NEET) were not in employment, education, or training; this level of unemployment has been continuously rising since 2012, matching trends in the global rate (Anon., 2021b). In 2021, the NEET rate is anticipated to rise 0.3 percentage points to 20.8 (Anon., 2021b). A gender discrepancy of almost 10 percentage points since 2018 indicates that young women are disproportionately more affected by the NEET status. The gender gap in Africa, however, is less than that of other continents, particularly the Asia Pacific and Arab States regions, where it is between 20 and 30 percentage points, respectively. An interesting finding is that, between 2012 and 2018, the NEET rate for young women decreased, although it has been rising for young men since 2012 (Anon., 2021b). The high rates in Northern Africa, where rates are observed to be above 26% compared to rates of around 20% in Sub-Saharan Africa, are primarily responsible for the high NEET rate in Africa. The level of employment to population ratio (EPR) gauges the economy's capacity to generate jobs. It displays the percentage of young people of working age who are employed, regardless of the nature of their employment. In Africa, almost two out of every five young people who are of working age are employed in some capacity. This number slightly decreased from 2012 to 2018 (a reduction of 0.7 percentage points), and it is anticipated to slightly decrease again between 2020 and 2021 (Anon., 2021b). Thus, the EPR in Africa has been largely stable with a minor reduction, unlike at the world level and in other regions. This could be explained by the fact that while many young people continue their education for extended periods of time, a growing population and labour force also means that more young people are entering the workforce. Due primarily to the decline in the male ratio, the gender gap has somewhat decreased from 8.3 percentage points in 2012 to 6.6 percentage points in 2020 and 2021. The fact that young people and their families work primarily in the informal economy and experience generally subpar wages is reflected in Africa's higher poverty rates. According to a recent study1 by the ILO (2018), the vast majority of young people between the ages of 15 and 24 (94.9%) are informally employed, have no or little schooling, are based in rural areas, and are mostly involved in subsistence agriculture. But educated young also seek safety in the informal economy; two thirds of those with secondary education and nearly a third of those with tertiary education worked in this field (Anon., 2021b). Sub-Saharan Africa (95.8%) has a higher percentage of young people working in the informal sector than Northern Africa (87.5%). Young adults (ages 25 to 29) are less likely than their younger peers to work in the informal economy (Anon., 2021b). While there is a minor gender disparity in the informal sector among young people, it widens as people get older. A concerted effort is required to put in place a suitable and individualised blend of employment policies and programmes at macro levels, sectoral, and labour market levels with an increased focus on demand side support for structural transformation and focusing particular attention to disadvantaged groups in the labour market with the goal to sustainably address the employment challenges Africa is facing, especially among the youth, and to benefit from the demographic dividend (Anon., 2017). In Nigeria, the issue of youth unemployment is both obvious and terrible. There are not enough jobs to accommodate the thousands of graduates who leave school each year (Kazeem *et al.*, 2018).

2.7 Unemployment and Employment Trends in Ghana

The problems of unemployment and vulnerable work have recently taken centre stage in both local and international settings as severe developmental difficulties that call for rapid, focused, and cooperative measures to address them. A phenomena of job-seeking brought on by unemployment is unemployment (Baah-Boateng, 2013). In the field of economic literature, there are numerous explanations for the phenomena of unemployment. The labour market is assumed to always be transparent based on the neoclassical framework's assumption of flexible wages and complete understanding (Baah-Boateng, 2013). Another factor that contributes to unemployment is when businesses opt to boost employee productivity by paying higher wages than the equilibrium wage (Baah-Boateng, 2013). A jobless person who is available for work but chooses not to hunt for employment is referred to as a "discouraged worker" rather than being referred to as unemployed (Baah-Boateng, 2013). Additionally, someone who is employed but engages in job searching as a side hustle for a variety of reasons, such as to earn extra money or broaden their career options, is not considered to be unemployed (Baah-Boateng *et al.*, 2013). The majority of workers who experienced layoffs and privatisation found refuge in other economic sectors and were forced to turn to side jobs because their pay in their new primary position was less than it had been before the reform. This forced them to maintain an income level closer to what it had been before the reform (Baah-Boateng *et al.*, 2013).

In Ghana, people are becoming more aware of the crucial role that employment can and must play in fostering economic expansion and job development. In recent years, the national development agenda has made creating jobs or creating employment a top priority (Anon., 2015c). Ghana's Shared Growth and Development Agenda, which was formed for the years 2014 to 2017, aims to utilise the country's natural resource endowments, agricultural potential, and human resource base to promote rapid economic growth and the creation of jobs in value-added industries, particularly manufacturing (Anon., 2015c). In order to adopt an evidence-based approach to employment, the government established the National Employment Policy to address employment concerns (Anon., 2015c). It is anticipated that the Policy will serve as a roadmap for implementing programmes in order to attain predetermined goals and developmental results.

The Ghanaian government is committed to upholding its duty to ensure that all of its citizens have access to good employment and has highlighted the risks that unemployment and precarious employment represent to national stability, economic growth, and development (Anon., 2015c). However, the 1992 Ghanaian constitution placed responsibility for monitoring the execution of development initiatives with implications for the generation of jobs for the country's labour force. A seven-year development plan known as the Workers Brigade Plan was created from 1963 to 1970 as a tactical reaction to the issues of post-independence development and unemployment during that time (Anon., 2015c). The initiative addressed the production of food through the installation of state farms, the building and upkeep of roads and community sewage systems, the establishment of agriculture warehouses, public buildings, and other initiatives for development. The Workers Brigade strategy was a successful mass employment intervention which gave Ghana's young jobs and took in active men and women who were engaged in the war for independence. Under the auspices of "Operation Feed Yourself" and the National Reconstruction Programme, the public was heavily mobilised during the National Redemption Council's (1972–1988) rule in the fields of agriculture and construction. Similar to this, the government provided temporary employment opportunities in agriculture, construction, and rehabilitation between 1982 and 1993 as part of the National Mobilisation Programme (Anon, 2015c). The Economic Reform Programme (ERP) and Structural Adjustment Programme (SAP), respectively, were enacted by the government in 1983 and 1985 with the primary goals of achieving economic liberalisation and the privatisation of ineffective state and quasi-governmental firms and commerce. The government was forced to develop the Programme of Action to Mitigate the Social Cost of Adjustment (PAMSCAD) after realising that the ERP and SAP had adverse employment consequences (Anon., 2015c). It entails the development of community projects to provide employment for low-income workers, vulnerable urban households without jobs, laid-off workers, and rural households in the northern part of Ghana (Anon., 2015c). It was also intended to provide women and small-scale mining enterprises with access to loans for their companies. In order to lessen the high rates of unemployment and precarious labour, the Vision 2020 programme, which intended to assist Ghana become a middle-income country, also aimed to integrate employment and promotion policies into all production plans halfway through the 1990s. The government has carried out several employment creation schemes and activities from the year 2000. These include, among many others, the Graduate Entrepreneurial and Business Support Scheme (GEBSS), the National Youth Employment Programme (NYEP), the Microfinance and Small Loans Centre, the Rural Enterprises Programme, and the Young Entrepreneurs Programme (Anon., 2015c). The Ghanaian government developed the Nation Builders Corps (NABCO) initiative in 2018 as an intervention system with the goal of creating jobs (Anon., 2019a).

According to Anon (2021c) on the population of Ghana from the 2021 PHC, there are currently 30.8 million people living there. The population of the nation has increased by over five times since the first post-independence era census was taken in 1960. The population of Ghana is increasing, but at a slower rate than in previous censuses. As has been the pattern for the last four censuses, more women make up the population in the PHC of 2021. In 10 of the 16 areas, they outnumber men. Greater Accra is now the country's most populous region, surpassing Ashanti (Anon., 2021c). The component of labour underutilization that is most generally understood and utilised in Ghana across time is the unemployment rate (Anon., 2019a). One of the most important indicators of the labour market, as well as a reliable indicator of the current state of the economy is unemployment (Anon., 2019a). The standard International Labour Organisation (ILO) definition of the unemployed population includes all individuals (15 years of age or older) who are available for work and actively seeking employment throughout the reference period (Anon., 2019a). But the criterion that the applicant must be available and actively looking for work is softened by the broad definition of unemployment adopted in this study. This is because there are few employment opportunities in Ghana, as there are in many underdeveloped countries, and potential workers may give up after a while of being unsuccessful in finding employment. As a result, the report adopts the looser definition of unemployment, which is defined as anyone 15 years of age or older who, during the reference period, was not working, had no connection with a job or company, and "potentially" available for employment (Anon., 2019a). People who were unemployed or did not have a job to return to, nonetheless, were available to work during the reference period. are included in the prospective labour force. Those who have declared their availability for employment but have not made any effort to look for a job include discouraged job seekers in this group (Anon., 2019a). The prospective labour force recognises instances of insufficient labour absorbing, and those individuals also exert pressure on the labour market (Anon., 2019a). In Ghana, unemployment rates fall as people become older, with the youth age groups observing the greatest rates of 5.0% in 1998/99 to 6.6% in 2005/06 as compared to less than 4% for all other age groups (Baah-Boateng, 2013). Because they have less experience in the work market than their older counterparts, young people are more susceptible to economic hardships, which accounts for this discovery (Baah-Boateng, 2013). Additionally, they lack the knowledge and expertise needed to successfully get a job in the labour market (Baah-Boateng, 2013). Even during economic upturns, young people are at a disadvantage for new employment chances due to a lack of work experience and social capital (Baah-Boateng, 2013). The 13.4% unemployment rate for people aged 15 and over is greater for females than males (11.6% vs. 15.5%). The unemployment rate for those between the ages of 15 and 35 is 19.7%, although it is significantly greater for young adults between the ages of 15 and 24 (32.8%) (Anon., 2021c). According to Anon (2021a), 1.5 million of the 11.5 million Ghanaians who are actively seeking employment are unemployed, according to the Ghana Statistical Service's 2021 Population and Housing Census. Thus, 13.4% of Ghanaians as a result are unemployed (Anon., 2021c). An estimated half a million of those without jobs are first-time job seekers who are at least 15 years old (Anon., 2021a). A lack of confidence in the government prevents 65.8% of the labour force from looking for work, according to the statistical service's survey of labour force participation (Anon., 2021a). According to Ghana's Statistical Service, the country's jobless rate has nearly tripled in a little over ten years (Anon., 2021a). "Male economic activity (63.5%) is significantly higher than female economic activity (53%)". Another half a million (487,470), approximately similar percentages of men (2.5%) and women (2.4%) who are 15 years of age or older and seeking employment for the first time (Anon., 2021a).

According to Baah-Boateng (2013), there is no discernible pattern of gender-specific differences in Ghana's gender-related unemployment statistics. In 1991–1992, compared to men, women experienced a higher rate, however, in 1998–1999 and 2005–2006, the opposite is true (Baah–Boateng, 2013). The population and housing census estimates of 2000 and 2010 show that the rate for females was 10.7% and 6.3 percent, respectively, while the rate for males was 10.1 percent and 5.4 percent (Baah-Boateng, 2013). The recent rise in the unemployment rate of women compared to men has been attributed to the growing desire of women to participate in the job market, as seen by the continually increasing number of women joining the job market despite fewer work possibilities (Baah-Boateng, 2013). According to Baah-Boateng (2013), additionally, it is said that urban rather than rural areas of Ghana experience higher rates of unemployment. According to Baah-Boateng (2013) analysis, the unemployment rate was thirteen times more than in urban regions in 1991–1992, while it was over four times greater in urban areas in 1998–1999 and 2005–2006. The continuous movement of people, especially the youth, from rural to urban regions in search of better economic opportunities, which are hard to come by, accounts in part for the issue of the high urban unemployment rate in Ghana (Baah-Boateng, 2013). The lack of facilities like power and water, as well as the poor income related to rural economic activity, which is characterised mainly by farming, also tend to drive a lot of rural adolescents into urban areas (Baah-Boateng, 2013). The educational component of unemployment in Ghana suggests that educated people have a greater rate than illiterate people. According to Baah-Boateng (2013), individuals with university education exhibited the highest levels of unemployment between 1991 and 1992 and 1998 and 1999, followed by those who had completed their secondary education, while the lowest rates were found among persons with no formal education. Secondary-educated individuals had the greatest unemployment rate in 2005–2006, followed by tertiary graduates, while those without education had the lowest rate (Baah-Boateng, 2013). The low unemployment rate among these groups is significantly influenced by the fact that those without a formal education and those with only a basic education have few options other than to settle for unstructured agricultural and non-technical jobs that don't require any formal education (Baah-Boateng, 2013). Nevertheless, the decreased employment rates among people who have completed their secondary and tertiary education may also be attributed to the fact that there aren't many openings in the official sector for them, as well as the unappealing nature of positions in the informal market (Baah-Boateng, 2013).

Job creation for the active job seeker in Ghana is of paramount interest to the government (Anon., 2019a). Even though attempts are being made by government policies in creating jobs for its labour force, yet it has not produced the required result (Anon., 2015c).

2.8 Ghana's Economic Sectors

Agriculture, industry, and services make up the three main economic sectors in Ghana. One of Ghana's main economic driving forces is its workforce, therefore sector indicators for employment development on job creation are crucial (Anon., 2015a). However, most of its economically active persons in the labour force are engaged in agriculture and services sectors (Anon., 2016a). Anon (2015a), report on national employment highlights gender's economic demarcation with a greater number of males dominating the workforce in all the three main sectors of the economy accounting for 60.3% to 39.7% females. The Ghana Statistical Service classification of these sectors and its sub sectors under the agriculture sector comprises: "crops, livestock and poultry, forestry and logging, fishing and aquaculture" (Anon., 2015a). Industry subsectors are "manufacturing, mining and quarrying, electricity and gas, water supply, sewerage, waste management and construction" (Anon., 2015a). Services comprises of "wholesale and retail trade, transportation and storage, accommodation and food, information and communication, financial and insurance, real estate, professional, scientific and technical, administrative and support service activities, public administration and defence, education, human health and social work, arts, entertainment and recreation, and other services" (Anon,, 2015a). The relationship of labour supply and demand results in employment and unemployment (Baah-Boateng, 2013). The demand for labour arises from businesses needing the engagement of staff to produce results, which leads to an increase in national output (Baah-Boateng, 2013). As a result, weak economic expansion and low employment levels may increase the unemployment rate (Baah-Boateng, 2013). On the other hand, the demand for labour is also influenced by an increase in the number of people who are working age and an improvement in human resources, that alternately promote both the development and quality of the labour force (Baah-Boateng, 2013). If there are not enough job openings to keep up with the increased expansion of the workforce, the rate of unemployment may also increase. If the quality of the labour pool does not fulfil the hiring requirements of the businesses, an economy may also experience structural unemployment. The discrepancy between economic growth and job creation can be partly attributed to the slow expansion of industries like mining and finance, which provide few jobs, compared to the fast growth of industries like agriculture and manufacturing, which absorb a lot of labour. An expanding work force relative to fewer employment options helps to explain the issue of unemployment in the nation with regard to the labour market's supply side (Baah-Boateng, 2013). Between 1984 and 2010, Ghana's labour force increased by nearly half, from 5.6 million people to 10.9 million, representing a yearly average increase of 2.6%, while employment increased by a mean annual growth of 2.48% during the same period (Baah-Boateng, 2013). A measure of unemployment is the resulting 0.12 percentage point average annual gap. Jobseekers increased from 157,624 in 1984 to 863,740 in 2000, then decreased to 632,994 in 2010, indicating an average yearly increase in unemployment of 5.5% between 1984 and 2010 (Baah-Boateng, 2013). According to Baah-Boateng (2013), the poor standard of the work force or its lack in relation to the skills required on the labour market also contributes to Ghana's high unemployment rate. In contrast to the more than a quarter of individuals of working age who had no formal education in 2010, less than a quarter of persons of working age had completed secondary education or higher in 2010. Additionally, with only a basic education, approximately half of the workforce lacks the transferrable skills needed to operate in the official sector of the labour market other than having the capacity to read and write (Baah-Boateng, 2013). While the proportion of people of working age without any sort of schooling has fallen from 47 percent in 1991/92 to 29 percent in 2010, it has increased from 11 percent to 24 percent for those with a secondary education or higher and from 42 percent to 48 percent for those with only a basic education (Baah-Boateng, 2013). This demonstrates that over time, the quality of the workforce has improved. In order to employ the growing number of highly educated and skilled workers whose primary employment goal is the formal sector, which has seen employment growth average roughly 1.8% yearly since 1984, and prevent increased unemployment in the formal sector, this calls for quicker economic expansion in the formal sector. The employment to population ratio gauges the economy's capacity to accommodate an increasing labour force (Anon., 2019a). It is the percentage of people who are employed in the population (Anon., 2019a).

2.8.1 Agriculture Sector

Even though the agricultural sector's proportion of overall employment decreased by 8% between 1984 and 2000, it still provides the majority of jobs for Ghana's expanding workforce. In 1991 and 1992, the agriculture industry employed almost 6 out of every 10 Ghanaians (Anon., 2015c). More than half of Ghana's workers are engaged in agriculture, despite a decline in the quantity of agricultural workers. In 2013, employment rates in manufacturing and agriculture both decreased by 3 and 6 percent, respectively, while services saw a 1.2 percent increase (Anon., 2015c). The primary source of income for Ghanaian employees is still, however, agriculture, which in 2012–2013 accounted for around 52% of all employment (Anon., 2015c). Although agriculture has lost its dominant position in the economy to the services sector, the decline in its employment share at the expense of industry and services have not been as severe. The agriculture industry continues to rely largely on rainfall, which has proven challenging to harness throughout the years. The selection of the wrong technology has thwarted the few attempts to put policies into place aimed at conserving and exploiting rainfall to boost agriculture. Land acquisition for extensive commercial farming is challenging. A lack of readily available financing for agricultural expansion, "widespread bushfires, post-harvest losses, a lack of storage facilities, expensive transportation, and marketing issues" are among additional obstacles. It is essential that the restrictions and obstacles be eliminated and the industry modernised if agriculture is going to continue to play an important economic role and to thrive, supply industry with high-quality raw materials to enable greater productive employment generation. Estimates of quarterly gross economic growth give a short-term (within three months) overview of the state of the economy (Anon., 2021e). Jobs are created and the labour market experiences steady expansion as a result of these economic changes. The crops sub-sector of the agriculture industry expanded by 1.4 percent in the first quarter of 2021 compared to 2.8 percent in the fourth quarter (Anon., 2021e). In comparison to the 1.7 percent growth seen in the fourth quarter of 2020, the cattle sub sector expanded by 1.3 percent (Anon., 2021e). In contrast to a 2.4 percent decline in the fourth quarter of 2020, the forestry and logging subsector expanded by 0.1 percent (Anon., 2021e). In contrast to a gain of 0.4 percent in the fourth quarter of 2020, the fishing sub sector shrank by -0.9 percent (Anon., 2021e). In accordance with Baah-Boateng (2013), the agricultural sector, which accounted for more than 50% of all employment until 2010, witnessed an average 3.3 percent growth in output.

According to a study from the Ghana statistical service's 2021 population and housing census, the agricultural industry currently employs 33% of the country's total employment, down from 60% in 1984 and 42% in 2010 (Anon., 2021a).

2.8.2 Industry Sector

The present private-sector-led industrialization programme in Ghana was developed from the import substitution industrialization (ISI) model after the country gained independence (Anon., 2015c). With the anticipation that it would have expanded above the 25% increase that it recorded in 2005, the industrial sector is in the forefront of structural transformation (Anon., 2015c). A higher chance of employment and the creation of new jobs is also suggested by the sector's greatest level of labour productivity. Despite this, the sector confronts significant operational and management challenges. Like in the other two sectors, land administration is a big issue in this one as well (Anon., 2015c). Ineffective land use practises, "numerous sales, multiple ownership, and delays in document processing" are all substantial barriers to investment that obstruct the development of jobs (Anon., 2015c). Although the ERP was generally effective in facilitating the economy, the anticipated expansion in industrial activity, particularly in producing goods, and investments from the private sector were not feasible because of the sector's continued difficulties. Foreign investment campaigns have received a lot of attention at the maximum levels of government since foreign direct investment (FDI) has significantly contributed to developments in the industrial sector. However, the most recent data shows a decline in FDI, which has a big impact on the growth of employment in the sector. A significant amount of value will be retained in-country if the emphasis is shifted from the long-standing development of FDI zones to models connecting FDI to domestic corporate partnerships, establishing connections with regional vendors of raw materials, components, and value-added services through subcontracting, collaboration exchanges, and other agreements. (Anon., 2015c). Over 80% of all employment in the industrial sector or 11% of all employment resides in the manufacturing subsector, which continues to be the largest industrial sector employer (Anon., 2015c). But according to the results of the population and housing census done in 2021, the industry only employs 14% of the working force (Anon., 2021a). With 14% of the workforce employed, industry is the sector of the economy that employs the least amount of individuals in Ghana. This contrasts with roughly 13% in 1984 and 15% in 2010 (Anon., 2021a). Construction had a 3.5 percent rise in the first quarter of 2021 within the Industry sector, as opposed to a 1.9 percent growth in the fourth quarter of 2020 (Anon., 2021e). In comparison to a growth of 2.9 percent in the fourth quarter of 2020, the manufacturing sub-sector rose by 2.0 percent in the first quarter of 2021 (Anon., 2021e). The sub-sector of “water supply, sewerage, waste management, and remediation activities” had growth in the first quarter of 2021 of 1.6 percent compared to 0.5 percent in the fourth quarter of 2020 (Anon., 2021e). After declining by -3.0 percent in the fourth quarter of 2020, the mining and quarrying sub sector continued to decline by 2.6 percent in the first quarter of 2021 (Anon., 2021e). Similar to this, between 1984 and 2010 the manufacturing sector, which employs roughly 10% of the nation's workers, achieved a 5.2 percent yearly average output increase (Baah-Boateng, 2013). By comparison, from 1984 to 2010, the oil sector; which directly employs less than 100 Ghanaians grew at a mean annual pace of 6.6%. In 2011, the oil sector, combined with mining—which accounts for less than 1% of Ghana's workforce—grew at a yearly mean of 6.6%, contributed 50% of the 15.2% growth (Baah-Boateng, 2013).

2.8.3 Services Sector

The services sector overtook manufacturing as the biggest contributor to GDP in 2010 as the economy turned around. A 4.2% growth rate was recorded in the industry in 2011, which contributed 48.1% of the GDP (Anon., 2015c). Transport, storage, and communications, wholesale and retail trade, hospitality and restaurant services, finance and insurance, real estate, and business services have experienced the largest size growth within the services sector, in that order (Anon., 2015c). The services industry, which has overtaken manufacturing as the largest economic sector, is anticipated to become the main driver of employment growth. With a share of 53% of GDP at base prices in the first quarter of 2021, the services sector was the largest sector of the Ghanaian economy (Anon., 2021e). The Upper West area has 36,676 workers, 24,699 of whom are permanent employees and 13,977 of whom are (Anon., 2015a) males, making it the region with the lowest proportion of permanent workers in the services industry. With 926,871 permanent employees, including 357,232 women and 569,639 men, the Greater Accra region has the nation's greatest concentration of workers in the services industry (Anon., 2015a). According to the population and housing census conducted in 2021 by the Ghana Statistical Service, the services sector, in contrast, employed 53% of the population, up from 43% in 2010 and 27% in 1984 (Anon., 2021a). Although the services sector continued to drive expansion for decades and increased by 13.4% in the third quarter of 2021, the country's annual economic growth was still mostly driven by this sector (Anon., 2021a). Experts start to wonder whether this expansion is the best course for sustaining economic growth for the nation as far as job creation is concerned when it becomes the norm as it has been for decades. Compared to a growth of 9.4% in real estate activities and a growth of 6.6 percent in information and communication in the 4th quarter of 2020, the first quarter of 2021 saw real estate and information and communication activities grow at a slower rate in the services sector, 2.4 percent and 4.3 percent, respectively (Anon., 2021e). In comparison to a growth of 1.4 percent in the fourth quarter of 2020, the trade sector, auto repair, and household goods all saw growth of 0.8 percent in the first quarter of 2021. In the first quarter of 2021 compared to the fourth quarter, Social Security grew by 1.4 percent of 2020, public administration, and defence (Anon., 2021e). The subsectors of professional, administrative (Powell, 2015) and support (-0.8%), as well as hotels and restaurants (-4.4%), saw declines (Anon., 2021e). The government of Ghana has made a concerted effort to promote the growth of the nation's services sector, whose quick development has accelerated overall economic growth and helped to reduce unemployment in Ghana (Powell, 2015). The service sector's economic influence is nevertheless constrained by a number of variables. Nearly half of Ghana's GDP and a sizeable portion of its commerce are accounted for by the services sector (Powell, 2015). Between 2005 and 2013, the percentage of the services sector in Ghana's GDP rose from 32.2 percent to 48.8 percent, overtaking agriculture as the largest economic sector in the nation (Powell, 2015). Furthermore, a sizable portion of Ghana's jobs—43.1% of all jobs—were in the service sector in 2010 (Powell, 2015). The tourism sector, which includes hotels and restaurants, experienced slower average annual growth from 2009 to 2013 than the country's overall GDP (9.6%), and its GDP share dropped from 6.2 to 5.2% (Powell, 2015). Despite being a significant source of foreign revenue, the World Trade Organisation (WTO) reported in its Trade Policy Review that from 2006 to 2013, receipts and employment in the tourism sector more than doubled, reaching over $1.9 billion and 319,000, respectively (Powell, 2015). In addition to accessibility and government backing, Ghana has a multitude of natural, cultural, and historical sights that are advantageous to the country's tourist business (Powell, 2015). The use of public-private partnerships to enhance investment in infrastructure, promoting Ghanaian travel to new audiences, and supporting worker training to generate employment are just a few of the strategies the government has in mind to foster growth in the tourism sector (Powell, 2015). The areas of particular importance are domestic travel, ecotourism, cultural tourism, and business travel. Additionally, the expansion of the tourism industry is restricted due to lack of competent employees, a lack of hotel rooms, and insufficient healthcare services in some regions (Powell, 2015). As a result, spending on related services and goods like building, ICT services, food, and fuel may be affected (Powell, 2015). One of the two sectors driving the services sector's spectacular development has been the banking, insurance, and real estate subsector, but it only employs 0.7% of the workforce nationwide (Baah-Boateng, 2013). According to Baah-Boateng (2013), trade employment, which also experienced strong GDP growth, was largely responsible for the enormous upsurge in the working population in the services sector between 1984 and 2010, going from 26 to 44 percent of all employment. In fact, one factor contributing to the issue of unemployment in

Ghana, both specifically and generally, can be explained by the slower expansion of sectors with low potential for job creation compared to faster growth of sectors with strong labour absorption.

2.9 Job Creation in Ghana

The process of creating new jobs, particularly for the unemployed, is known as job creation. It measures the net hiring of adult workers on a full- and part-time basis. Numerous possibilities for paid employment are being made available, especially for individuals who are unemployed. A total of 207,492 jobs were added nationwide in 2014 (Anon., 2015b). Household agriculture and other domestic tasks are not included in the jobs. Both Ghana's public and private sectors economy contribute new jobs by expanding non-household establishments. Due to this, as of 2014, the agriculture industry had added 1,756 jobs overall. The Ghana Statistical Service reports that the services sector added 181,641 new jobs (Anon., 2015b). A total of 24,095 new jobs were added to the industrial sector during that time. The number of jobs produced by informal businesses in the services sector was significantly higher than the 79.6% created by legal businesses. The number of employments produced in the industrial sector was also larger in informal firms than in formal ones. Contrarily, the number of employments produced in the agriculture sector's formal and informal establishments was larger, with the former accounting for 57.1% and the latter for 42.9% of all positions.

There were 27,931 unskilled jobs and 17,9561 skilled jobs out of the total number of employments produced in Ghana's three major economic sectors. With 151,041 skilled positions and 30,600 unskilled employments, the services sector had the most jobs overall. 1 185 skilled jobs and 571 unskilled jobs were identified in the agricultural industry. Overall, 179,561 skilled positions or 86.5% of all new jobs—were generated throughout the three major economic sectors, compared to just 27,93 unskilled jobs representing 13.0% (Anon., 2015b).

The overall ownership and size classification of establishments is in the private sector, which includes establishing public-private partnerships. the highest number of jobs, which was 182,856 during the period, which accounts for nine times the number of jobs created by state-owned enterprises of 24,636. The majority of the jobs created by the state-owned institutions were in the services sector, taking 93.6%. Arguably, the part of the private sector as the engine room in the country's economic growth process cannot be ignored. Given this anticipation, it is crucial for policymakers and researchers to know whether the private sector is expanding and creating the required number of jobs to combat the increasing unemployment (Anon., 2015b). Every country's economic development process depends on its citizens' skill sets. As economies move from being dependent on agriculture to being dependent on industry and service provision, workers and firms must be able to learn new social, technical, and entrepreneurial skills. The inability to develop new skills due to a lack of opportunity slows down the transfer of all production aspects from lower to higher value added activities (Anon., 2015b).

CHAPTER 3

REVIEW OF THEORY AND SOME FUNDAMENTAL CONCEPTS

3.1 Background

This section reviews relevant materials on theoretical concepts used in this research. It reviews theories on linear and nonlinear systems of ordinary differential equations, existence and uniqueness, equilibrium analysis, linearization and some theorems on stability analysis.

3.2 Systems of Linear and Nonlinear Ordinary Differential Equations (ODEs)

The study of linear systems of ODEs is a foundational topic in differential equations (Hirsch *et al.*, 2012). A linear system of ordinary differential equations (ODEs) is a collection of first-order differential equations that exhibit a linear relationship between the dependent variables and their derivatives. Mathematically, a linear system of ODEs is represented as:

$$\frac{dx_i}{dt} = Ax_i + c \tag{3.1}$$

where dx_i/dx_i is a vector of the derivatives of the dependent variables with respect to the independent variable t, A is a constant matrix that defines the coefficients of the derivatives in terms of the dependent variables, and c is a vector representing a constant term, often associated with external influences or sources. The system is homogenous if $c = 0$.

Linear systems of ODEs provide a structured framework for modelling various phenomena and can be analysed using linear algebraic techniques.

A system of nonlinear ordinary differential equations (ODEs) refers to a set of equations that describe the behavior of multiple dependent variables and their derivatives with respect to an independent variable, where the relationships between variables are nonlinear in nature. Nonlinear systems of ODEs play a significant role in various scientific and engineering applications (Strogatz, 2018). Systems of nonlinear ODEs are characterized by their rich and often intricate behavior, which can include phenomena such as chaos, bifurcations, and complex dynamics. Mathematically, a system of n nonlinear ODEs can be expressed as

$$\frac{dx_i}{dt} = f_i(x_i, t); i = 1, 2, \ldots, n \quad (3.2)$$

or its expanded form as

$$\begin{cases} \frac{dx_1}{dt} = f_1(t, x_1, x_2, \ldots x_n) \\ \frac{dx_2}{dt} = f_2(t, x_1, x_2, \ldots x_n) \\ \vdots \quad \vdots \quad \quad \vdots \\ \frac{dx_n}{dt} = f_n(t, x_1, x_2, \ldots x_n) \end{cases} \quad (3.3)$$

where $x_i(i = 1, 2, \ldots, n)$ is a vector representing the n dependent variables, dx_i/dt is a vector representing the derivatives of the various dependent variables with respect to time t, which represents the independent variable, $f_i(i = 1, 2, \ldots, n)$ are nonlinear functions defining the relationships among the variables.

The study of nonlinear dynamical systems, including nonlinear ODEs, is a cornerstone of modern mathematics (Perko, 2001). This probably explains why systems of nonlinear ODEs have been applied as a modelling tool in various applications.

3.3 Existence and Uniqueness of Solution to ODEs

A significant thing to consider before finding solution to ODEs is to attempt to address key mathematical issues of existence and uniqueness (Olver, 2017). As practically as feasible, it is significant to find out if a solution exists else, it does not sound rational to attempting to find one. Next, it is also important to know if the solution is unique, else, the given differential equation possibly has negligible significance in respect of its physical or pragmatic applications as it loses its relevance as a tool for prediction. Since differential equations inevitably have lots of solutions, "the only way in which we can deduce uniqueness is by imposing suitable initial (or boundary) conditions.

As already noted, a system of ordinary differential equations (ODEs) can be broadly categorized into linear and nonlinear systems based on their mathematical properties. Linear systems exhibit a linear relationship between the dependent variables and their derivatives, while nonlinear systems involve more complex relationships that are not strictly linear. The major task in this section is to explain existence and uniqueness with regard to linear and nonlinear systems of ODEs.

3.3.1 Existence

For linear systems of ODEs, the existence of solutions is well-established. Linear systems are characterized by equations of the form: $dx_i/dt = Ax_i$, where x_i is a vector of dependent variables, and A is a constant matrix. The existence and uniqueness of solutions for linear systems are rooted in foundational results such as the matrix exponential and linear algebra techniques (Hirsch *et al.*, 2012).

However, the existence of solutions for nonlinear systems of ODEs is a more intricate matter. Nonlinear systems can exhibit a wide range of behaviors, including chaos and multiple steady states. The Picard-Lindelöf theorem, as extended to the Banach space setting, is a powerful tool for establishing the existence of solutions for certain classes of nonlinear systems. The existence of solutions for nonlinear systems of ordinary differential equations (ODEs) hinges on the Picard-Lindelöf theorem, extended to the Banach space setting. The theorem establishes the conditions under which solutions are guaranteed to exist and be unique for systems of the form Equation (3.2) or (3.3). Significant to the applicability of the Picard-Lindelöf theorem is the Lipschitz condition.

Definition 3.1 (Lipschitz condition) Let the vector valued function $f(x_i,t)$ be defined within a domain $D \subset \Re^n \times R$. Then $f(x_i,t)$ is said to satisfy the Lipschitz condition if there exists a constant $L > 0$ such that for all x_1 and x_2 in D : $\|f(x_1,t) - f(x_2,t)\| \le L\|x_1 - x_2\|$ where $\|\cdot\|$ denotes a suitable norm, such as the Euclidean norm. In simpler terms, the Lipschitz condition ensures that the rate of change of the system's components does not vary too rapidly as the variables change. When this condition holds, the Picard-Lindelöf theorem guarantees the existence of solutions for the given nonlinear system of ODEs.

3.3.2 Uniqueness

In the realm of linear systems of ordinary differential equations (ODEs), the concept of uniqueness of solution plays a pivotal role. Uniqueness assures that for a given initial condition, there exists only one solution trajectory in the state space that satisfies the ODE system. This property is particularly notable in linear systems due to their well-defined and deterministic behaviour. For a linear system of ODEs of the form Equation (3.1), the uniqueness of solution asserts that if $x_1(t)$ and $x_2(t)$ are two solutions for the system that share the same initial condition $x(t_0)=x_0$, then they must be identical, that is, $x_1(t)=x_2(t)$, for all time t. In other words, there must exist a unique solution $x(t)$ that satisfy the initial condition $x(t_0)=x_0$.

This property arises due to the linearity of the system, which ensures that any linear combination of solutions is itself a solution. Thus, if two solutions coincide at a single point in time, they will coincide at all points in time. The uniqueness of solution to linear systems of ODEs is a fundamental characteristic in their analysis (Hirsch *et al.,* 2012).

However, in the realm of nonlinear systems of ODEs, uniqueness of solutions is a fundamental property. It guarantees that for a given set of initial conditions, there exists only one trajectory that the system follows. This concept is pivotal in ensuring the well-posedness and predictability of nonlinear dynamical systems. The uniqueness theorem for nonlinear systems of ODEs builds on the Lipschitz condition and is often referred to as the Picard-Lindelöf theorem. It states that if a vector-valued function $f(x_i,t)$ satisfies the Lipschitz condition in the variable x_i within a certain domain D, then for any initial condition $x(t_0)=x_0$ within D, there exists a unique solution $x(t)$ defined on an interval containing t_0.

3.4 Equilibrium Solution

An equilibrium solution (also known as a fixed point or steady state) of a linear system of ordinary differential equations is a solution in which the derivatives of all variables are zero (Aström and Murray, 2008). In mathematical notation, a linear system of the following form Equation (3.1), an equilibrium solution x_i^* satisfies the condition $Ax_i^*=0$

In other words, the equilibrium solution is the vector x_i^*, where a variation in the rate of each state variable is zero, leading toward a steady state.

It is eminent to note that this definition applies specifically to linear systems of ODEs. Nonlinear systems have equilibrium solutions as well, but they are typically found by solving for points where all derivatives are zero using nonlinear equations (Smith, 2000).

For a nonlinear system of ODEs represented as: $dx_i/dt = f_i(x_i)$, where x exists as a vector of state variables and $f_i(x_i)$ is a vector-valued function, then an equilibrium solution x_i^* satisfies the condition $dx_i/dt = f_i(x_i) = 0$.

In other words, the equilibrium solution is the vector x_i^* for a derivative of which each state variable in relation to time is zero, resulting in a steady state. Notably, this definition extends the concept of equilibrium solutions to nonlinear systems. Here, the equilibrium is achieved when the rate of change of each state variable is zero Equilibrium solutions of ordinary differential equations (ODEs) can be classified based on their stability properties. Stability refers to the behaviour of solutions at or around an equilibrium point. The main classifications are:

Stable Equilibrium (Asymptotically Stable):

An equilibrium solution x_i^* is stable if, for any small perturbation $\varepsilon > 0$, there exists a $\delta > 0$ in such a way that whenever $\|x_i(0) - x_i^*\| < \delta$, the solution $x_i(t)$ approaches x_i^* as t goes to infinity. Mathematically, this is represented as: $\lim_{t\to\infty} \|x_i(t) - x_i^*\| = 0$.

Unstable Equilibrium: An equilibrium solution x_i^* is unstable if, for any small perturbation $\varepsilon > 0$, no matter how small $\delta > 0$ is chosen, there exist initial conditions $x_i(0)$ such that $\|x_i(0) - x_i^*\| < \delta$ but the solution $x_i(t)$ diverges away from x_i^* as t goes to infinity. Mathematically, this is represented as: $\lim_{t\to\infty} \|x_i(t) - x_i^*\| = \infty$.

Semi-Stable Equilibrium:

An equilibrium solution x_i^* is semi-stable if it is stable in some directions (perturbations) but unstable in others. This means that solutions starting near x_i^* can approach x_i^* along some trajectories while diverging along others.

Intuitively, to say an equilibrium solution x_i^* of a nonlinear system of ODE is stable, it implies that the system has inbuilt mechanism that ensures that any perturbation or disturbance that causes the orbits of the system to move far away from x_i^* are restored back to x_i^*. However, if the movement of the solution from x_i^* cannot be restored back to x_i^*, then the system is unstable.

It is important to note that these classifications are particularly relevant for nonlinear systems of ODEs. Linear systems have a simple-criteria for stability based on the eigenvalues of the coefficient matrix. Nonlinear systems can exhibit more complex behaviours, and stability analysis often involves advanced techniques which will be discussed in the ensuing sections (Smith, 2000; Aström and Murray, 2008).

3.5 Positivity of Solution to a System of ODE

The positivity theorem is often used in the study of dynamical systems to establish the positivity or non-negativity of solutions to certain nonlinear ordinary differential equation (ODEs) systems. This theorem is commonly used within the context of systems that represent quantities that cannot be negative, such as populations or concentrations (Perko, 2001; Stogatz, 2014).

Theorem 3.1 (Positivity of Solution to ODE) Consider a system of ODEs of the form (3.2), where x_i is a state variable vector and $f(x_i,t)$ is a continuous vector-valued function with non-negative components. If $x_i(0)$ is a non-negative initial condition, then the solution $x_i(t)$ remains non-negative for t in specified domain where the solution exists.

The proof of the positivity theorem depends on the notion that all components of the vector $f(x_i,t)$ are non-negative, which means that each component of the derivative dx_i/dt is influenced only by non-negative terms. This prevents the solution from crossing into negative territory. Here's a general sketch of the proof:

Assumption: Assume that the vector-valued function $f_i(x_i,t)$ has non-negative components, i.e., $f_i(x_i,t) \geq 0$ for all i

Non-Negative Initial Condition: Assume that $x(0)$ is a non-negative initial condition, i. e. $x_i(0) \geq 0$ for all i.

Non-Negativity of Derivative: Since $f_i(x_i,t) \geq 0$ and $x_i(0)$, it follows that $dx_i/dt \geq 0$ for all i.

Integral Inequality: Consider the integral of dx_i/dt over time t from 0 to t such that

$$\int_0^t \frac{dx_i}{dt} dt - x_i(t) - x_i(0)$$

Since dx_i/dt, the integral is non-negative.

Non-Negativity of Solution: The integral inequality implies that $x_i(t) - x_i(0) \geq 0$, which means $x_i(t) \geq x_i(0)$. Since $x_i(0)$ is non-negative, it follows that $x_i(t)$ is also non-negative for all t.

Conclusion: Since each component $x_i(t)$ of the solution $x(t)$ is non-negative for all t, the entire vector $x(t)$ stays as non-negative over the given range of time t.

This proof strategy is a basic outline of how the Positivity Theorem can be demonstrated. However, specific details may vary based on the nature of the system and the assumptions made about the properties of $f_i(x_i, t)$. For a rigorous proof, there is a need to consider the behaviour of each component of the solution and establish the non-negativity property step by step. This will be demonstrated rigorously in the next chapter (Perko, 2001; Stogatz, 2014).

3.6 Boundedness of a Solution to Systems of ODE

The Boundedness Theorem is a result used in the study of dynamical systems that examines the dynamics of solutions to ordinary differential equations (ODEs). It states that under certain conditions, the solutions to a system of ODEs remain bounded over a specified interval.

Theorem 3.2 Define a system of ODEs of the form Equation (3.2), where x_i is a vector of state variables and $f_i(x_i, t)$ is a continuous vector-valued function. If there is a constant K such that $\|f_i(x_i, t)\| \leq K$ for all x within some bounded region, then the solutions to the system are bound for all t within intervals over which the solutions are present.

A sketchy proof can be demonstrated to illustrate how the boundedness theorem can be established using basic properties of norms, inequalities, and integration. The theorem provides a valuable insight into the behaviour of solutions in terms of their boundedness, which is important for understanding the dynamical system's characteristic overtime. This is important to note again that the proof is system-specific or depends on the properties of the system model and its assumptions (Perko, 2001; Strogatz, 2014). A full proof of the boundedness theorem will be accomplished in the light of the nonlinear unemployment model in Chapter 4.

Definition 3.2 Let Ω be a region that absorbs all the positive values of the system Equation (3.2), then Ω is called a positive invariant region given by

$$\Omega = \left\{ x_i(t) \in \Re_+^n : \sum_{i=1}^{n} x_i(t) \le K \right\} \tag{3.4}$$

where K is the infimum expressed by the relation $\lim_{t \to 0} N \le K$.

Definition 3.3 (Well-posedness) A system of ODE $\frac{dx_i}{dt} = f(x_i)$ is said to be well-posed if its solution $x_i(t)$ are all positive and bounded within an invariant region Equation (3.4) (Perko, 2001; Strogatz, 2014).

3.7 Linearization of Nonlinear Systems of ODEs

The approach to establishing the stability or otherwise of linear systems of ODEs is very simple; since the systems are already linear, the characteristics of the eigenvalues are analysed.

Theorem 3.3 Let $dx_i/dt = 0$ be a linear system of ODE of dimension n, and $\lambda_i (i = 1, 2, ..., n)$ be the eigenvalues corresponding to the given system. If the real part of the eigenvalues $\lambda_i (i = 1, 2, ..., n)$ are all positive, the system is stable, and unstable otherwise.

More significantly, the characteristics of linear systems are very easy and simple to analyse compared to nonlinear systems of ODEs. To simplify the analytical techniques of nonlinear systems, it is crucial to linearize the nonlinear systems to their corresponding linear forms. Before the linearization is treated, this section will first examine the following:

Isolated Equilibria: The property of most nonlinear systems is quite remarkable. Around the neighbourhood of the equilibrium solution, they act like the linear systems that approximate such nonlinear systems. To comprehend what is meant by a linear system that approximates a system that is nonlinear near a point of equilibrium, consider the autonomous system of ODE of the form Equation (3.2) with the usual definition of variables; it is important to recall that the equilibrium solution to the above system remains a fixed function x_i^* in which $f\left(x_i^*\right)=0$. A point is the phase space representation of the equilibrium solution in this regard. Most nonlinear systems have at least one solution or none at all, unlike linear systems that have just a single solution. In situations where there are many equilibria, it is important to avoid such situation by dealing with only isolated equilibria (Guckenheimer, and Holmes, 1994; Strogatz, 1994).

Definition 3.4 Isolated equilibria refer to those equilibrium solutions that consists of neighbourhood independent of the other equilibrium solution.

Change of Variables: This is a technique used to execute the process of isolating equilibrium solutions. Since the interest is centred on the character of the flow near an isolated equilibrium, the origin of the frame will be shifted to x_0, which can be accomplished through the change of variables; $v_i = x_i - x_{i0}$. In coordinate system, the change of variable yields $v_i' = Q(v_i)$; where G is the new representation of the vector field and the equilibrium is represented by the origin, that is $Q(0)=0$.

3.8 The Linearization Technique

In general, in analyzing the flow near an isolated equilibrium, the first point of action is to move the origin of the frame to the equilibrium, then the resultant system is linearized about the origin 0. Describing the process in detail, do:

Step 1: Transform the system $\frac{dx_i}{dt} = f\left(x_i\right)$ using the change of variable, $u = x_i - x_{i0}$, to give an equivalent system $v_i' = Q(v_i)$, which have its isolated equilibrium at 0.

Step 2: Linearize through finding the partial derivatives of $Q(v_i)$ in terms of the constituents of v_i at the equilibrium 0 to obtain a matrix A given by

$$A=\begin{pmatrix} \frac{\partial Q_1}{\partial v_1}(0,0,\cdots,0) & \frac{\partial Q_1}{\partial v_2}(0,0,\cdots,0) & \cdots & \frac{\partial Q_1}{\partial v_n}(0,0,\cdots,0) \\ \frac{\partial Q_2}{\partial v_1}(0,0,\cdots,0) & \frac{\partial Q_2}{\partial v_2}(0,0,\cdots,0) & \cdots & \frac{\partial Q_2}{\partial v_n}(0,0,\cdots,0) \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial Q_n}{\partial v_1}(0,0,\cdots,0) & \frac{\partial Q_n}{\partial v_2}(0,0,\cdots,0) & \cdots & \frac{\partial Q_n}{\partial v_n}(0,0,\cdots,0) \end{pmatrix}$$

Step 3: The resulting linearized system around $(0,0,\cdots,0)$ is written as

$$\begin{cases} v_1' = \frac{\partial Q_1}{\partial v_1}(0,0,\cdots,0)v_1 + \frac{\partial Q_1}{\partial v_2}(0,0,\cdots,0)v_2 + \cdots + \frac{\partial Q_1}{\partial v_n}(0,0,\cdots,0)v_n \\ v_2' = \frac{\partial Q_2}{\partial v_1}(0,0,\cdots,0)v_1 + \frac{\partial Q_2}{\partial v_2}(0,0,\cdots,0)v_2 + \cdots + \frac{\partial Q_2}{\partial v_n}(0,0,\cdots,0)v_n \\ \vdots \\ v_n' = \frac{\partial Q_n}{\partial v_1}(0,0,\cdots,0)v_1 + \frac{\partial Q_n}{\partial v_2}(0,0,\cdots,0)v_2 + \cdots + \frac{\partial Q_n}{\partial v_n}(0,0,\cdots,0)v_n \end{cases} \quad (3.5)$$

Modified in the literature (Guckenheimer and Holmes, 1983; Strogatz, 1994).

Theorem 3.4 (Hartman-Grobman) Let x_0 be an isolated equilibrium of the nonlinear system of form Equation (3.2), corresponding to the equilibrium 0 of its linearized system $v_i' = Q(v_i)$ (as detailed in Equation 3.5), then if

i. 0 is the hyperbolic equilibrium of the linear system, the behaviour of the solution to the nonlinear system near x_0 mimics the characteristic of the linear system near 0.

ii. 0 is the nonhyperbolic equilibrium solution to the linear system, no conclusion can be made concerning the behaviour of the flow of the nonlinear system near x_0.

Definition 3.5

i. An equilibrium of the nonlinear system $dx_i/dt = f(x_i)$ is referred to as hyperbolic (source, sink, saddle) provided that the equilibrium corresponding to its linearized form is hyperbolic.

ii. An equilibrium of the nonlinear system $dx_i/dt = f(x_i)$ is referred to as nonhyperbolic provided the equilibrium corresponding to its linearized form is nonhyperbolic.

The equilibrium solutions of linear systems differ markedly from those of nonlinear systems in the sense that the equilibrium solutions of linear systems confirm the global behaviour of the solution, whereas the equilibrium solutions of nonlinear systems impact the qualitative representation of the flow only locally. (Katkok and Hasselblatt, 1995).

3.9 Techniques of Stability Analysis

Stability analysis using the eigenvalue criterion requires finding the roots of the characteristic or behavioural equation epitomizing the system. This auxiliary equation is obtained from the variational matrix resulting from the evaluation of the partial derivatives of the system around the equilibrium points. This is the matrix A above.

Definition 3.6 Let the variational matrix that represents the nonlinear system $dx_i/dt = f(x_i)$ be $J(x_i^*)$, where x_i^* is a vector that represents the equilibrium solution of the given nonlinear system. Then we can obtain a characteristic polynomial $p_n(\lambda_i)$ (of degree n) with $\lambda_i(i=1,2,\ldots,n)$ being the eigenvalues corresponding to the system such that

$$p_n(\lambda_i) = \left|J(x_i^*) - \lambda I\right| = k_n\lambda^n + k_{n-1}\lambda^{n-1} + \ldots + k_1\lambda + k_0 \qquad (3.6)$$

The limitation with the eigenvalue stability criterion has to do with the difficulty in finding the roots of the polynomial in Equation (3.6) for large n or under complex situation. To overcome this limitation, there are other advanced techniques of performing stability analysis without necessarily finding the roots of the polynomial [Equation (3.6)], in other cases, we can circumvent linearization of the linear system in which case the roots of the polynomial [Equation (3.6)] will not be computed. Some of these techniques are discussed below.

3.9.1 The Routh-Hurwitz Stability Criterion

The stability criterion developed by Routh-Hurwitz is a mathematical technique for determining the stability of linear time-invariant systems by examining the coefficients of their characteristic polynomial. This standard allows a suitable means of analysing the stability of dynamic systems without having to calculate their roots directly. It's widely employed in control systems engineering and related fields to ensure the stable and reliable operation of systems. Consider a polynomial in the form of Equation (3.6), where

λ is a complex variable, and $k_n, k_{n-1}, \ldots, k_1, k_0$ are the coefficients of the polynomial. The stability criterion of Routh-Hurwitz examines system stability using polynomial coefficients. The technique involves constructing a table called the Routh-Hurwitz table.

Table 3.1 Routh-Hurwitz Table

s^n	k_n	k_{n-2}	k_{n-4}	• • •	k_0
s^{n-1}	k_{n-1}	k_{n-3}	k_{n-5}	• • •	k_1
s^{n-2}	b_1	b_2	b_3	• • •	b_i
s^{n-3}	c_1	c_2	c_3	• • •	c_i
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
s^0					

The third row will be formed using the first and second rows. Let us see how

$$b_1 = \frac{\begin{vmatrix} k_{n-1} & k_{n-3} \\ k_n & k_{n-2} \end{vmatrix}}{k_{n-1}}, b_2 = \frac{\begin{vmatrix} k_{n-1} & k_{n-5} \\ k_n & k_{n-4} \end{vmatrix}}{k_{n-1}}, b_3 = \frac{\begin{vmatrix} k_{n-1} & k_{n-7} \\ k_n & k_{n-6} \end{vmatrix}}{k_{n-1}}, \text{ etc.}$$

In a similar way after forming the 3^{rd} row, it can use 2^{nd} and 3^{rd} row to form the 4^{th}row

$$c_1 = \frac{\begin{vmatrix} b_1 & b_2 \\ k_{n-1} & k_{n-3} \end{vmatrix}}{b_1}, c_2 = \frac{\begin{vmatrix} b_1 & b_3 \\ k_{n-1} & k_{n-5} \end{vmatrix}}{b_1}, c_3 = \frac{\begin{vmatrix} b_1 & b_4 \\ k_{n-1} & k_{n-7} \end{vmatrix}}{b_1}, \text{ etc.}$$

Step-by-Step Procedure:

Formation of Routh Array: Write down the polynomial coefficients in the first row of the Routh-Hurwitz table. If any coefficient is missing, replace it with zeros. Then, based on these coefficients, construct the subsequent rows of the table by following a given rule.

Checking for Sign Changes: Determine the number of signs variations in the Routh's first column array. This frequency corresponds to the number of roots in the right side of the polynomial of the complex plane (i.e., the number of unstable roots). If no sign variation occurs, the system does not have any unstable root.

Determining Stability: The number of important changes and the subsequent rows of the Routh-Hurwitz table can be used to determine the system's stability. The frequency of significant variations in the first column of the table, in particular, indicates the number of unstable roots. If there are no sign changes, all roots are stable. If the sign varies, then the number of roots that are stable depends on the frequency of sign changes in the table's second column, and so forth.

If all of the elements are zero on row of a Routh array, it implies that there is a repeated root in the system. This method can't determine the exact location of the root in this case.
If there are elements that are very close to zero, but not exactly zero, it may indicate a marginally stable root.

The stability method based on Routh-Hurwitz is a powerful technique for quickly assessing the stability of a system without explicitly finding the polynomial's roots.
It can handle high-degree polynomials efficiently.
It is however significant to note that the Routh-Hurwitz criterion only works for polynomial equations with real coefficients. It can't provide information about the damping ratio or the frequency of oscillation of the system.
To summarise, the stability criterion proposed by Routh-Hurwitz is a valuable mathematical technique in control systems analysis to analyse the stability of linear time-invariant systems using the coefficients of the auxiliary polynomials. It provides useful intuitions regarding the stability characteristics of these types of systems without explicitly calculating their roots. (Dorf, 2010; Ogata, 2010; Franklinn and Powell, 2014)

3.9.2 Poincare Stability Criterion

Poincaré stability refers to the stability of periodic solutions in a dynamical system. It is a concept named after the French mathematician Henri Poincaré and is crucial in understanding the behaviour of systems that exhibit periodic or repetitive motion. Poincaré stability analysis how small perturbations or deviations from a periodic orbit evolve over time. A system is considered Poincaré stable if these perturbations do not lead to significant divergence from the periodic orbit, indicating that the system's oscillatory behaviour remains bounded over time. Poincaré stability is particularly relevant in fields such as celestial mechanics, nonlinear dynamics, and chaos theory.
Mathematically, let's consider a dynamical system described the Equations (3.2),

where x_i signifies the variables of the state system, and $f_i(x_i, t)$ defines a vector field characterizing the dynamics of the system. Assuming a system exhibits a periodic orbit or limit cycle, denoted as

$x_p(t)$, the Poincaré stability analysis involves examining the behaviour of small deviations $\delta x(t)$ from this orbit. The linearized dynamics around the periodic orbit are given by

$$\frac{d\delta}{dt}x = J\delta x \tag{3.7}$$

where in Equation (3.7), J is the vector field's Jacobian matrix $f(x)$ evaluated at the periodic orbit $x_p(t)$.

Poincaré stability analysis assesses the eigenvalues J of the matrix to find the behaviour of perturbations. If all eigenvalues have magnitudes fewer than or equal to 1 (in absolute value), the periodic orbit is Poincaré stable, indicating that small perturbations remain bounded and do not grow exponentially over time. In mathematical terms, a periodic orbit $x_p(t)$ is Poincaré stable if all the eigenvalues λ of J, satisfy $|\lambda| \leq 1$.

This condition ensures that any small perturbations introduced to the system will not lead to unbounded growth and will eventually converge back to the periodic orbit. In summary, Poincaré stability focuses on the behaviour of small perturbations around a periodic orbit in a dynamical system. It provides insights into the long-term stability of oscillatory solutions and is an essential concept in the study of complex and nonlinear systems. (Strogatz, 2015).

3.9.3 Lyapunov Stability Criterion

Lyapunov stability criterion," named after the Russian mathematician Aleksandr Lyapunov. Lyapunov stability is a concept used in the field of dynamical systems and control theory to analyse the stability of equilibrium points in a system.

The Lyapunov Stability Criterion is a fundamental concept in the field pertaining to dynamical systems and control theory. This is used to analyse the stability of equilibrium points (also known as fixed points) of nonlinear systems. This criterion helps determine whether a system's trajectory, when perturbed from an equilibrium point, will stay close to that point over time or diverge away from it.

Given below is a brief sketch of the Lyapunov Stability Criterion.

The Lyapunov Stability Criterion is a method to assess the stability of equilibrium points in dynamical systems. Given a differential equation-based system $dx/dt = f(x)$, in which x represents the state variables and $f(x)$ represents their rates of change, an equilibrium point x^* is a state, where $f(x^*) = 0$.

The criterion employs the concept of a Lyapunov function, which is a scalar function $V(x)$ that quantifies the "energy" or "distance" of the system from the steady state or point of equilibrium. A Lyapunov function must satisfy the following properties:

$V(x^*) = 0$: The function should be zero at the point of equilibrium.

$V(x) > 0$ for $\neq x^*$: The function should be positive-definite for points other than the equilibrium.

$dV/dt \leq 0$: The time differential of the Lyapunov function along the system's paths or trajectories must be non-positive (or negative) for all of the points within the path of the system's state space except possibly at x^*.

If a valid Lyapunov function can be found for an equilibrium point, and its time derivative satisfies $dV/dt \leq 0$ in the region around the equilibrium (except possibly at the equilibrium itself), the point of equilibrium is then Lyapunov stable. Additionally, $dV/dt < 0$ for all points except at x^*, the point of equilibrium is thus, asymptotically stable, indicating that trajectories initiated near the equilibrium point converge to it over time. It is imperative to note that Lyapunov's criterion doesn't guarantee stability for all possible trajectories of a system, but rather for a specific equilibrium point and the surrounding region in the state space. If no valid Lyapunov function can be found, the criterion doesn't provide conclusive information about stability.

In summary, the Lyapunov Stability Criterion offers an indispensable tool for assessing the behaviour, around the equilibrium points, of dynamics systems that are nonlinear, thereby enabling engineers and researchers to determine the stability characteristics of complex systems in various fields, including engineering, physics, and biology. (Khakil, 2002; Slotine and Li, 1991; Sastry, 1999).

CHAPTER 4

A NONLINEAR ODE MODEL FOR UNEMPLOYMENT DYNAMICS IN GHANA'S LABOUR MARKET

4.1 Background

In this section, a system of the nonlinear differential equation model to study the unemployment dynamics in Ghana's labour market in the three main economic sectors is proposed; a seven-state model will be developed, the equilibrium points will be derived, and stability (local and global) of the equilibrium points will be analysed. The objective is to develop a nonlinear differential equation model to study the unemployment dynamics in Ghana's labour market with regard to the three main economic sectors to reflect the significance of sectoral contribution of the Ghanaian economy through economic growth which stimulates vacancy creation with both employed and unemployed class.

4.2 Developing the Nonlinear ODE Unemployment Model

The nonlinear differential equation model which is an extension of (Pathan and Bhathawala, 2017a) model was adapted and developed to suit this study. Entrants to the unemployment state are completely legitimate to engage in any job at any time t. Available employment in the agriculture sector by both government and private sectors denoted by A, available employment in the industry sector by both government and private sectors as I and available employment in the services sector by government and private sectors as S. Denoted by U_A, U_I and U_S are, respectively, the number of unemployed persons in Agriculture, Industry and Service. Likewise, E_A, E_I and E_S denote the number of employed people in Agriculture, Industry and Service respectively. Newly created vacancies by governments and private sector in the agriculture, industry and services sectors are represented as C. The number of available vacancies in the market is $A + C - E_A$, $I + C - E_I$ and $S + C - E_S$. The number of unemployed persons in all sectors grows at a constant rate a_1. The rate of transition from the unemployment state to the employment state is represented by a_i where $i = 2,3,4\ldots$, thus, the rate of transition of persons from the unemployment class to the employment class in the Agriculture Sector is represented by a_2, the rate of transition of individuals from the unemployment class to the employment class in the Industry Sector given by a_3, the rate at which unemployed persons transits to the employment class in the Services Sector given by a_4. The rate constant a_5 is a positive constant which proxies the retirement and death rates of employed persons in a sector, a_6 signifies the rate at which individuals in the employment class become unemployed, the resignation rate of persons in the employment state out of a sector is represented as a_7. Finally, the creation of vacancies greatly depends on the number of individuals in unemployment and employment states in the specified sectors. Signified by α and δ are the rate at which new vacancies are created by the government and private sectors in favour of individuals in the unemployment and employment classes respectively, β signifies the rate of decadence (diminution) of newly created vacancies.

4.3 Assumptions of the Model

The model assumes that;

i. Entrants to labour force refer persons in both the employment and unemployment classes who legitimately fall within the working age group (18 to 60 years).
ii. Individuals who transit to the unemployment state are completely legitimate to engage in work at any time t.
iii. Subject to the number of persons: unemployed and employed in the agriculture, industry and service, vacancies are jointly created.
iv. The rate of transition of persons from the unemployment state to the employment state is collectively proportional to U_A, U_I , U_S and $A + C - E_A$, $I + C - E_I$, $S + C - E_S$ respectively.
v. The model does not allow movement across different sectors. Thus, an employed person in a given sector cannot be employed in a different sector at the same time.
vi. The death rate remains the same for unemployed and employed individuals.
vii. The rate at which employed individuals resign from a sector is unique for all given sectors of the economy.
viii. The rate of unemployed persons in all sectors increases with a constant rate
ix. Vacancy creation emanates from resignation, retirement or death of individuals across the specified economic sectors.
x. Individuals in the employed compartment across the three sectors can be fired (dismissed) by employers or resign from their job.

4.3.1 Flowchart for the Unemployment Model

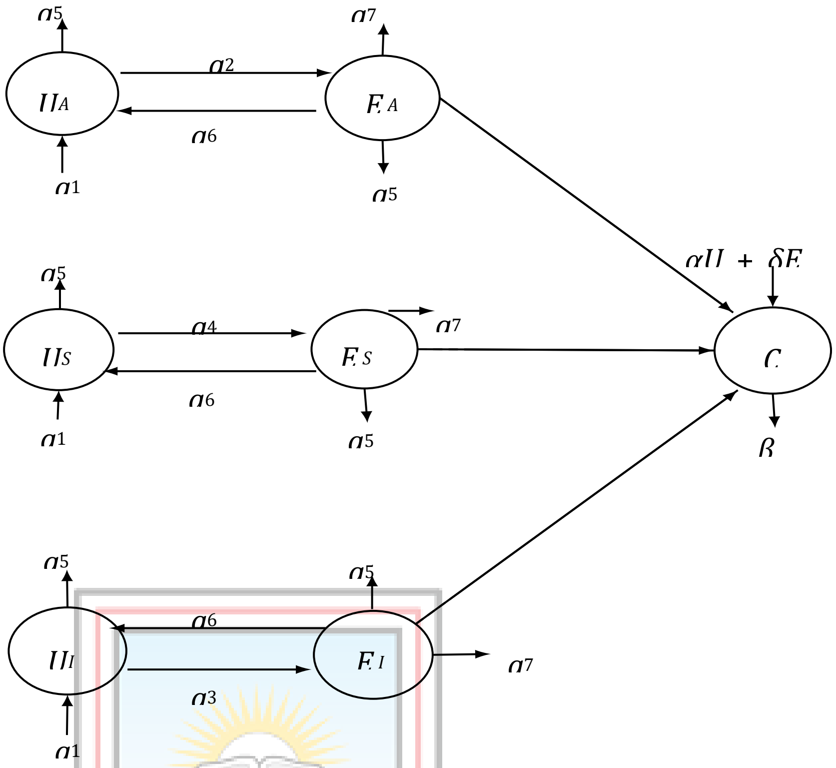

Fig. 4.1 Schematic Diagram of the Model

4.4 The Model Formulation

Resulting from the assumptions made and the schematic compartmental model depicted, the equations that govern the model are given by the nonlinear ODEs in Equations (4.1) to (4.7).

$$\frac{dU_A}{dt} = a_1 - a_2U_A\left(A + C - E_A\right) - a_5U_A + a_6E_A \tag{4.1}$$

$$\frac{dU_I}{dt} = a_1 - a_3U_I\left(I + C - E_I\right) - a_5U_I + a_6E_I \tag{4.2}$$

$$\frac{dU_S}{dt} = a_1 - a_4U_S\left(S + C - E_S\right) - a_5U_S + a_6E_S \tag{4.3}$$

$$\frac{dE_A}{dt} = a_2U_A\left(A + C - E_A\right) - a_5E_A - a_6E_A - a_7E_A \tag{4.4}$$

$$\frac{dE_I}{dt} = a_3U_I\left(I + C - E_I\right) - a_5E_I - a_6E_I - a_7E_I \tag{4.5}$$

$$\frac{dE_S}{dt} = a_4 U_A (S + C - E_S) - a_5 E_S - a_6 E_S - a_7 E_S \quad (4.6)$$

$$\frac{dC}{dt} = \alpha (U_A + U_I + U_S) + \delta (E_A + E_I + E_S) - \beta C \quad (4.7)$$

Table 4.1 Parameters and Variables Definition

Variable and Parameter	Definition
U_A	Number of unemployed individuals in the Agriculture Sector at time t
U_I	Number of individuals unemployed at time t in the Industry Sector
U_S	Number of individuals unemployed at time t in the Service Sector
E_A	Number of individuals employed at time t in the Agriculture Sector
E_I	Number of individuals employed at time t in the Industry Sector
E_S	Number of individuals employed at time t in the Service Sector
C	New vacancies created at time t in all three sectors
A	Number of vacancies presently created in the agriculture sector by government and private individuals
I	Number of vacancies presently created in the industry sector by government and private individuals
S	Number of vacancies presently created in the Services sector by government and private individuals
a_1	Constant rate at the number of unemployed individuals increases in all sectors
a_2	Rate of transition of persons from the unemployment to the employment class in Agriculture Sector.
a_3	The rate of transition of individuals from the unemployment class to the employment class in the Industry Sector.
a_4	The rate of transition of persons from the unemployment class to the employment class in the Services Sector.
a_5	The rate of death and retirement of the employed from a sector
a_6	Rate of transition persons from the employment to unemployment
a_7	Rate at which employed persons resign from a sector
α	Rate at which new vacancies are created in favour of the unemployed

δ	The rate at which new vacancies are created in favour of the employed
β	The rate of decadence of newly created vacancies in all sectors

4.5 Qualitative Analysis of the Nonlinear ODE Unemployment Model

4.5.1 Positivity of the Solutions

In order to establish the characteristics of the model over time, we show that the solution to this model is non-negative and bounded. Thus, this is because the model is about a human population and hence cannot be negative. We ascertain boundedness to show the solution stays in the defined region or domain prescribed but does not increase to infinity. First, we need to show the following lemma which ensures that, under suitable conditions; all the solutions of the system of Equations (4.1) to (4.7) are non-negative. We also identify a set Ω in $\mathbb{R}_+^7$ such that all solutions starting from Ω remain bounded.

Theorem 4.1 Let all parameters in Equations (4.1) to(4.7) be positive and define Ω to be a region in $\mathbf{R}_+^7$ defined as given below.

$$\Omega = \left\{ U_A, U_I, U_S, E_A, E_I, E_S, C \in \mathbb{R}^7 \leq U_A + U_I + U_S + E_A + E_I + E_S + C \leq \frac{3a_1}{\gamma} \right\},$$

where$\gamma = min\{(a_5), (a_5 + a_7 - \delta), \beta\}$ is a region of attraction for the system Equations $(4.1) - (4.7)$ and every solution that originates within the positive octant is attracted into this region. Thus, the solution is positive and bounded

Positivity. First, we establish the positive invariant part of the theorem. By the method of contradiction, let $[U_A(0), U_I(0), U_S(0), E_A(0), E_I(0), E_S(0), C(0)] \in \Omega$. Suppose $U_A(t)$ be non-positive. Hence, there is $t_0 > 0$ which ensures that $U_A(t_0) = 0$ and $U_A(t) > 0$ for any t satisfying $0 \leq t \leq t_0$ such that

$$\left. \frac{dU_A}{dt} \right|_{t=t_0} \leq 0 \tag{4.8}$$

This is a contradiction, because

$$\left. \frac{dU_A}{dt} \right|_{t=t_0} = a_1 - a_2 U_A(t_0)\left(A + C(t_0) - E_A(t_0)\right) - a_5 U_A(t_0) + a_6 E_A(t_0) = a_1 > 0$$

Hence, $U_A(t)$ is positive $\forall t \geq 0$. Similarly, $U_I(t), U_S(t)$ are positive $\forall t \geq 0$.

Suppose $U_I(t)$ be non-positive. Following this, there is $t_0 > 0$ so that $U_I(t_0) = 0$ and $U_I(t) > 0$ for any t satisfying $0 \leq t \leq t_0$. Hence,

$$\left.\frac{dU_I}{dt}\right|_{t=t_0} \leq 0 \tag{4.9}$$

This is a contradiction, because

$$\left.\frac{dU_I}{dt}\right|_{t=t_0} = a_1 - a_3 U_I(t_0)\left(I + C(t_0) - E_I(t_0)\right) - a_5 U_I(t_0) + a_6 E_I(t_0) > 0 \tag{4.10}$$

Hence, $U_I(t)$ is positive $\forall t \geq 0$.

Suppose $U_S(t)$ is non positive. Then, there exists $t_0 > 0$ to guarantee that $U_S(t_0) = 0$ and $U_S(t) > 0$ for any t satisfying $0 \leq t \leq t_0$. Then,

$$\left.\frac{dU_S}{dt}\right|_{t=t_0} \leq 0 \tag{4.11}$$

This is a contradiction, because

$$\left.\frac{dU_S}{dt}\right|_{t=t_0} = a_1 - a_4 U_S(t_0)\left(S + C(t_0) - E_S(t_0)\right) - a_5 U_S(t_0) + a_6 E_S(t_0) = a_1 > 0 \tag{4.12}$$

Hence, $U_S(t)$ is positive $\forall t \geq 0$.

Next suppose $E_A(t)$ be non-positive. Then, there exists $t_0 > 0$ to guarantee that $E_A(t_0) = 0$ and $E_A(t) > 0$ for any $t \in [0, t_0]$. Consequently,

$$\left.\frac{dE_A}{dt}\right|_{t=t_0} \leq 0 \tag{4.13}$$

This is a contradiction, because

$$\left.\begin{aligned} \left.\frac{dE_A}{dt}\right|_{t=t_0} &= a_2 U_A(t_0)\left(A + C(t_0) - E_A(t_0)\right) - a_5 E_A(t_0) - a_6 E_A(t_0) - a_7 E_A(t_0) \\ \Rightarrow \left.\frac{dE_A}{dt}\right|_{t=t_0} &= a_2 U_A(t_0) A \geq 0 \end{aligned}\right\} \tag{4.14}$$

Hence $E_A(t)$ is positive $\forall t \geq 0$.

Suppose $E_I(t)$ be non-positive. Then there exists $t_0 > 0$, which ensures that $E_I(t) = 0$ and $E_I(t) > 0$ for any $t \in [0, t_0]$. Consequently,

$$\left.\frac{dE_I}{dt}\right|_{t=t_0} \leq 0 \tag{4.15}$$

This is a contradiction, because

$$\left.\begin{aligned} \left.\frac{dE_I}{dt}\right|_{t=t_0} &= a_3U_A(t_0)\big(I+C(t_0)-E_I(t_0)\big)-a_5E_I(t_0)-a_6E_I(t_0)-a_7E_I(t_0) \\ \Rightarrow \left.\frac{dE_I}{dt}\right|_{t=t_0} &= a_3U_A(t_0)I \geq 0 \end{aligned}\right\} \quad (4.16)$$

Hence $E_I(t)$ is positive$\forall t \geq 0$.

Suppose $E_S(t)$ be non-positive. Then there exists $t_0 > 0$such that $E_S(t) = 0$ and $E_S(t) > 0$ for any $t \in [0, t_0]$. Then,

$$\left.\frac{dE_S}{dt}\right|_{t=t_0} \leq 0 \quad (4.17)$$

This is a contradiction, because

$$\left.\begin{aligned} \left.\frac{dE_S}{dt}\right|_{t=t_0} &= a_4U_S(t_0)\big(S+C(t_0)-E_S(t_0)\big)-a_5E_S(t_0)-a_6E_S(t_0)-a_7E_S(t_0) \\ \Rightarrow \left.\frac{dE_S}{dt}\right|_{t=t_0} &= a_4U_S(t_0)S \geq 0 \end{aligned}\right\} \quad (4.18)$$

Hence, $E_S[t]$ is positive $\forall t \geq 0$. Thus, in general, $E_A(t), E_I(t), E_S(t)$ is positive$\forall t \geq 0$. Lastly,

$$\left.\frac{dC}{dt}\right|_{t=t_0} \leq 0 \quad (4.19)$$

But

$$\left.\begin{aligned} \left.\frac{dC}{dt}\right|_{t=t_0} &= \alpha(t_0)\big(U_A(t_0)+U_I(t_0)+U_S(t_0)\big)+\delta\big(E_A(t_0)+E_I(t_0)+E_S(t_0)\big)-\beta C(t_0) \\ &= \alpha(t_0)\big(U_A(t_0)+U_I(t_0)+U_S(t_0)\big)+\delta\big(E_A(t_0)+E_I(t_0)+E_S(t_0)\big) \geq 0 \end{aligned}\right\} \quad (4.20)$$

Hence, C_t is positive$\forall t \geq 0$. Therefore, all solutions to the system from Equations (4.1) to (4.7) are positive.

4.5.2 Boundedness

Let

$$\begin{aligned} U_A + U_I + U_S &= U \\ E_A + E_I + E_S &= E \end{aligned}$$

Thus, from Equations (4.1) to (4.7), it can be shown that

$$\frac{d}{dt}(U_A+U_I+U_S+E_A+E_I+E_S+C)=\frac{d}{dt}(U+E+C)$$

This is further expanded to obtain

$$\frac{d}{dt}(U+E+C)=D_1+D_2+D_3+D_4 \tag{4.21}$$

where;

$$D_1=a_1-a_2U_A(A+C-E_A)-a_5U_A+a_6E_A+a_1-a_3U_I(I+C-E_I)-a_5U_I+a_6E_I$$

$$D_2=+a_1-a_4U_S(S+C-E_S)-a_5U_S+a_6E_S+a_2U_A(A+C-E_A)-a_5E_A-a_6E_A$$

$$D_3=-a_7E_A+a_3U_I(I+C-E_I)-a_5E_I-a_6E_I-a_7E_I+a_4U_S(S+C-E_S)-a_5E_S$$

$$D_4=-a_6E_S-a_7E_S+\alpha(U_A+U_I+U_S)+\delta(E_A+E_I+E_S)-\beta C$$

The expression in the right-hand side of Equation (4.21) can be simplified to give

$$\left.\begin{aligned}\frac{d}{dt}(U+E+C)&=3a_1-a_5U-a_5E-a_7E+\alpha U+\delta E-\beta C\\&=3a_1+(\alpha-a_5)U+(\delta-a_5-a_7)E-\beta C\\&=3a_1-(a_5-\alpha)U-(a_5+a_7-\delta)E-\beta C\\&\le 3a_1-\gamma(U+E+C)\end{aligned}\right\} \tag{4.22}$$

where $\gamma=\min\{a_5,(a_5+a_7-\delta),\beta\}$

Taking the limit supremum

$$\lim_{t\to\infty}\left[\sup(U_A+U_I+U_S+E_A+E_I+E_S+C)\right]\le\frac{3a_1}{\gamma} \tag{4.23}$$

4.5.3 Computation of the Model's Equilibrium Point

The model Equations (4.1) to (4.7) has only one equilibrium point which is non-negative given by

$$U_A^1=\frac{a_1a_2\delta+\beta\left[Aa_2a_5-a_1a_2+a_5{}^2+a_7(Aa_2+a_5)\right]+\sqrt{X_1}}{2\left[a_2a_5(\beta-\delta)+\alpha(a_2a_5+a_2a_7)\right]} \tag{4.24}$$

$$U_I^1=\frac{a_1a_3\delta+\beta\left[Ia_3a_5-a_1a_3+a_5{}^2+a_5a_6+a_7(Ia_3+a_5)\right]+\sqrt{X_2}}{2\left[a_3a_5(\beta-\delta)+\alpha(a_3a_5+a_3a_7)\right]} \tag{4.25}$$

$$U_S^1=\frac{a_1a_4\delta+\beta\left[Sa_4a_5-a_1a_4+a_5{}^2+a_5a_6+a_7(Sa_4+a_5)\right]+\sqrt{X_3}}{2\left[a_4a_5(\beta-\delta)+\alpha(a_4a_5+a_4a_7)\right]} \tag{4.26}$$

$$E_A^1 = \frac{a_1a_2a_5\delta - 2\alpha\left(a_1a_2a_5 - a_1a_2a_7\right) + \beta\left[Aa_2a_5^{\,2} + a_1a_2a_5 + a_5^{\,3} + a_5^{\,2}a_6 + a_7\left(Aa_2a_5 + a_5^{\,2}\right)\right] + \sqrt{X_4a_5}}{2\left[\alpha\left(a_2a_5^{\,2} + 2a_2a_5a_7 + a_2a_7^{\,2}\right) + \left(a_2a_5^{\,2} + a_2a_5a_7\right)\left(\beta-\delta\right)\right]} \quad (4.27)$$

$$E_I^1 = \frac{a_1a_3a_5\delta - 2\alpha\left(a_1a_3a_5 - a_1a_3a_7\right) + \beta\left[Ia_3a_5^{\,2} + a_1a_3a_5 + a_5^{\,3} + a_5^{\,2}a_6 + a_7\left(Aa_3a_5 + a_5^{\,2}\right)\right] + \sqrt{X_5a_5}}{2\left[\alpha\left(a_3a_5^{\,2} + 2a_3a_5a_7 + a_3a_7^{\,2}\right) + \left(a_3a_5^{\,2} + a_3a_5a_7\right)\left(\beta-\delta\right)\right]} \quad (4.28)$$

$$E_S^1 = \frac{a_1a_4a_5\delta - 2\alpha\left(a_1a_4a_5 - a_1a_4a_7\right) + \beta\left[Sa_4a_5^{\,2} + a_1a_4a_5 + a_5^{\,3} + a_5^{\,2}a_6 + a_7\left(Aa_4a_5 + a_5^{\,2}\right)\right] + \sqrt{X_6a_5}}{2\left[\alpha\left(a_4a_5^{\,2} + 2a_4a_5a_7 + a_4a_7^{\,2}\right) + \left(a_4a_5^{\,2} + a_4a_5a_7\right)\left(\beta-\delta\right)\right]} \quad (4.29)$$

$$C^1 = \frac{X_7 + \sqrt{X_8\left[\alpha\left(a_5 + a_7\right) - a_5\delta\right]}}{2\left[\alpha\beta\left(a_2a_5^{\,2} + 2a_2a_5a_7 + a_2a_7^{\,2}\right) + \left(a_2a_5^{\,2} + a_2a_5a_7\right)\beta\left(\beta-\delta\right)\right]} \quad (4.30)$$

where;

$$\begin{aligned}
X_1 = {} & a_1^{\,2}a_2^{\,2}\delta^2 + 4\alpha\beta\left[a_1a_2a_5^{\,2} + a_1a_2a_5a_6 + a_1a_2a_7^{\,2} + \left(2a_1a_2a_5 + a_1a_2a_6\right)a_7\right] \\
& -\left[2Aa_1a_2^{\,2}a_5 - 2Aa_2a_5^{\,2} - a_1^{\,2}a_2^{\,2} - a_5^{\,4} - a_5^{\,2}a_6^{\,2} - \left(A^2a_2^{\,2} + 2a_1a_2\right)a_5^{\,2}\right]\beta^2 \\
& -\left[\left(A^2a_2^{\,2} + 2Aa_2a_5 + a_5^{\,2}\right)a_7^{\,2} - 2\left(Aa_2a_5^{\,2} + a_1a_2a_5 + a_5^{\,3}\right)a_6\right]\beta^2 \\
& +2\left\{\left[Aa_1a_2^{\,2} - 2Aa_2a_5^{\,2} - a_5^{\,3} - \left(A^2a_2^{\,2} + a_1a_2\right)a_5 - \left(Aa_2a_5 + a_5^{\,2}\right)a_6\right]a_7\right\}\beta^2 \\
& +2\left[Aa_1a_2^{\,2}a_5 - a_1^{\,2}a_2^{\,2} - a_1a_2a_5^{\,2} - a_1a_2a_5a_6 + \left(Aa_1a_2^{\,2} - a_1a_2a_5\right)a_7\right]\beta\delta
\end{aligned}$$

$$\begin{aligned}
X_2 = {} & a_1^{\,2}a_3^{\,2}\delta^2 + 4\alpha\beta\left[a_1a_3a_5^{\,2} + a_1a_3a_5a_6 + a_1a_3a_7^{\,2} + \left(2a_1a_3a_5 + a_1a_3a_6\right)a_7\right] \\
& -\left[2Ia_1a_3^{\,2}a_5 - 2Ia_3a_5^{\,2} - a_1^{\,2}a_3^{\,2} - a_5^{\,4} - a_5^{\,2}a_6^{\,2} - \left(I^2a_3^{\,2} + 2a_1a_3\right)a_5^{\,2}\right]\beta^2 \\
& -\left[\left(I^2a_3^{\,2} + 2Ia_3a_5 + a_5^{\,2}\right)a_7^{\,2} - 2\left(Ia_3a_5^{\,2} + a_1a_3a_5 + a_5^{\,3}\right)a_6\right]\beta^2 \\
& +2\left\{\left[Ia_1a_3^{\,2} - 2Ia_3a_5^{\,2} - a_5^{\,3} - \left(I^2a_3^{\,2} + a_1a_3\right)a_5 - \left(Ia_3a_5 + a_5^{\,2}\right)a_6\right]a_7\right\}\beta^2 \\
& +2\left[Ia_1a_3^{\,2}a_5 - a_1^{\,2}a_3^{\,2} - a_1a_3a_5^{\,2} - a_1a_3a_5a_6 + \left(Ia_1a_3^{\,2} - a_1a_3a_5\right)a_7\right]\beta\delta
\end{aligned}$$

$$\begin{aligned}
X_3 = {} & a_1^{\,2}a_4^{\,2}\delta^2 + 4\alpha\beta\left[a_1a_4a_5^{\,2} + a_1a_4a_5a_6 + a_1a_4a_7^{\,2} + \left(2a_1a_4a_5 + a_1a_4a_6\right)a_7\right] \\
& -\left[2Sa_1a_4^{\,2}a_5 - 2Sa_4a_5^{\,2} - a_1^{\,2}a_4^{\,2} - a_5^{\,4} - a_5^{\,2}a_6^{\,2} - \left(S^2a_4^{\,2} + 2a_1a_4\right)a_5^{\,2}\right]\beta^2 \\
& -\left[\left(S^2a_4^{\,2} + 2Sa_4a_5 + a_5^{\,2}\right)a_7^{\,2} - 2\left(Sa_4a_5^{\,2} + a_1a_4a_5 + a_5^{\,3}\right)a_6\right]\beta^2 \\
& +2\left\{\left[Sa_1a_4^{\,2} - 2Sa_4a_5^{\,2} - a_5^{\,3} - \left(S^2a_4^{\,2} + a_1a_4\right)a_5 - \left(Sa_4a_5 + a_5^{\,2}\right)a_6\right]a_7\right\}\beta^2 \\
& +2\left[Sa_1a_4^{\,2}a_5 - a_1^{\,2}a_4^{\,2} - a_1a_4a_5^{\,2} - a_1a_4a_5a_6 + \left(Sa_1a_4^{\,2} - a_1a_4a_5\right)a_7\right]\beta\delta
\end{aligned}$$

$$X_4 = a_1^2 a_2^2 \delta^2 + 4\alpha\beta\left[a_1 a_2 a_5^2 + a_1 a_2 a_5 a_6 + a_1 a_2 a_7^2 + \left(2a_1 a_2 a_5 + a_1 a_2 a_6\right)a_7\right]$$
$$-\left[2Aa_1 a_2^2 a_5 - 2Aa_2 a_5^3 - a_1^2 a_2^2 - a_5^4 - a_5^2 a_6^2 - \left(A^2 a_2^2 + 2a_1 a_2\right)a_5^2\right]\beta^2$$
$$-\left[\left(A^2 a_2^2 + 2Aa_2 a_5 + a_5^2\right)a_7^2 - 2\left(Aa_2 a_5^2 + a_1 a_2 a_5 + a_5^3\right)a_6\right]\beta^2$$
$$+2\left\{\left[Aa_1 a_2^2 - 2Aa_2 a_5^2 - a_5^3 - \left(A^2 a_2^2 + a_1 a_2\right)a_5 - \left(Aa_2 a_5 + a_5^2\right)a_6\right]a_7\right\}\beta^2$$
$$+2\left[Aa_1 a_2^2 a_5 - a_1^2 a_2^2 - a_1 a_2 a_5^2 - a_1 a_2 a_5 a_6 + \left(Aa_1 a_2^2 - a_1 a_2 a_5\right)a_7\right]\beta\delta$$

$$X_5 = a_1^2 a_3^2 \delta^2 + 4\alpha\beta\left[a_1 a_3 a_5^2 + a_1 a_3 a_5 a_6 + a_1 a_3 a_7^2 + \left(2a_1 a_3 a_5 + a_1 a_3 a_6\right)a_7\right]$$
$$-\left[2Ia_1 a_3^2 a_5 - 2Ia_3 a_5^3 - a_1^2 a_3^2 - a_5^4 - a_5^2 a_6^2 - \left(I^2 a_3^2 + 2a_1 a_3\right)a_5^2\right]\beta^2$$
$$-\left[\left(I^2 a_3^2 + 2Ia_3 a_5 + a_5^2\right)a_7^2 - 2\left(Ia_3 a_5^2 + a_1 a_3 a_5 + a_5^3\right)a_6\right]\beta^2$$
$$+2\left\{\left[Ia_1 a_3^2 - 2Ia_3 a_5^2 - a_5^3 - \left(I^2 a_3^2 + a_1 a_3\right)a_5 - \left(Ia_3 a_5 + a_5^2\right)a_6\right]a_7\right\}\beta^2$$
$$+2\left[Ia_1 a_3^2 a_5 - a_1^2 a_3^2 - a_1 a_3 a_5^2 - a_1 a_3 a_5 a_6 + \left(Ia_1 a_3^2 - a_1 a_3 a_5\right)a_7\right]\beta\delta$$

$$X_6 = a_1^2 a_4^2 \delta^2 + 4\alpha\beta\left[a_1 a_4 a_5^2 + a_1 a_4 a_5 a_6 + a_1 a_4 a_7^2 + \left(2a_1 a_4 a_5 + a_1 a_4 a_6\right)a_7\right]$$
$$-\left[2Sa_1 a_4^2 a_5 - 2Sa_4 a_5^3 - a_1^2 a_4^2 - a_5^4 - a_5^2 a_6^2 - \left(S^2 a_4^2 + 2a_1 a_4\right)a_5^2\right]\beta^2$$
$$-\left[\left(S^2 a_4^2 + 2Sa_4 a_5 + a_5^2\right)a_7^2 - 2\left(Sa_4 a_5^2 + a_1 a_4 a_5 + a_5^3\right)a_6\right]\beta^2$$
$$+2\left\{\left[Sa_1 a_4^2 - 2Sa_4 a_5^2 - a_5^3 - \left(S^2 a_4^2 + a_1 a_4\right)a_5 - \left(Sa_4 a_5 + a_5^2\right)a_6\right]a_7\right\}\beta^2$$
$$+2\left[Sa_1 a_4^2 a_5 - a_1^2 a_4^2 - a_1 a_4 a_5^2 - a_1 a_4 a_5 a_6 + \left(Sa_1 a_4^2 - a_1 a_4 a_5\right)a_7\right]\beta\delta$$

$$X_7 = a_1 a_2 a_5 \delta^2 + \left[a_2 a_5^2 - a_1 a_2 a_5 + a_5^3 + a_5^2 a_6 + \left(a_2 + a_5\right)a_7^2 + \left(2a_2 a_5 - a_1 a_2\right)a_7\right]\alpha\beta$$
$$+\left(2a_5^2 + a_5 a_6\right)a_7\alpha\beta - \left[\alpha\left(a_1 a_2 a_5 + a_1 a_2 a_7\right) + \beta\left(a_2 a_5^2 + a_1 a_2 a_5 + a_5^3 + a_5^2 a_6\right)\right]\delta$$
$$+\left(a_2 a_5 + a_5^2\right)a_7\delta$$

$$X_8 = a_1^2 a_2^2 \delta^2 + 4\alpha\beta\left[a_1 a_3 a_5^2 + a_1 a_4 a_5 a_6 + a_1 a_4 a_7^2 + \left(2a_1 a_2 a_5 + a_1 a_2 a_6\right)a_7\right]$$
$$-\left[2a_1 a_3^2 a_5 - 2a_2 a_5^3 - a_1^2 a_2^2 - a_5^4 - a_5^2 a_6^2 - \left(a_2^2 + 2a_1 a_2\right)a_5^2\right]\beta^2$$
$$-\left[\left(a_2^2 + 2a_2 a_5 + a_5^2\right)a_7^2 - 2\left(a_2 a_5^2 + a_1 a_2 a_5 + a_5^3\right)a_6\right]\beta^2$$
$$+2\left\{\left[a_1 a_4^2 - 2a_2 a_5^2 - a_5^3 - \left(a_2^2 + a_1 a_2\right)a_5 - \left(a_2 a_5 + a_5^2\right)a_6\right]a_7\right\}\beta^2$$
$$+2\left[a_1 a_3^2 a_5 - a_1^2 a_2^2 - a_1 a_4 a_5^2 - a_1 a_2 a_5 a_6 + \left(a_1 a_2^2 - a_1 a_4 a_5\right)a_7\right]\beta\delta$$

The set of algebraic equations given below were solved to obtain the expressions above.

$$F_1 = a_1 - a_2 U_A\left(A + C - E_A\right) - a_5 U_A + a_6 E_A = 0 \qquad (4.31)$$

$$F_2 = a_1 - a_3 U_I \left(I + C - E_I\right) - a_5 U_I + a_6 E_I = 0 \quad (4.32)$$

$$F_3 = a_1 - a_4 U_s \left(S + C - E_S\right) - a_5 U_S + a_6 E_S = 0 \quad (4.33)$$

$$F_4 = a_2 U_A \left(A + C - E_A\right) - a_5 E_A - a_6 E_A - a_7 E_E = 0 \quad (4.34)$$

$$F_5 = a_3 U_I \left(I + C - E_I\right) - a_5 E_I - a_6 E_I - a_7 E_I = 0 \quad (4.35)$$

$$F_6 = a_4 U_S \left(S + C - E_S\right) - a_5 E_S - a_6 E_S - a_7 E_S = 0 \quad (4.36)$$

$$F_7 = \alpha\left(U_A + U_I + U_S\right) + \delta\left(E_A + E_I + E_S\right) - \beta C = 0 \quad (4.37)$$

Hence the equilibrium point will inform one about the dynamics of the system over time.

4.5.4 Linearisation of the Model

In determining the stability analysis, we implore the linearisation technique which gives the Jacobian matrix. The matrix of partial derivatives of the system Equations (4.31) to (4.37) is given as:

$$J = \begin{bmatrix}
\frac{\partial F_1}{\partial U_A} & \frac{\partial F_1}{\partial U_I} & \frac{\partial F_1}{\partial U_S} & \frac{\partial F_1}{\partial E_A} & \frac{\partial F_1}{\partial E_I} & \frac{\partial F_1}{\partial E_S} & \frac{\partial F_1}{\partial C} \\
\frac{\partial F_2}{\partial U_A} & \frac{\partial F_2}{\partial U_I} & \frac{\partial F_2}{\partial U_S} & \frac{\partial F_2}{\partial E_A} & \frac{\partial F_2}{\partial E_I} & \frac{\partial F_2}{\partial E_S} & \frac{\partial F_2}{\partial C} \\
\frac{\partial F_3}{\partial U_A} & \frac{\partial F_3}{\partial U_I} & \frac{\partial F_3}{\partial U_S} & \frac{\partial F_3}{\partial E_A} & \frac{\partial F_3}{\partial E_I} & \frac{\partial F_3}{\partial E_S} & \frac{\partial F_3}{\partial C} \\
\frac{\partial F_4}{\partial U_A} & \frac{\partial F_4}{\partial U_I} & \frac{\partial F_4}{\partial U_S} & \frac{\partial F_4}{\partial E_A} & \frac{\partial F_4}{\partial E_I} & \frac{\partial F_4}{\partial E_S} & \frac{\partial F_4}{\partial C} \\
\frac{\partial F_5}{\partial U_A} & \frac{\partial F_5}{\partial U_I} & \frac{\partial F_5}{\partial U_S} & \frac{\partial F_5}{\partial E_A} & \frac{\partial F_5}{\partial E_I} & \frac{\partial F_5}{\partial E_S} & \frac{\partial F_5}{\partial C} \\
\frac{\partial F_6}{\partial U_A} & \frac{\partial F_6}{\partial U_I} & \frac{\partial F_6}{\partial U_S} & \frac{\partial F_6}{\partial E_A} & \frac{\partial F_6}{\partial E_I} & \frac{\partial F_6}{\partial E_S} & \frac{\partial F_6}{\partial C} \\
\frac{\partial F_7}{\partial U_A} & \frac{\partial F_7}{\partial U_I} & \frac{\partial F_7}{\partial U_S} & \frac{\partial F_7}{\partial E_A} & \frac{\partial F_7}{\partial E_I} & \frac{\partial F_7}{\partial E_S} & \frac{\partial F_7}{\partial C}
\end{bmatrix}$$

Using Equations (4.31) to (4.37), gives

$$J = \begin{bmatrix}
\lambda_1 & 0 & 0 & U_A a_2 + a_6 & 0 & 0 & -U_A a_2 \\
0 & \lambda_2 & 0 & 0 & U_I a_3 + a_6 & 0 & -U_I a_3 \\
0 & 0 & \lambda_3 & 0 & 0 & U_S a_4 + a_6 & -U_S a_4 \\
\lambda_7 & 0 & 0 & \lambda_4 & 0 & 0 & U_A a_2 \\
0 & \lambda_8 & 0 & 0 & \lambda_5 & 0 & U_I a_3 \\
0 & 0 & \lambda_9 & 0 & 0 & \lambda_6 & U_S a_4 \\
\alpha & \alpha & \alpha & \delta & \delta & \delta & -\beta
\end{bmatrix}$$

where;

$\lambda_1 = -(A + C - E_A)a_2 - a_5$

$\lambda_2 = -(I + C - E_I)a_3 - a_5$

$\lambda_3 = -(S + C - E_S)a_4 - a_5$

$\lambda_4 = -U_A a_2 - a_5 - a_6 - a_7$

$\lambda_5 = -U_I a_3 - a_5 - a_6 - a_7$

$\lambda_6 = -U_S a_4 - a_5 - a_6 - a_7$

$\lambda_7 = (A + C - E_A)a_2$

$\lambda_8 = (I + C - E_I)a_3$

$\lambda_9 = (S + C - E_S)a_4$

4.5.5 Local Stability Analysis

Theorem 4.2 The model represented by Equations (4.1) to(4.7) is locally asymptotically stable at $(U_A^1, U_I^1, U_S^1, E_A^1, E_I^1, E_S^1, C^1)$ if and only if the conditions given below can be fulfilled.

i. $A_0 > 0$

ii. $A_1 > 0$

iii. $A_0 A_3 > A_1 A_2$

iv. $B_1 A_3 > A_1 B_2$

v. $C_1 B_2 > B_1 C_2$

vi. $C_1 D_2 > D_1 C_2$

Proof: Evaluating the equilibrium point in the Jacobian matrix (J) obtains

$$J_1 = \begin{bmatrix} \lambda_1 & 0 & 0 & U_A^1 a_2 + a_6 & 0 & 0 & -U_A^1 a_2 \\ 0 & \lambda_2 & 0 & 0 & U_I^1 a_3 + a_6 & 0 & -U_I^1 a_3 \\ 0 & 0 & \lambda_3 & 0 & 0 & U_S^1 a_4 + a_6 & -U_S^1 a_4 \\ \lambda_7 & 0 & 0 & \lambda_4 & 0 & 0 & U_A^1 a_2 \\ 0 & \lambda_8 & 0 & 0 & \lambda_5 & 0 & U_I^1 a_3 \\ 0 & 0 & \lambda_9 & 0 & 0 & \lambda_6 & U_S^1 a_4 \\ \alpha & \alpha & \alpha & \delta & \delta & \delta & -\beta \end{bmatrix} \quad (4.38)$$

Where;

$\lambda_1 = -(A + C^1 - E_A^1)a_2 - a_5$

$$\lambda_2 = -\left(I + C^1 - E_I^1\right)a_3 - a_5$$
$$\lambda_3 = -\left(S + C^1 - E_S^1\right)a_4 - a_5$$
$$\lambda_4 = -U_A^1 a_2 - a_5 - a_6 - a_7$$
$$\lambda_5 = -U_I^1 a_3 - a_5 - a_6 - a_7$$
$$\lambda_6 = -U_S^1 a_4 - a_5 - a_6 - a_7$$
$$\lambda_7 = \left(A + C^1 - E_A^1\right)a_2$$
$$\lambda_8 = \left(I + C^1 - E_I^1\right)a_3$$
$$\lambda_9 = \left(S + C^1 - E_S^I\right)a_4$$

$$|\mathcal{J} - sI| = 0$$

$$\left\| \begin{bmatrix} \lambda_1 & 0 & 0 & U_A^1 a_2 + a_6 & 0 & 0 & -U_A^1 a_2 \\ 0 & \lambda_2 & 0 & 0 & U_I^1 a_3 + a_6 & 0 & -U_I^1 a_3 \\ 0 & 0 & \lambda_3 & 0 & 0 & U_S^1 a_4 + a_6 & -U_S^1 a_4 \\ \lambda_7 & 0 & 0 & \lambda_4 & 0 & 0 & U_A^1 a_2 \\ 0 & \lambda_8 & 0 & 0 & \lambda_5 & 0 & U_I^1 a_3 \\ 0 & 0 & \lambda_9 & 0 & 0 & \lambda_6 & U_S^1 a_4 \\ \alpha & \alpha & \alpha & \delta & \delta & \delta & -\beta \end{bmatrix} - s \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\| = 0$$

The characteristic equation is obtained as:

$$K_0 s^7 + K_1 s^6 + K_2 s^5 + K_4 s^4 + K_5 s^3 + K_6 s^2 + K_7 s^1 + K_0 \tag{3.39}$$

$$s^7 - \operatorname{tr}(J_1)s^6 + \operatorname{tr}(J_1)s^5 - \operatorname{tr}(J_1)A_4 s^4 + \operatorname{tr}(J_1)A_5 s^3 - \operatorname{tr}(J_1)A_6 s^2 + \operatorname{tr}(J_1)A_7 s^1 - \det(J_1) = 0 \tag{3.40}$$

where;

$$\operatorname{tr}(J_1) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 - \beta$$

$$\det(J_1) = \lambda_1\lambda_2\lambda_3\Delta_1\left(\Delta_2 - \Delta_3\right)$$

$$\Delta_1 = \left(\lambda_4 - \frac{\lambda_7 U_A^1 a_2 + a_6}{\lambda_1}\right)\left(\lambda_5 - \frac{\lambda_8 U_I^1 a_3 + a_6}{\lambda_2}\right)\left(\lambda_6 - \frac{\lambda_9 U_S^1 a_4 + a_6}{\lambda_3}\right)$$

$$\Delta_2 = \left[-\beta - \frac{\alpha\left(a_2 U_A^1 - a_3 U_I^1\right)}{\lambda_2} + \frac{\alpha U_S^1}{\lambda_3} + \left(\frac{\delta - \dfrac{\alpha a_2 U_A^1 + a_6}{\lambda_1}}{\lambda_4 - \dfrac{\lambda_7 a_2 U_A^1 + a_6}{\lambda_1}}\right)\left(-a_2 U_A^1 - \frac{\lambda_7 a_2 U_A^1}{\lambda_2}\right)\right]$$

$$\Delta_3 = \left(\frac{\delta - \dfrac{\alpha a_2 U_A^1 + a_6}{\lambda_1}}{\lambda_4 - \dfrac{\lambda_7 a_2 U_A^1 + a_6}{\lambda_1}} \right) \left(-a_3 U_I^1 - \frac{\lambda_8 a_2 U_A^1}{\lambda_2} \right) + \left(\frac{\delta - \dfrac{\alpha a_4 U_S^1 + a_6}{\lambda_3}}{\lambda_6 - \dfrac{\lambda_9 a_4 U_S^1 + a_6}{\lambda_3}} \right) \left(a_3 U_S^1 - \frac{\lambda_9 a_4 U_S^1}{\lambda_3} \right)$$

4.5.6 Global Stability Using Routh-Hurwitz Stability Criterion

Using the Routh-Hurwitz criterion to establish the global stability of the non-negative equilibrium points, we develop the Routh array from the characteristic Equation (4.33) as

Table 4.2 Routh Array

K_0	K_2	K_4	K_6	0
K_1	K_3	K_5	K_7	0
M_1	M_2	M_3	0	0
N_1	N_2	K_7	0	0
P_1	P_2	0	0	0
Q_1	K_7	0	0	0

where;

$$M_1 = \frac{K_0 K_3 - K_1 K_2}{K_1}$$

$$M_2 = \frac{K_1 K_4 - K_0 K_5}{K_1}$$

$$M_3 = \frac{K_1 K_6 - K_0 K_7}{K_1}$$

$$N_1 = \frac{M_1 K_3 - K_1 M_2}{M_1}$$

$$N_2 = \frac{M_1 K_5 - K_1 M_3}{M_1}$$

$$P_1 = \frac{N_1 M_2 - M_1 N_2}{N_1}$$

$$P_2 = \frac{N_1 M_3 - M_1 K_7}{N_1}$$

$$Q_1 = \frac{N_1 P_2 - P_1 N_2}{P_1}$$

The Routh-Hurwitz (RH) stability condition postulates that if and only if all first-column elements of the Routh array have the same sign, then a polynomial of degree n has all its roots in the open left half plane. Thus,

$$\left.\begin{aligned} &K_0 > 0 \\ &K_1 > 0 \\ &K_0K_3 > K_1K_2 \\ &M_1K_3 > K_1M_2 \\ &N_1M_2 > M_1N_2 \\ &N_1P_2 > P_1N_2 \end{aligned}\right\} \tag{4.41}$$

Since the coefficients of the characteristic equation can be established and by algebraic manipulation, the RH conditions in Equation (4.41) are satisfied. By the Routh Hurwitz condition, all the roots of the characteristic equation are negative or have negative real parts. Therefore, the equilibrium point is both locally and globally asymptotically stable.

Local asymptotic stability implies that while the solution near to the equilibrium points remains close, they also eventually converge to the equilibrium. On the other hand, global asymptotic stability simply implies that solutions that starts remotely from the equilibrium point, eventually converge to the equilibrium point. Practically, the implication is that the model has inbuilt mechanism to restore back to the equilibrium point any perturbation (internal and external shocks) that distorts the equilibrium point to positions both close and afar in relation to the equilibrium point. There is therefore, a balance in the various compartments such that, the ratios of number of unemployed to that of the employed are in relative proportions. Vacancies are optimally created to be filled. This behaviour continues over a certain period of time.

4.6 Characteristics of the Equilibrium Values

Here, the impact of perturbation of some parameters' values on the number of individuals in the unemployment class is analysed. The parameters under consideration include:

i. The rate of transition of persons from the unemployment class to employment class respectively in Agriculture, Industry and Services, thus: a_2,a_3, a_4.

ii. The rate at which new vacancies are created in favour of the unemployed α.

iii. The rate at which new vacancies are created in favour the employed δ.

4.6.1 Characteristics of the Equilibrium Values of Unemployed Persons and Employed Persons with Respect to $\boldsymbol{a_2}$

The behaviour of the equilibrium values of unemployed persons and employed persons with respect to a_2 (the rate of transition of persons from the unemployment class to employment class in agriculture sector) are analysed to assess its characteristics in the system.

From Equations (4.31) and (4.34),

$$F_1\left(U_A^1, U_I^1, U_S^1, E_A^1, E_I^1, E_S^1, C^1\right) = a_1 - a_2 U_A^1\left(A + C^1 - E_A^1\right) - a_5 U_A^1 + a_6 E_A^1$$

$$F_4\left(U_A^1, U_I^1, U_S^1, E_A^1, E_I^1, E_S^1, C^1\right) = a_2 U_A^1\left(A + C^1 - E_A^1\right) - a_5 E_A^1 - a_6 E_A^1 - a_7 E_A^1$$

$$\frac{dU_A^1}{da_2} = \frac{\begin{vmatrix} \frac{\partial F_1}{\partial E_A^1} & \frac{\partial F_1}{\partial a_2} \\ \frac{\partial F_4}{\partial E_A^1} & \frac{\partial F_1}{\partial a_2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1}{\partial U_A^1} & \frac{\partial F_1}{\partial E_A^1} \\ \frac{\partial F_4}{\partial U_A^1} & \frac{\partial F_4}{\partial E_A^1} \end{vmatrix}} = \frac{\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial a_2} - \frac{\partial F_1}{\partial a_2}\frac{\partial F_4}{\partial E_A^1}}{\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial E_A^1} - \frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial U_A^1}} \tag{4.42}$$

Applying Equation (4.42) gives

$$\frac{\partial F_1}{\partial U_A^1} = -a_2\left(A + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta}\right) - a_5 \tag{4.43}$$

$$\frac{\partial F_1}{\partial E_A^1} = a_2 U_A^1 + a_6 - \frac{a_2\delta}{\beta} U_A^1 \tag{4.44}$$

$$\frac{\partial F_1}{\partial a_2} = -U_A^1\left(A - E_A^1 + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta}\right) \tag{4.45}$$

$$\frac{\partial F_4}{\partial U_A^1} = a_2\left(A + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta}\right) \tag{4.46}$$

$$\frac{\partial F_4}{\partial E_A^1} = -a_2 U_A^1 + \frac{a_2\delta}{\beta} U_A^1 - \left(a_5 + a_6 + a_7\right) \tag{4.47}$$

$$\frac{\partial F_4}{\partial a_2} = U_A^1\left(A - E_A^1 + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta}\right) \tag{4.48}$$

Using Equations (4.43) to (4.48) yields;

$$\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial a_2}-\frac{\partial F_1}{\partial a_2}\frac{\partial F_4}{\partial E_A^1}=-U_A^1\left(a_5+a_7\right)\left(A-E_A^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)<0 \quad (4.49)$$

$$\left.\begin{aligned}\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial E_A^1}-\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial U_A^1}=\left(a_5+a_7\right)\left(A-E_A^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\+a_5\left(a_2U_A^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.50)$$

Simplifying further will lead to

$$\frac{dU_A^1}{da_2}<0$$

Thus, as a_2 increases, U_A^1 decreases indicating that a_2 is a positive correlate of U_A^1.

Now,

$$\frac{dE_A^1}{da_2}=\frac{\begin{vmatrix}\frac{\partial F_1}{\partial a_2} & \frac{\partial F_1}{\partial U_A^1}\\ \frac{\partial F_4}{\partial U_A^1} & \frac{\partial F_4}{\partial a_2}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_1}{\partial U_A^1} & \frac{\partial F_1}{\partial E_A^1}\\ \frac{\partial F_4}{\partial U_A^1} & \frac{\partial F_4}{\partial E_A^1}\end{vmatrix}}=\frac{\frac{\partial F_1}{\partial a_2}\frac{\partial F_4}{\partial a_2}-\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial U_A^1}}{\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial E_A^1}-\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial U_A^1}} \quad (4.51)$$

$$\frac{\partial F_1}{\partial a_2}\frac{\partial F_4}{\partial U_A^1}-\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial a_2}=a_5U_A^1\left(A-E_A^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)>0 \quad (4.52)$$

$$\left.\begin{aligned}\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial E_A^1}-\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial U_A^1}=\left(a_5+a_7\right)\left(A-E_A^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\+a_5\left(a_2U_A^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.53)$$

Again, Equations (4.52) and (4.53) are further simplified according to Equation (4.51), the result is

$$\frac{dE_A^1}{da_2}>0$$

Thus, as a_2 increases, E_A^1 increases, reflecting that a_2 is a direct determinant of E_A^1.

4.6.2 Properties of the Equilibrium Values of Unemployed Persons and Employed Persons with Respect to $\boldsymbol{a_3}$

The behaviour of the equilibrium values of unemployed persons and employed persons with respect to a_3 (the rate of transition of persons from the unemployment class to the employment class in the industry sector) are investigated to establish its characteristics in the system.

From Equations (4.32) *and* (4.35),

$$F_2\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_1-a_3U_I^1\left(I+C^1-E_I^1\right)-a_5U_I^1+a_6E_I^1$$

$$F_5\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_3U_I^1\left(I+C^1-E_I^1\right)-a_5E_I^1-a_6E_A^1-a_7E_A^1$$

$$\frac{dU_I^1}{da_3}=\frac{\begin{vmatrix}\frac{\partial F_2}{\partial E_I^1} & \frac{\partial F_2}{\partial a_3}\\ \frac{\partial F_5}{\partial E_I^1} & \frac{\partial F_5}{\partial a_3}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_2}{\partial U_I^1} & \frac{\partial F_2}{\partial E_I^1}\\ \frac{\partial F_5}{\partial U_I^1} & \frac{\partial F_5}{\partial E_I^1}\end{vmatrix}}=\frac{\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial a_3}-\frac{\partial F_2}{\partial a_3}\frac{\partial F_5}{\partial E_I^1}}{\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}} \qquad (4.54)$$

Applying Equation (4.54) gives

$$\frac{\partial F_2}{\partial U_I^1}=-a_3\left(I+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)-a_5 \qquad (4.55)$$

$$\frac{\partial F_2}{\partial E_I^1}=a_3U_I^1+a_6-\frac{a_3\delta}{\beta}U_I^1 \qquad (4.56)$$

$$\frac{\partial F_2}{\partial a_3}=-U_I^1\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \qquad (4.57)$$

$$\frac{\partial F_5}{\partial U_I^1}=a_3\left(I+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \qquad (4.58)$$

$$\frac{\partial F_5}{\partial E_I^1}=-a_3U_I^1+\frac{a_3\delta}{\beta}U_I^1-\left(a_5+a_6+a_7\right) \qquad (4.59)$$

$$\frac{\partial F_5}{\partial a_3}=U_1^1\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \qquad (4.60)$$

Using Equations (4.55) to (4.60) yields;

$$\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial a_3}-\frac{\partial F_2}{\partial a_3}\frac{\partial F_5}{\partial E_I^1}=-U_I^1\left(a_5+a_7\right)\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)<0 \quad (4.61)$$

$$\left.\begin{aligned}\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}=\left(a_5+a_7\right)\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\+a_5\left(a_3U_I^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.62)$$

Simplifying further will lead to

$$\frac{dU_I^1}{da_3}<0$$

Thus, a_3 is an inverse determinant of U_I^1.

Now,

$$\frac{dE_I^1}{da_3}=\frac{\begin{vmatrix}\frac{\partial F_2}{\partial a_3} & \frac{\partial F_2}{\partial U_I^1}\\ \frac{\partial F_5}{\partial a_3} & \frac{\partial F_5}{\partial U_I^1}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_2}{\partial U_I^1} & \frac{\partial F_2}{\partial E_I^1}\\ \frac{\partial F_5}{\partial U_I^1} & \frac{\partial F_5}{\partial E_I^1}\end{vmatrix}}=\frac{\frac{\partial F_2}{\partial a_3}\frac{\partial F_5}{\partial U_I^1}-\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial a_3}}{\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}} \quad (4.63)$$

$$\frac{\partial F_2}{\partial a_3}\frac{\partial F_5}{\partial U_I^1}-\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial a_3}=a_5U_I^1\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)>0 \quad (4.64)$$

$$\left.\begin{aligned}\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}=\left(a_5+a_7\right)\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\+a_5\left(a_3U_I^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.65)$$

Similarly, Equations (4.64) and (4.65) are further simplified to obtain

$$\frac{dE_I^1}{da_3}>0$$

Thus, a_3 is a positive correlate of E_I^1.

4.6.3 Properties of the Equilibrium Values of Unemployed Persons and Employed Persons with Respect to $\boldsymbol{a_4}$

The properties of the equilibrium values of unemployed persons and employed persons with respect to a_4 (the rate of transition of persons from the unemployment class to the employment class in the services sector) are evaluated to establish its characteristics in the system. From Equations (4.33) and (4.36),

$$F_3\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_1-a_4U_S^1\left(S+C^1-E_s^1\right)-a_5U_s^1+a_6E_s^1$$

$$F_6\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_4U_S^1\left(S+C^1-E_S^1\right)-a_5E_s^1-a_6E_s^1-a_7E_S^1$$

$$\frac{dU_S^1}{da_4}=\frac{\begin{vmatrix}\frac{\partial F_3}{\partial E_S^1} & \frac{\partial F_3}{\partial a_4}\\ \frac{\partial F_6}{\partial E_S^1} & \frac{\partial F_6}{\partial a_4}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_3}{\partial U_S^1} & \frac{\partial F_3}{\partial E_S^1}\\ \frac{\partial F_6}{\partial U_S^1} & \frac{\partial F_6}{\partial E_S^1}\end{vmatrix}}=\frac{\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial a_4}-\frac{\partial F_3}{\partial a_4}\frac{\partial F_6}{\partial E_S^1}}{\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}} \tag{4.66}$$

From Equation (4.66) gives

$$\frac{\partial F_3}{\partial U_S^1}=-a_4\left(S+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)-a_5 \tag{4.67}$$

$$\frac{\partial F_3}{\partial E_S^1}=a_4U_S^1+a_6-\frac{a_4\delta}{\beta}U_S^1 \tag{4.68}$$

$$\frac{\partial F_3}{\partial a_4}=-U_S^1\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \tag{4.69}$$

$$\frac{\partial F_6}{\partial U_S^1}=a_4\left(S+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \tag{4.70}$$

$$\frac{\partial F_6}{\partial E_S^1}=-a_4U_S^1+\frac{a_4\delta}{\beta}U_S^1-\left(a_5+a_6+a_7\right) \tag{4.71}$$

$$\frac{\partial F_6}{\partial a_4}=U_S^1\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \tag{4.72}$$

Using Equations (4.67) to (4.72) yields;

$$\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial a_4}-\frac{\partial F_3}{\partial a_4}\frac{\partial F_6}{\partial E_S^1}=-U_S^1\left(a_5+a_7\right)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)<0 \quad (4.73)$$

$$\left.\begin{aligned}\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}=\left(a_5+a_7\right)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\ +a_5\left(a_4U_S^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.74)$$

Simplifying further will lead to

$$\frac{dU_S^1}{da_4}<0$$

Therefore, a_4 is an inverse correlate of U_S^1.

Now,

$$\frac{dE_S^1}{da_4}=\frac{\begin{vmatrix}\frac{\partial F_3}{\partial a_4} & \frac{\partial F_3}{\partial U_S^1}\\ \frac{\partial F_6}{\partial a_4} & \frac{\partial F_6}{\partial U_S^1}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_3}{\partial U_S^1} & \frac{\partial F_3}{\partial E_S^1}\\ \frac{\partial F_6}{\partial U_S^1} & \frac{\partial F_6}{\partial E_S^1}\end{vmatrix}}=\frac{\frac{\partial F_3}{\partial a_4}\frac{\partial F_6}{\partial U_S^1}-\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial a_4}}{\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}} \quad (4.75)$$

$$\frac{\partial F_3}{\partial a_4}\frac{\partial F_6}{\partial U_S^1}-\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial a_4}=a_5U_S^1\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)>0 \quad (4.76)$$

$$\left.\begin{aligned}\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}=\left(a_5+a_7\right)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\ +a_5\left(a_4U_S^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.77)$$

Simplifying Equations (4.76) and (4.77) further according to Equation (4.75), yields

$$\frac{dE_S^1}{da_4}>0$$

Hence, a_4 is a positive determinant of E_S^1.

4.6.4 Behaviour of the Equilibrium Value of Unemployed Persons with Respect to $\boldsymbol{\alpha}$

Properties of the equilibrium value of unemployed persons with respect to α (rate at which new vacancies are created in favour of the unemployed) are assessed to confirm its characteristics in the system. From Equations (4.31) and (4.34),

$$F_1\left(\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right),\alpha\right)=a_1-a_2U_A^1\left(A+C^1-E_A^1\right)-a_5U_A^1+a_6E_A^1$$

$$F_4\left(\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right),\alpha\right)=a_2U_A^1\left(A+C^1-E_A^1\right)-a_5E_A^1-a_6E_A^1-a_7E_A^1$$

$$\frac{dU_A^1}{d\alpha}=\frac{\begin{vmatrix}\dfrac{\partial F_1}{\partial E_A^1} & \dfrac{\partial F_1}{\partial \alpha}\\ \dfrac{\partial F_4}{\partial E_A^1} & \dfrac{\partial F_4}{\partial \alpha}\end{vmatrix}}{\begin{vmatrix}\dfrac{\partial F_1}{\partial U_A^1} & \dfrac{\partial F_1}{\partial E_A^1}\\ \dfrac{\partial F_4}{\partial U_A^1} & \dfrac{\partial F_4}{\partial E_A^1}\end{vmatrix}}=\frac{\dfrac{\partial F_1}{\partial E_A^1}\dfrac{\partial F_4}{\partial \alpha}-\dfrac{\partial F_1}{\partial \alpha}\dfrac{\partial F_4}{\partial E_A^1}}{\dfrac{\partial F_1}{\partial U_A^1}\dfrac{\partial F_4}{\partial E_A^1}-\dfrac{\partial F_1}{\partial E_A^1}\dfrac{\partial F_4}{\partial U_A^1}} \tag{4.78}$$

From Equation (4.78) gives

$$\frac{\partial F_1}{\partial U_A^1}=-a_2\left(A+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)-a_5 \tag{4.79}$$

$$\frac{\partial F_1}{\partial E_A^1}=a_2U_A^1+a_6-\frac{a_2\delta}{\beta}U_A^1 \tag{4.80}$$

$$\frac{\partial F_1}{\partial \alpha}=-a_2U_A^1\left(\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right) \tag{4.81}$$

$$\frac{\partial F_4}{\partial U_A^1}=a_2\left(A+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \tag{4.82}$$

$$\frac{\partial F_4}{\partial E_A^1}=-a_2U_A^1+\frac{a_2\delta}{\beta}U_A^1-\left(a_5+a_6+a_7\right) \tag{4.83}$$

$$\frac{\partial F_4}{\partial \alpha}=a_2U_A^1\left(\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right) \tag{4.84}$$

Using Equations (4.79) to (4.84) results in

$$\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial \alpha}-\frac{\partial F_1}{\partial \alpha}\frac{\partial F_4}{\partial E_A^1}=a_2\left(a_5+a_7\right)U_A^1\left(\frac{\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right)<0 \tag{4.85}$$

$$\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial E_A^1}-\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial U_A^1}=\left.\begin{array}{l}(a_5+a_7)\left(A-E_A^1+\dfrac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\ +a_5\left(a_2U_A^1+a_5+a_7\right)>0\end{array}\right\} \tag{4.86}$$

Simplifying further results in

$$\frac{dU_A^1}{d\alpha}<0$$

Thus, is α a negative determinant of U_A^1.

4.6.5 Characteristics of the Equilibrium Values of Unemployed Persons with Respect to $\boldsymbol{\delta}$

The properties of the values of the equilibrium point for employed individuals with respect to δ (rate at which new vacancies are created in favour of the employed) is assessed to establish it characteristic in the system.

Now from Equations (4.31) *and* (4.34),

$$\frac{dE_A^1}{d\delta}=\frac{\begin{vmatrix}\dfrac{\partial F_1}{\partial\delta} & \dfrac{\partial F_1}{\partial U_A^1}\\ \dfrac{\partial F_4}{\partial\delta} & \dfrac{\partial F_4}{\partial U_A^1}\end{vmatrix}}{\begin{vmatrix}\dfrac{\partial F_1}{\partial U_A^1} & \dfrac{\partial F_1}{\partial E_A^1}\\ \dfrac{\partial F_4}{\partial U_A^1} & \dfrac{\partial F_4}{\partial E_A^1}\end{vmatrix}}=\frac{\dfrac{\partial F_1}{\partial\delta}\dfrac{\partial F_4}{\partial U_A^1}-\dfrac{\partial F_1}{\partial U_A^1}\dfrac{\partial F_4}{\partial\delta}}{\dfrac{\partial F_1}{\partial U_A^1}\dfrac{\partial F_4}{\partial E_A^1}-\dfrac{\partial F_1}{\partial E_A^1}\dfrac{\partial F_4}{\partial U_A^1}} \tag{4.87}$$

From Equation (4.87), these results can be obtained.

$$\frac{\partial F_1}{\partial\delta}=-a_2U_A^1\left(\frac{\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right) \tag{4.88}$$

$$\frac{\partial F_4}{\partial\delta}=a_2U_A^1\left(\frac{\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right) \tag{4.89}$$

Substitute Equations (4.88) and (4.89) in addition to previously computed expressions, to get

$$\frac{\partial F_1}{\partial\delta}\frac{\partial F_4}{\partial U_A^1}-\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial\delta}=a_5U_A^1\left(A-E_A^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right)>0 \tag{4.90}$$

$$\frac{\partial F_1}{\partial U_A^1}\frac{\partial F_4}{\partial E_A^1}-\frac{\partial F_1}{\partial E_A^1}\frac{\partial F_4}{\partial U_A^1}=\left(a_5+a_7\right)\left(A-E_A^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) + a_5\left(a_2U_A^1+a_5+a_7\right)>0 \tag{4.91}$$

This is further simplified to obtain

$$\frac{dE_A^1}{d\delta}>0$$

Therefore, δ is a positive correlate of E_A^1.

4.6.6 Properties of the Equilibrium Values of Unemployed Persons with Respect to $\boldsymbol{\alpha}$

The properties of the values of the equilibrium point for unemployed persons with respect to α (rate of at which new vacancies are created in favour of the unemployed) are investigated to ascertain its properties in the system. From Equations (4.32) and (4.35),

$$F_2\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_1-a_3U_I^1\left(I+C^1-E_I^1\right)-a_5U_I^1+a_6E_I^1$$

$$F_5\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_3U_I^1\left(I+C^1-E_I^1\right)-a_5E_I^1-a_6E_A^1-a_7E_A^1$$

$$\frac{dU_I^1}{d\alpha}=\frac{\begin{vmatrix}\frac{\partial F_2}{\partial E_I^1} & \frac{\partial F_2}{\partial \alpha}\\ \frac{\partial F_5}{\partial E_I^1} & \frac{\partial F_5}{\partial \alpha}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_2}{\partial U_I^1} & \frac{\partial F_2}{\partial E_I^1}\\ \frac{\partial F_5}{\partial U_I^1} & \frac{\partial F_5}{\partial E_I^1}\end{vmatrix}}=\frac{\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial \alpha}-\frac{\partial F_2}{\partial \alpha}\frac{\partial F_5}{\partial E_I^1}}{\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}} \tag{4.92}$$

Applying Equation (4.54) gives

$$\frac{\partial F_2}{\partial U_I^1}=-a_3\left(I+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)-a_5 \tag{4.93}$$

$$\frac{\partial F_2}{\partial E_I^1}=a_3U_I^1+a_6-\frac{a_3\delta}{\beta}U_I^1 \tag{4.94}$$

$$\frac{\partial F_2}{\partial \alpha} = -a_3 U_I^1 \left(\frac{\left(U_A^1, U_I^1, U_S^1\right)}{\beta} \right) \tag{4.95}$$

$$\frac{\partial F_5}{\partial U_I^1} = a_3 \left(I + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta} \right) \tag{4.96}$$

$$\frac{\partial F_5}{\partial E_I^1} = -a_3 U_I^1 + \frac{a_3 \delta}{\beta} U_I^1 - \left(a_5 + a_6 + a_7\right) \tag{4.97}$$

$$\frac{\partial F_5}{\partial \alpha} = -a_3 U_1^1 \left(\frac{\left(U_A^1, U_I^1, U_S^1\right)}{\beta} \right) \tag{4.98}$$

Applying Equations (4.93) to (4.98) gives the following:

$$\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial \alpha} - \frac{\partial F_2}{\partial \alpha}\frac{\partial F_5}{\partial E_I^1} = -U_I^1\left(a_5 + a_7\right)\left(I - E_I^1 + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta} \right) < 0 \tag{4.99}$$

$$\left.\begin{aligned} \frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1} - \frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1} = \left(a_5 + a_7\right)\left(I - E_I^1 + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta} \right) \\ + a_5\left(a_3 U_I^1 + a_5 + a_7\right) > 0 \end{aligned}\right\} \tag{4.100}$$

Simplifying further Equations (4.99) and (4.100) as in Equation (4.92) yields

$$\frac{dU_I^1}{d\alpha} < 0$$

Hence, α is a negative determinant of U_I^1.

4.6.7 Behaviour of the Equilibrium Values of Employed Persons with Respect to $\boldsymbol{\delta}$

The behaviour of the values of the equilibrium points for employed persons with respect to δ (rate of at which new vacancies are created in favour of the employed) is investigated to ascertain its behaviour in the system.

Now from Equations(4.32) *and* (4.35),

$$\frac{dE_I^1}{d\delta}=\frac{\begin{vmatrix}\frac{\partial F_2}{\partial \delta} & \frac{\partial F_2}{\partial U_I^1}\\ \frac{\partial F_5}{\partial \delta} & \frac{\partial F_5}{\partial U_I^1}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_2}{\partial U_I^1} & \frac{\partial F_2}{\partial E_I^1}\\ \frac{\partial F_5}{\partial U_I^1} & \frac{\partial F_5}{\partial E_I^1}\end{vmatrix}}=\frac{\frac{\partial F_2}{\partial \delta}\frac{\partial F_5}{\partial U_I^1}-\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial \delta}}{\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}} \qquad (4.102)$$

Since all other derivatives have already been evaluated above in the previous sections, only the following are computed in Equation (4.102):

$$\frac{\partial F_2}{\partial \delta}=-a_3E_I^1\left(\frac{\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \qquad (4.103)$$

$$\frac{\partial F_5}{\partial \alpha}=-a_3E_1^1\left(\frac{\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \qquad (4.104)$$

Applying Equations (4.103) and (4.104) leads to the following:

$$\frac{\partial F_2}{\partial \delta}\frac{\partial F_5}{\partial U_I^1}-\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial \delta}=a_5U_I^1\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)>0 \quad (4.105)$$

$$\left.\begin{array}{l}\frac{\partial F_2}{\partial U_I^1}\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\frac{\partial F_5}{\partial U_I^1}=\left(a_5+a_7\right)\left(I-E_I^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\ +a_5\left(a_3U_I^1+a_5+a_7\right)>0\end{array}\right\} \quad (4.106)$$

Simplifying further Equations (4.105) and (4.106) as in Equation (4.102), the result is

$$\frac{dE_I^1}{d\delta}>0$$

Thus, δ is a positive determinant of E_I^1.

4.6.8 Behaviour of the Equilibrium Values of Unemployed Persons with Respect to $\boldsymbol{\alpha}$

The behaviour of the values of the equilibrium points for unemployed persons with respect to α (rate of at which new vacancies are created in favour of the unemployed) is investigated to establish its characteristics in the system. From Equations (4.33) and (4.36).

$$F_3\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_1-a_4U_S^1\left(S+C^1-E_s^1\right)-a_5U_s^1+a_6E_s^1$$

$$F_6\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=a_4U_S^1\left(S+C^1-E_S^1\right)-a_5E_s^1-a_6E_s^1-a_7E_S^1$$

$$\frac{dU_S^1}{d\alpha}=\frac{\begin{vmatrix}\frac{\partial F_3}{\partial E_S^1} & \frac{\partial F_3}{\partial \alpha}\\ \frac{\partial F_6}{\partial E_S^1} & \frac{\partial F_6}{\partial \alpha}\end{vmatrix}}{\begin{vmatrix}\frac{\partial F_3}{\partial U_S^1} & \frac{\partial F_3}{\partial E_S^1}\\ \frac{\partial F_6}{\partial U_S^1} & \frac{\partial F_6}{\partial E_S^1}\end{vmatrix}}=\frac{\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial \alpha}-\frac{\partial F_3}{\partial \alpha}\frac{\partial F_6}{\partial E_S^1}}{\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}} \quad (4.107)$$

From Equation (4.107), the following derivatives are evaluated:

$$\frac{\partial F_3}{\partial U_S^1}=-a_4\left(S+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)-a_5 \quad (4.108)$$

$$\frac{\partial F_3}{\partial E_S^1}=a_4U_S^1+a_6-\frac{a_4\delta}{\beta}U_S^1 \quad (4.109)$$

$$\frac{\partial F_3}{\partial \alpha}=-a_4U_S^1\left(\frac{\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right) \quad (4.110)$$

$$\frac{\partial F_6}{\partial U_S^1}=a_4\left(S+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right) \quad (4.111)$$

$$\frac{\partial F_6}{\partial E_S^1}=-a_4U_S^1+\frac{a_4\delta}{\beta}U_S^1-\left(a_5+a_6+a_7\right) \quad (4.112)$$

$$\frac{\partial F_6}{\partial \alpha}=a_4U_S^1\left(\frac{\left(U_A^1,U_I^1,U_S^1\right)}{\beta}\right) \quad (4.113)$$

Using Equations (4.108) to (4.113) results in

$$\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial \alpha}-\frac{\partial F_3}{\partial \alpha}\frac{\partial F_6}{\partial E_S^1}=-U_S^1\left(a_5+a_7\right)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)<0 \quad (4.114)$$

$$\left.\begin{aligned}\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}&=\left(a_5+a_7\right)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\&\quad+a_5\left(a_4U_S^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.115)$$

Then, simplifying further will lead to

$$\frac{dU_S^1}{d\alpha} < 0$$

Thus, α is an inverse determinant of U_S^1.

4.6.9 Properties of the Values of the Equilibrium of Employed Persons with Respect to δ

The properties of the values of the equilibrium points for persons employed with respect to δ (rate of at which new vacancies are created in favour the persons employed) is evaluated to establish its characteristics in the system.

Now from Equations (4.33) and (4.36),

$$\frac{dE_S^1}{d\delta} = \frac{\begin{vmatrix} \frac{\partial F_3}{\partial \delta} & \frac{\partial F_3}{\partial U_S^1} \\ \frac{\partial F_6}{\partial \delta} & \frac{\partial F_6}{\partial U_S^1} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_3}{\partial U_S^1} & \frac{\partial F_3}{\partial E_S^1} \\ \frac{\partial F_6}{\partial U_S^1} & \frac{\partial F_6}{\partial E_S^1} \end{vmatrix}} = \frac{\frac{\partial F_3}{\partial \delta}\frac{\partial F_6}{\partial U_S^1} - \frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial \delta}}{\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1} - \frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}} \qquad (4.116)$$

From Equation (4.116) the only derivatives required are given in the following two expressions:

$$\frac{\partial F_3}{\partial \delta} = -a_4 U_S^1 \left(\frac{\left(E_A^1, E_I^1, E_S^1\right)}{\beta} \right) \qquad (4.117)$$

$$\frac{\partial F_6}{\partial \delta} = a_4 U_S^1 \left(\frac{\left(E_A^1, E_I^1, E_S^1\right)}{\beta} \right) \qquad (4.118)$$

Substituting further, gives

$$\frac{\partial F_3}{\partial \delta}\frac{\partial F_6}{\partial U_S^1} - \frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial \delta} = a_5 U_S^1 \left(S - E_S^1 + \frac{\alpha\left(U_A^1, U_I^1, U_S^1\right) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta} \right) > 0 \qquad (4.119)$$

$$\left.\begin{aligned}\frac{\partial F_3}{\partial U_S^1}\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\frac{\partial F_6}{\partial U_S^1}=\left(a_5+a_7\right)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)\\+a_5\left(a_4U_S^1+a_5+a_7\right)>0\end{aligned}\right\} \quad (4.200)$$

Putting Equations (4.119) and (4.200) into Equation (4.116), the result is

$$\frac{dE_S^1}{d\delta}>0$$

Therefore, δ and E_S^1 are directly related.

Also, from Equation (4.37),

$$F_7\left(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1\right)=\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta\left(E_A^1,E_I^1,E_S^1\right)-\beta C^1$$

where;

$$C^1=\frac{\alpha\left(U_A^1+U_I^1+U_S^1\right)+\delta\left(E_A^1+E_I^1+E_S^1\right)}{\beta} \quad (4.210)$$

Thus, the derivative of C^1 with respect to α and δ is

$$\left.\begin{aligned}\frac{dC^1}{d\alpha}=\frac{U_A^1+U_I^1+U_S^1}{\beta}>0\\ \frac{dC^1}{d\delta}=\frac{E_A^1+E_I^1+E_S^1}{\beta}>0\end{aligned}\right\} \quad (4.211)$$

Therefore, it is conclusive from Equation (4.211) that both α and δ are positive correlates of C^1.

4.7 Analytical Deduction

It can be construed that in equilibrium, the number of persons unemployed declines as the rate of transition of persons from the unemployment to the employment class a_2 rises. Hence, in equilibrium, the rate of transition of individuals from the unemployment to the employment class increases with a decrease in the number of unemployed persons. For the number of employed persons, we observe an increase when the rate of transition of persons from the unemployment class to the employment class a_2 increases. The behaviour conforms to expectation since a decline in the number of unemployed persons can be attributed to an increase in the number of individuals employed.

Subjecting the parameter α to further analysis, it can be established that, an increase in the rate at which new vacancies are created induces the number of unemployed persons to decrease, but an upsurge in the number of persons employed. It is a general expectation that the number of persons in the unemployed class will decrease as more vacancies are created since they can be absorbed into employment in agriculture, industry or service sectors by the government or private individuals.

CHAPTER 5

SIMULATIONS AND DISCUSSION OF RESULTS

5.1 Background

This chapter presents the simulations, sensitivity analysis using the employment elasticities of the sectors, and discussions of the result. The data used were obtained from the Ghana Statistical Service (Anon., 2019) and the Ministry of Finance (Anon., 2022a).

5.2 Numerical Method

For the numerical simulation, through the Euler method, the exploration of the model with data on the variables and the parameters was achieved. Simulations of the model were done on the effect of varying different rate values of a_2 , a_3 , a_4 , α , and β on unemployment, employment, newly created vacancies in relation to the unemployed and newly created vacancies in relation to the employed among the three main economic sectors (agriculture, industry and services) on Ghana's labour market. The table below shows the parameter values and the descriptions of the model.

Table 5.1 Definition of Parameters and Simulation Values

Parameter	Value	Source
a_1	30000	Computed based on GSS Anon (2019)
a_2	0.001	Estimated based on GSS Anon (2019)
a_3	0.05	Estimated based on GSS Anon (2019)
a_4	0.0001	Estimated based on GSS Anon (2019)
a_5	0.007	Computed
a_6	0.05	Computed
a_7	0.0056	Computed
α	0.02	Computed
δ	0.2	Estimated based on (Kazeem *et al.*, 2018)
β	0.62	Computed

5.3 Feasibility Analysis of the Equilibrium Values of the Model

For the numerical simulation through the Euler method, we consider the data in Table 5.1. To check the feasibility of our analysis, we have conducted some numerical computation using MATLAB 9.4.0 by choosing the following set of parameter values in the model system. So, the equilibrium values of the model are presented in Table 5.2.

The computed eigenvalues of the matrix corresponding to the equilibrium (U_A', $U_I', U_S', E_A', E_I', E_S', C', A', I', S'$) of the model system (1) are $12.817, 0.1776, 245.73, -207796, -0.20698, -82.676, -12.887, -2.055, -245.8$ which are either negative or have a negative real part, thus, the equilibrium is locally asymptotically stable. Using the above data, Fig. 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, 5.11 and 5.12 below were simulated on the variables unemployment, employment and newly created vacancies in agriculture, services and industry sectors.

5.4 Values of Parameters and Initial Values

The values of the model's parameters and initial values together with their sources are summarized in Table 5.2.

Table 5.2 Summary of Initial Values of Variables and Sources

Variable	Initial Value	Source
U_A	20959	GSS, Anon (2019)
U_I	10733	GSS, Anon (2019)
U_S	30240	GSS, Anon (2019)
E_A	50040	GSS, Anon (2019)
E_I	30267	GSS, Anon (2019)
E_S	16060	GSS, Anon (2019)
C	100	GSS, Anon (2019)
A	200	GSS, Anon (2019)
I	450	GSS, Anon (2019)
S	454	GSS, Anon (2019)

5.5 Results and Discussion

The model's simulations have been carried out, and the findings are adequately discussed as illustrated below.

5.5.1 Simulation of the Effect of $\boldsymbol{a_2}$ on Unemployment in Agriculture Sector

The effect of varying the rate of transition of persons from the unemployment to the employment class in agriculture sector ($\boldsymbol{a_2}$) is depicted in Fig. 5.1.

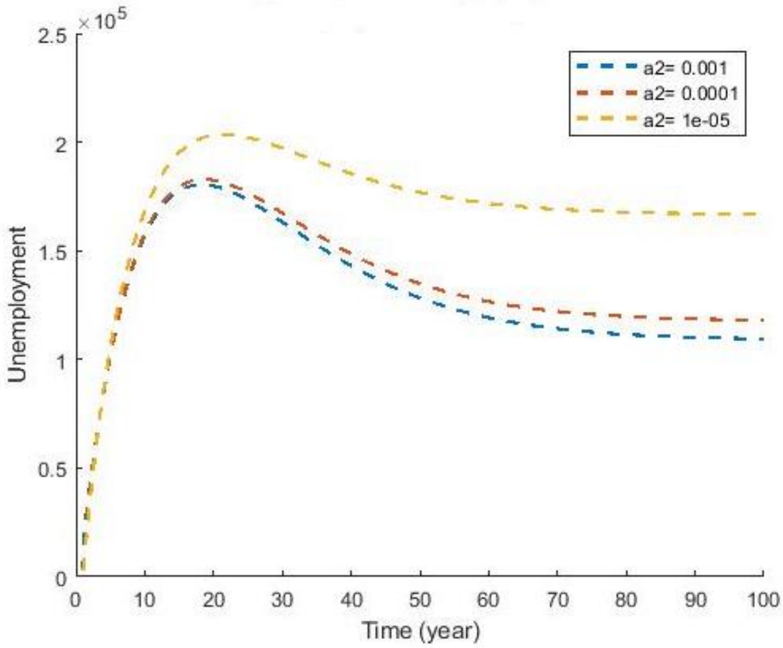

Fig. 5.1 Simulation of Unemployment in Agriculture at Different Rates of $\boldsymbol{a_2}$

Varying the rate of transition of persons from the unemployment to the employment class in agriculture sector do not have a significant impact on unemployment in the agriculture over the first ten years, however, the impacts were more pronounced after 10 years of unemployment in the agricultural sector. The simulation indicates that when the a_2 increases, then the number of unemployed persons reduces. It can be seen that if a_2 increases, then the number of unemployed persons in the agriculture sector decreases with time. For instance, at a rate of 0.001, total unemployment increases at an increasing rate to attain its maximum growth at year 16, it declines and remains almost constant after year 80. Relatively low-rate value (that is 0.00001) of movement of unemployed persons joining the employed in the agriculture sector increased the unemployed persons within the labour force over the years.

5.5.2 Simulation of the Effect of $\boldsymbol{a_3}$ on Unemployment in the Industry Sector

The impact of changing the rate at which individuals from the unemployment class transits to the employment class in the industry sector ($\boldsymbol{a_3}$) over time on unemployment in the industry is shown in Fig. 5.2.

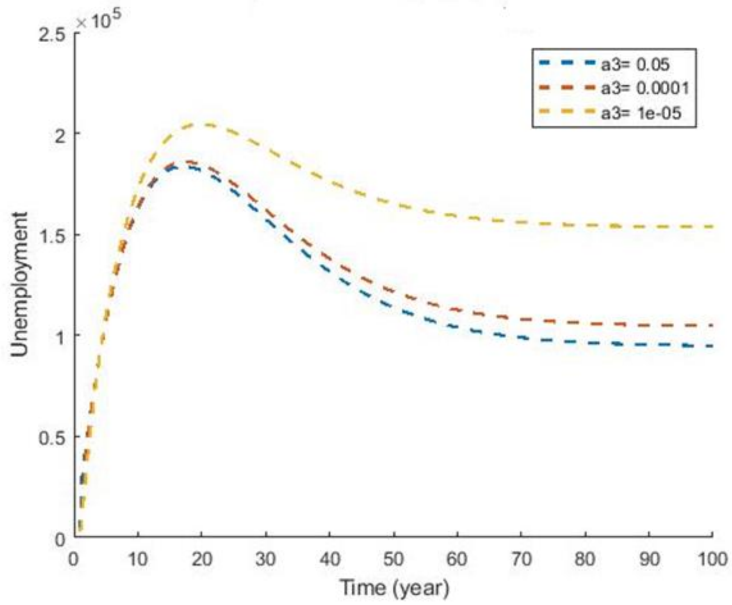

Fig. 5.2 Simulation of Unemployment in Industry at different Rates of $\boldsymbol{a_3}$

Generally, unemployment reduces when the value of the rate of transition of individuals from the unemployment class to the employment class in the industry sector (a_3) was at the highest rate of 0.05. Remarkably, at different rates of a_3, the simulation indicated that, at the initial years between 0-10 years, a_3 , do not have a significant impact on unemployment in the industry, but the impacts were more pronounced after 15 years in the industry sector. The effect of unemployment reduction can be seen as the rate of transition of persons from the unemployment to the employment class in industry sectors goes higher than the rate at which unemployed persons reduces. Unemployment increases only when a_3 is at the smallest value of 0.00001. For instance, at a rate 0.05, total unemployment increases at an increasing rate to attain its maximum at year 16, declines at the decreasing rate and remains almost constant after year 80. A very low value (that is 0.00001) of movement of unemployed persons joining the employed in the industry sector, resulted in the highest total unemployment and slowly stabilised after 70 years.

5.5.3 Simulation of the Effects of $\boldsymbol{a_4}$ on Unemployment in the Services Sector

In Fig. 5.3, a simulation of unemployment in the services sector at various rate of transition of persons from the unemployment class to the employment class in the services sector $\boldsymbol{a_4}$ is shown.

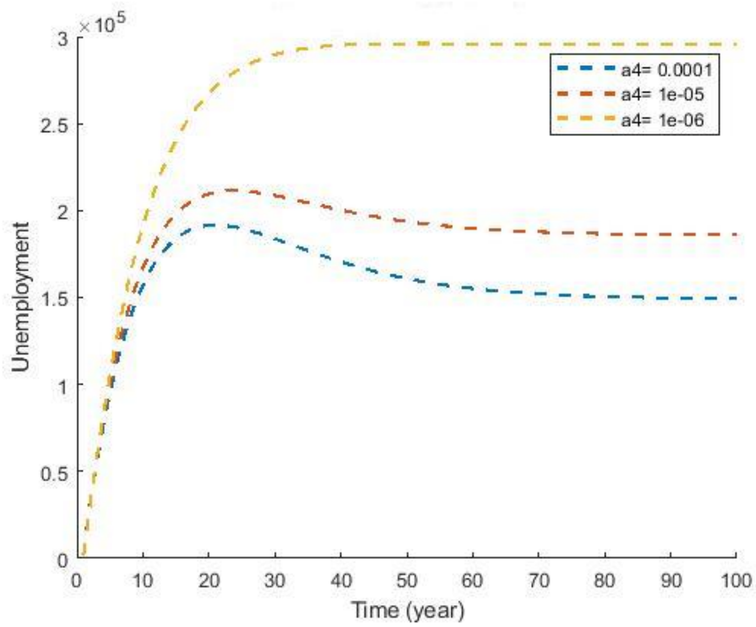

Fig. 5.3 Simulation of Unemployment in Services at Different Rates of a_4

When more unemployed people are joining the workforce in the services, unemployment decreases. The initial results show that there were no significant impacts on unemployment in the services over the first ten years at different parameter values, however, the impacts were more pronounced after 20 years. Total unemployment, a rate of a_4= 0.00001 increases at an increasing rate till it attains maximum at year 24, declines at a decreasing rate and remains almost constant after year 60. It can be seen that if the rate of transition of persons from the unemployment class to the employment class in the services sector increases, the number of total unemployed persons in the services sector decreases. For instance, at a rate 0.0001, total unemployment increases at an increasing rate to attain its maximum at year 22, declines at the decreasing rate and remains almost constant after year 70. Comparatively, the lowest rate value of 0.000001 increased unemployment over the years.

5.5.4 Simulation of the Impact of a_2 on Employment in the Agriculture Sector

In Fig. 5.4, the model's simulation result for the parameter a_2 and its employment behaviour is shown.

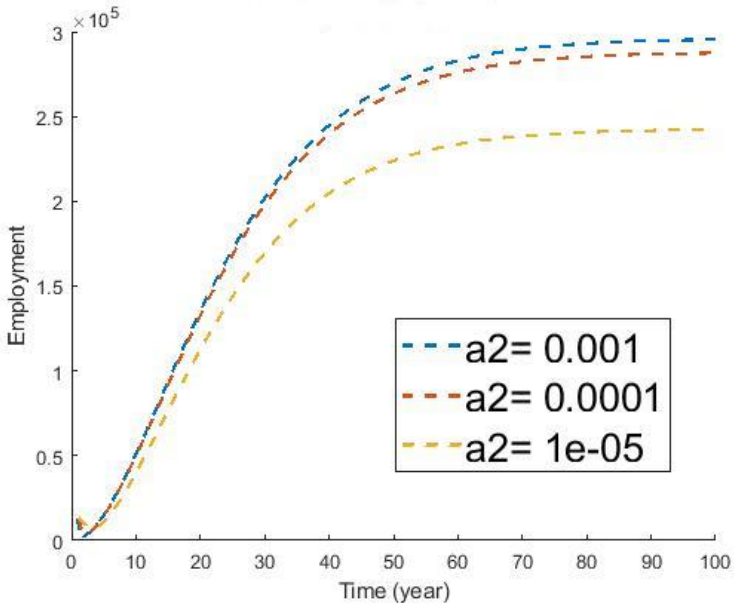

Fig. 5.4 Simulation of Employment in Agriculture at Different rates of a_2

By modelling employment on the rate at which jobless people join the workforce in the agricultural sector, more people in the labour force were able to find employment. Different parameter values of the rate of transition of persons from the unemployment to the employment class in agriculture sector do have a significant impact on total employment over the years. The number of persons getting employment increases when the rate value of a_2 is the highest value. Consequently, it can be said that for employment to observe a boost in the economy, the rate of transition of persons from the unemployment to the employment class in agriculture sector should be at a high rate of 0.001 in the agricultural sector over the years. At the lowest rate 0.00001 produced the minimum number of persons getting employed in the agriculture sector. On the rate value of 0.0001, movement of unemployed persons joining the employed in the agriculture sector resulted in total employment expansions at year 60 and continues to grow at a constant rate till the end of the 100 years. In total, as the of transition of persons from the unemployment to the employment class in agriculture sector parameter value increases, total employment into the agriculture sector proliferates tremendously from the beginning of the year till it reaches the highest total employment.

5.5.5 Simulation on the Impact of a_3 on Employment in the Industry Sector

From Fig. 5.5, it can be seen that as the rate of transition of individuals from the unemployment class to the employment class in the industry sector increases, the number of employed persons in the industry sector increases.

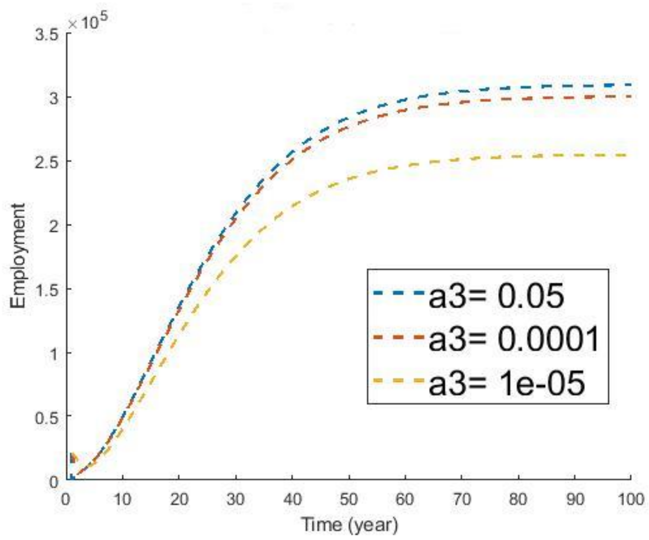

Fig. 5.5 Simulation of Employment in Industry at Different Rates of a_3

Varying the rate of transition of individuals from the unemployment class to the employment class in the industry sector does not have a significant impact on total employment for the initial thirty years. However, the effect on industrial employment was more pronounced after the year 40. At the lowest rate of 0.00001, total employment increases at an increasing rate to reach its maximum at year 70, continues to increase at an increasing rate and remains nearly constant thereafter. On the rate value of 0.0001, movement of unemployed persons joining the employed in the agriculture sector resulted in total employment expansions at year 60 and continues to grow at a constant rate till the end of the 100 years. Comparatively the highest rate value of 0.05 (movement of unemployed persons joining the employed in the industry sector on employment) resulted in the highest total employment and continues to expand throughout the 100-year period. In total, as the of transition of persons from the unemployment to the employment class in agriculture sector increases, total employment in the industry sector proliferates tremendously from the beginning of the year till it reaches the highest total employment.

5.5.6 Simulation of the Impact of a_4 on Employment in the Services Sector

The variational impact of a_4 on employment in the service sector is depicted in Fig. 5.6. It can be seen that as the a_4, then employment in the services sector increases.

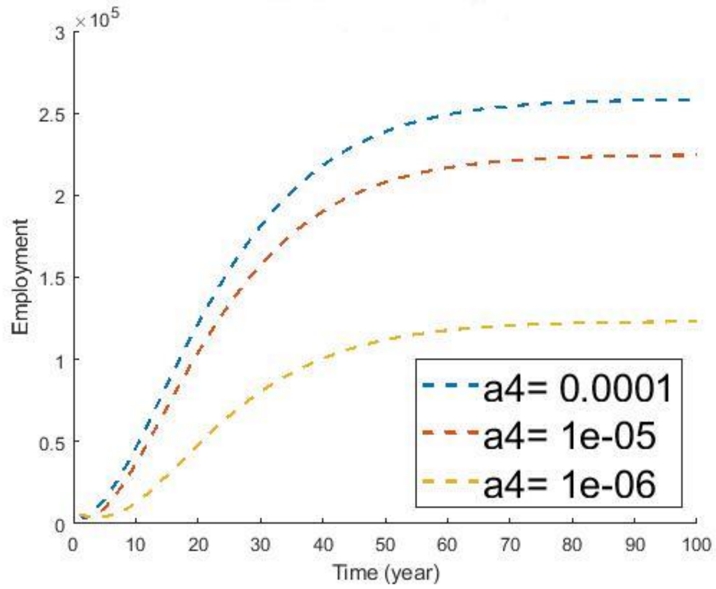

Fig. 5.6 Simulation of Employment in Services at Different Rates of a_4

Varying the rate of transition of persons from the unemployment class to the employment class in the services sector does not have a significant impact on total employment for the initial thirty years. At the lowest rate 0.000001, total employment increases at an increasing rate to attain its maximum at year 70, grows at an increasing rate and remains almost constant afterwards. On the rate value of 0.00001, movement of unemployed persons joining the employed in services sector resulted in total employment expansions at year 60 and continues to grow at a constant rate till the end of the 100 years. Comparatively the highest rate value 0.0001, movement of unemployed persons joining the employed in services sector resulted in the highest total employment and continues to expand throughout the 100 years period. In total, as the rate of transition of persons from the unemployment class to the employment class in the services sector increases, total employment in the services sector proliferates tremendously from the beginning of the year till it reaches the highest total employment.

5.5.7 Simulation of Unemployment in the Agricultural Sector at Different Rates of α

The simulation result of the effect of newly created vacancies in favour the unemployed (α) on the agricultural sector is analysed at varying rate values (Fig. 5.7). It turns out that when the number of new jobs in agriculture rises, unemployment falls. In other words, if the public or private sector creates more new jobs in proportion to the number of jobless people, unemployment can be substantially controlled. Generally, the result shows that higher rate values of newly created vacancies reduce unemployment in the agriculture sector.

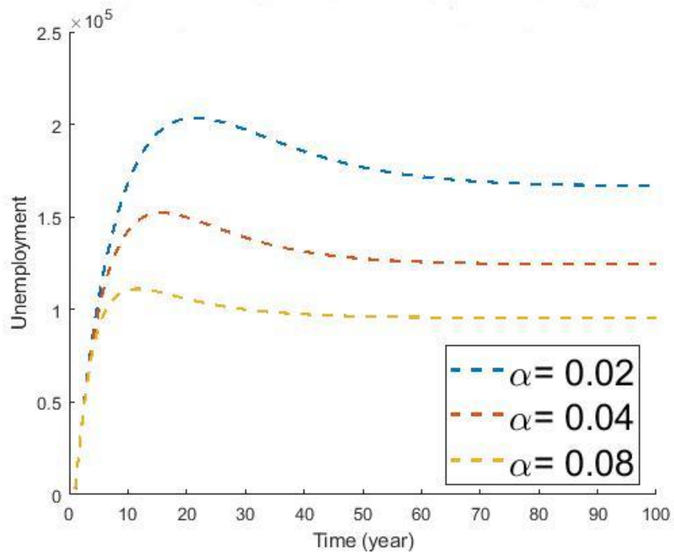

Fig. 5.7 Simulation of Unemployment at Different Rates of α in Agriculture

Varying the rate of transition of unemployed persons to the newly created vacancies in favour of the unemployed does not have a significant impact on unemployment in the agriculture over the first ten years, however, the impacts were more pronounced after 10 years of unemployment in the agricultural sector. It is clear that if the rate at which new vacancies are created in favour of the unemployed persons increases, the amount of unemployment in the agricultural sector decreases with time. For instance, at a rate 0.02, total unemployment increases at an increasing rate to attain its maximum at year 20, declines at the decreasing rate and remains almost constant after year 70. Relatively high-rate value (that is 0.08) of newly created vacancies in favour of the unemployed persons in the agriculture sector reduced total unemployment and slowly stabilised after 40 years. It is therefore recommended in line with Pathan and Bhathawala (2015) and Pathan and Bhathawala (2017a) that if the public and private sectors fill more jobs in accordance to the number of unemployed people, unemployment can be controlled to some extent.

5.5.8 Simulation of Unemployment at Different Rates of α in the Services Sector

Fig. 5.8 shows the simulation result of the effect of the rate at which new vacancies are created at different parameter values for the unemployed persons (α) in the services sector.

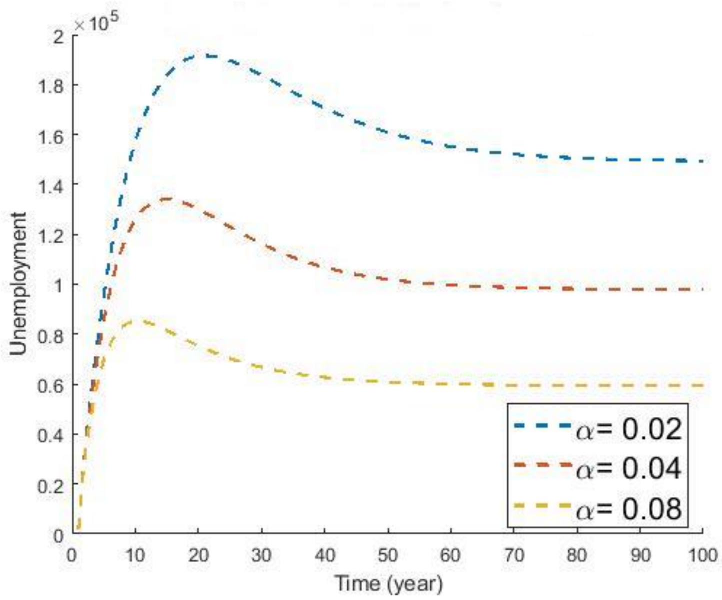

Fig. 5.8 A Simulation of Unemployment in Service Sector at Different Rates of α

Obviously, unemployed persons tend to decrease with respect to higher rate values of newly created vacancies by both private and government sectors. That is if the government or private sector creates more new vacancies proportional to the number of unemployed persons, then unemployment can be controlled at some level. However, the total amount of unemployment within the services sector reduces at varying rate values. The varying rate values of newly created vacancies in favour of unemployment do not have a significant impact on reducing unemployment as it reaches the peak 10 years, 16 years and 20 years respectively. For instance, at a rate 0.02, total unemployment increases at an increasing rate to attain its maximum at year 10, declines at the decreasing rate and remains almost constant after year 40. Relatively high-rate value (that is 0.08) of newly created vacancies in favour of the unemployed persons in the services sector reduced total unemployment and slowly stabilised after 40 years. At a rate of 0.04 of newly created vacancies in favour of the unemployed persons does not have a significant impact on unemployment in the services over the first 16 years, however, the impacts were more pronounced after 20 years of total unemployment in the services sector. Hence a reduced rate value will take longer for unemployment to reduce in the newly created vacancies compartment. In accordance with Pathan and Bhathawala (2015) and Pathan and Bhathawala (2017a), they suggested that unemployment can be somewhat controlled if the public and private sectors provide more job vacancies in proportion to the number of unemployed persons.

5.5.9 Simulation of Unemployment at Different Rates α in the Industry Sector

Fig. 5.9 shows the result of simulating the effect of the rate of newly created job vacancies in favour of unemployment at varying values on unemployment in the industrial sector. Unemployment tends to decrease with respect to higher rate values of newly created vacancies in respect of unemployment by both private and government sectors. However, the total amount of unemployment within the services sector reduces at varying rate values.

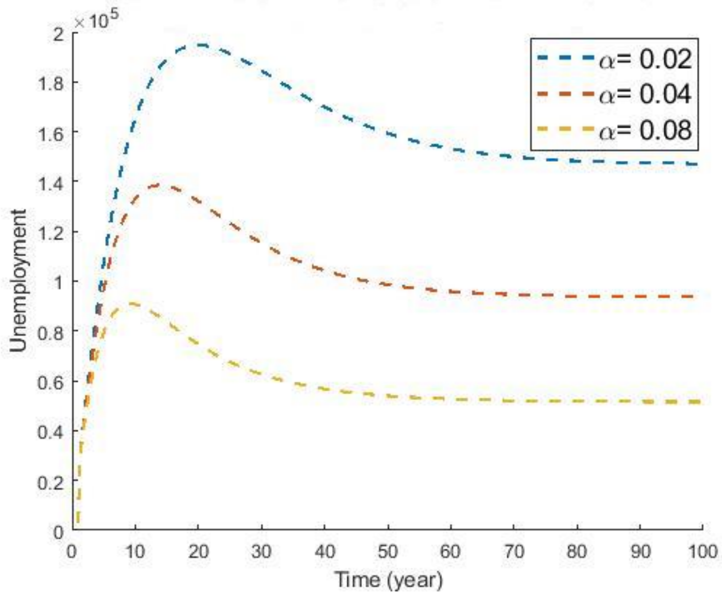

Fig. 5.9 Simulation of Unemployment in Industry at Different Rates of α

The varying rate values of newly created vacancies in favour of unemployment do not have a significant impact on reducing unemployment as it reaches the peak 10 years, 14 years and 20 years respectively. For instance, at a rate 0.02, total unemployment increases at an increasing rate to attain its maximum at year 10, declines at the decreasing rate and remains almost constant after year 40. Relatively high-rate value (that is 0.08) of newly created vacancies with respect to the unemployment in the industry sector reduces total unemployment and slowly remains constant over 60 years. At a rate of 0.04 newly created vacancies in relation to the unemployed persons does not have a significant impact on unemployment in the industry over the first 14 years. However, the impacts were more pronounced after 20 years of total unemployment in the industry sector. Therefore, the lower rate value will take longer for unemployment to have a meaningful reduction in new positions to be filled. It is consequently advised in accordance with Pathan and Bhathawala (2015) and Pathan and Bhathawala (2017a) that if the government and private sector produce more vacancies proportional to the number of unemployed persons than unemployment can control at certain levels.

5.5.10 Simulation of Employment at Different Rates of $\boldsymbol{\delta}$ in the Agriculture Sector

The impact of varying the rate at which new vacancies are created in favour of the employed persons ($\boldsymbol{\delta}$) in agriculture is presented (Fig. 5.10). It can be seen that despite the difference in the parameter δ (rate of at which new vacancies are created in favour the employed) the graph of employed persons with respect to time is very close.

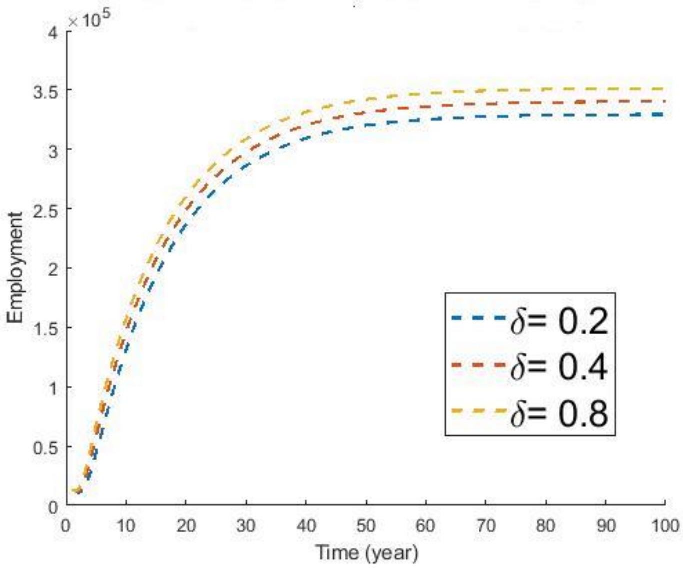

Fig. 5.10 Simulation of Employment in Agriculture Different Rates of δ

It suggests that if employment varies then the rate of newly created vacancies in favour of the employed should be high to create employment.

Varying the rates of newly created vacancies in relation to employment does have an important impact on total employment in the agriculture sector largely. However, the impacts were more pronounced after 30 years of employment in the agriculture sector. Though, the gains were more pronounced after 50 years of employment in agriculture. At the lowest rate 0.02, total employment increases at an increasing rate to attain its maximum at year 70, grows at an increasing rate and remains almost constant afterwards. It can be seen that when the rate of newly created vacancies in favour of the unemployed persons with regard to employment in agriculture increases, then the number of employed persons in the agriculture sector increases with time. At a rate 0.04, total employment increases at an increasing rate to attain its maximum at year 50, remaining almost constant afterwards. The high-rate value of 0.08 of the newly created vacancies in relation to the employed persons with regard to employment in agriculture resulted in the highest total employment gains and gradually remains constant over 60 years. It can be observed that employment growth for persons in the agriculture sector spikes from the beginning year till 60 years that the curve experienced stabilisation in employment for all the three parameter values of δ. However, a higher rate of newly created vacancies in favour of the employed greatly improves total employment of the labour force in agriculture. In accordance with Kazeem *et al.* (2018), as employment exists, vacancies also exist.

5.5.11 Simulation of Employment in the Service Sector at Different Rates of $\boldsymbol{\delta}$ in the Services Sector

Fig. 5.11 shows the impact of varying the rate of newly created vacancies in favour of persons already in employment ($\boldsymbol{\delta}$) in the service sector. Varying the rate of newly created vacancies in favour of the employment does have a significant impact on total employment in the services generally, nonetheless, the impacts were more pronounced after 30 years of employment across all rates values in the services sector. At the lowest rate value of 0.02, total employment increases at an increasing rate to attain its maximum at year 50, grows at an increasing rate and remains almost constant afterwards. It can be seen that when the rate of newly created vacancies in favour of the employed persons with regard to employment in services increases, the number of employed persons in the services sector increases with time.

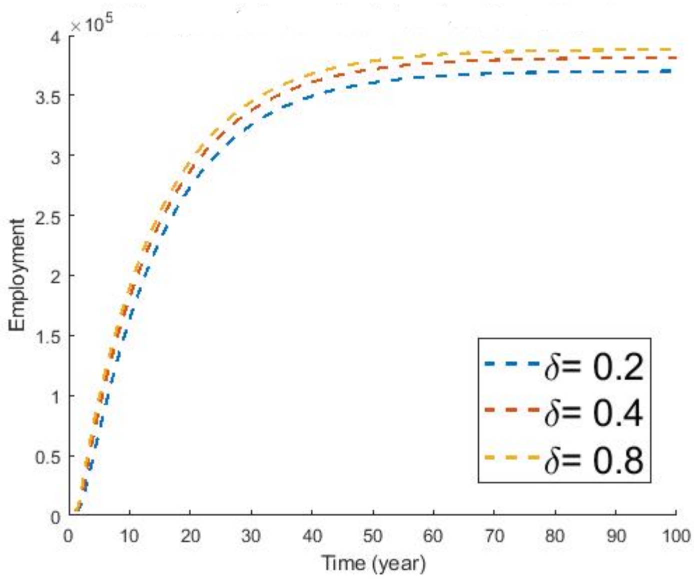

Fig. 5.11 Simulation of Employment at Different Rates of $\boldsymbol{\delta}$ in the Service Sector

At a rate of 0.04, total employment increases at a growing rate, reaching its peak by the year 50, and remains almost constant thereafter. The highest value of 0.08 in employment of the services sector resulted in the highest total employment absorption numbers of the labour force after reaching its peak and remained constant years subsequently. Thus, employment for persons in the services sector spikes till 60 years. However, higher rate of newly created vacancies in favour of the employed persons improves total employment of the labour force in the services sector. Under the services sector, more persons are expected to move into employment as newly vacancies have been created over the years at higher rates. Kazeem *et al.* (2018) asserts that there are both open roles and available jobs.

5.5.12 Simulation of Employment in Industry Due to Variation of $\boldsymbol{\delta}$ in Industry

Fig. 5.12 shows the simulation of newly created vacancies in industry at different rates of newly created vacancies in favour of the employed ($\boldsymbol{\delta}$) in industry. Varying the rate of newly created vacancies in favour of employment have improved employment significantly within the industry sector with an increased number of people in the labour force. Nonetheless, rates values of 0.4 and 0.8 resulted in almost equal employment gains over the time period. However, the impacts were more pronounced after 30 years of employment. Also, the increases were more pronounced after 50 years of employment in the industry sector.

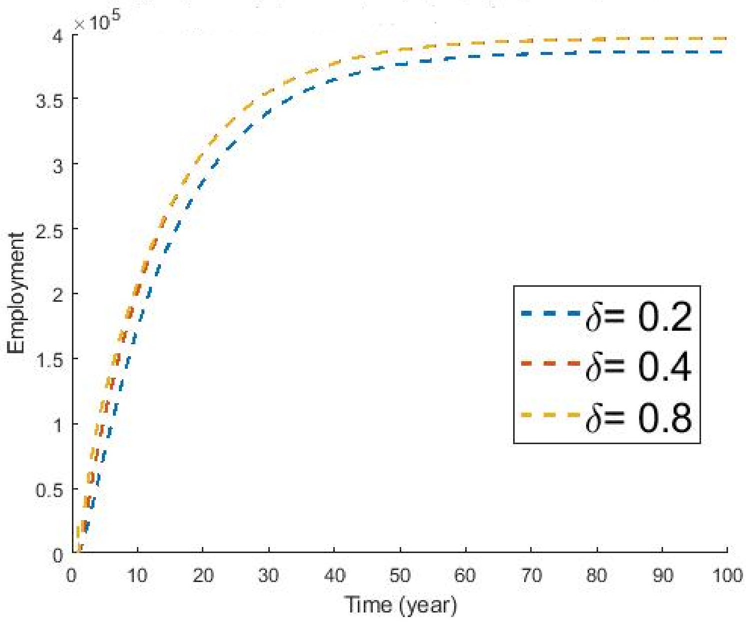

Fig. 5.12 Simulation of Employment in Industry at Different Rates of δ

At the lowest rate 0.02, total employment impact was more pronounced to attain its maximum at year 60, grows at an increasing rate and remains almost constant afterwards. At a rate 0.04, total employment increases at an increasing rate to attain its maximum at year 60, remaining almost constant afterwards. Relatively high-rate value (that is 0.08) of newly created vacancies in favour of the unemployed with regard to employment in the industry sector results in the highest total employment gains and slowly remains constant over 60 years. It is recorded however, that higher rates of newly created vacancies in favour of the employed improve total employment of the labour force. Therefore, existing employed persons do contribute to employment of the labour force.

5.6 Sensitivity of the Model's Variables to Variations in the Parameters of the Model

Economic employment elasticity method is used to calculate the elasticities of the parameters to establish the sensitivity of relevant parameters in the model Morén and Wändal (2018). Table 5.3 summarises the computations of the elasticities for the employment, unemployment and the newly created vacancies across the three main economic sectors.

Table 5.3 Results of Sensitivity Analysis of the Model's Parameters

Variables	Sensitivity of Parameters		
	$a_2 = 0.001$	$a_2 = 0.0001$	$a_2 = 0.00001$
U_A	7.8325408	3.304360	5.892212
E_A	-0.215828	-1.258514	-3.760062
	$a_3 = 0.05$	$a_3 = 0.0001$	$a_3 = 0.00001$
U_I	0.819309	3.740061	6.643396
E_I	-0.233359	-1.209857	-3.71397139
	$a_4 = 0.0001$	$a_4 = 0.00001$	$a_4 = 0.000001$
U_S	1.954901	4.707425	2.799458
E_S	-1.048043	-3.614202	-6.228246
C	$\alpha = 0.02$	$\alpha = 0.04$	$\alpha = 0.08$
	1.515668	1.005647	0.240137
	$\delta = 0.2$	$\delta = 0.4$	$\delta = 0.8$
	0.49866	0.600935	0.215314

A unit change in Ghana's growth rate will cause the rate of movement of unemployment to employment in agriculture (a_2) to increase by the margin 7.8325408. It is clear that the unemployed persons in all three sectors and newly created vacancies by all three sectors, on average, have the highest elasticities across all unemployed persons in the three economic sectors than the number of employed persons in all three sectors which recorded. These negative elasticities indicate that the rate of transition of individuals from the unemployment to the employment class in all three main economic sectors with regard to employment is inversely linked to economic growth. Hence, in addressing the issue of unemployment in Ghana, it is necessary to integrate the essential role Ghana's economic growth in policy formulation and decision-making as the rate of unemployed persons joining the employed in all three main economic sectors changes overtime. At $a_2 = 0.001$ with its elasticity value of 7.8325408, which is the most sensitive to growth has the highest elasticity for the agriculture sector with regard to economic growth. This signifies that more jobs are being created per unit of output growth in the agriculture sector at that rate. At $a_3 = 0.00001$, the highest elasticity value of 6.64339671 is attained in the industry sector. This is the most sensitive parameter value of growth in the industry sector which signifies that more jobs are created per unit of output growth in the industry at the rate 0.00001. However, in the services sector, the most sensitive parameter at $a_4 = 0.00001$ had the highest elasticity value of 4.707425923 among its varied parameters. This indicates the rate of growth of employment relative to output growth in the services sector is most significant when $a_4 = 0.00001$ as more employment is filled by the unemployed persons. For the newly created vacancies by all three sectors, at $\alpha = 0.02$, elasticity is 1.515668, which is the highest indicating that the rate at which new vacancies are created in favour of the unemployed is most sensitive to economic growth as more vacancies will be created to be filled by the unemployed persons per Ghana's growth rate for all three economic sectors. The economic growth will be driven by increased newly created vacancies in respect (favour) of the unemployed in addition to other factors (other parameters) which will consequently result in increase in new vacancies. At $\delta = 0.4$, the elasticity value is 0.600935307, thus the most sensitive among the rates. This indicates that the rate of newly created vacancies to be filled with employed persons will also, at that rate, contribute to economic growth. However, for the parameters a_2, a_3, a_4; elasticities obtained were weak and negative for all sectors. This is an indication of their weak sensitivity to growth. Also, the negative elasticities signal that employment is falling at all levels, which therefore, has a negative impact on the economic growth. Employment at this level has no sensitivity towards growth across all sector levels.

It is also very clear that $a_2 = 0.001$, $a_3 = 0.00001$ and $a_4 = 0.00001$inU_A (in U_I and U_S respectively) directly reduces unemployment resulting in economic growth as elasticities are at the highest at these rates. It is also evident that the rate at which new vacancies are created in favour of the unemployed persons at $\alpha = 1.515668$ and the rate at which new vacancies are created in favour of the the employed at $\delta = 0.4$ directly improve vacancies to be filled across the three main economic sectors.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

The nonlinear ODE unemployment model has been developed. It establishes the behaviour of the system over time, which shows that the solution to the system is positive and bounded. The equations of the system have a non-negative equilibrium point. Both local and global stability analysis was achieved. The Routh-Hurwitz stability criterion established that the equilibrium point is globally asymptotically stable. It is observed that the stability of the model is feasible when $A_0 > 0$, $A_1 > 0$, $A_0A_3 > A_1A_2$, $B_1A_3 > A_1B_2$, $C_1B_2 > B_1C_2$ and $C_1D_2 > D_1C_2$. The model was simulated and the results show that the rate of getting employment is an inverse predictor of the number of unemployed persons.

The extended nonlinear ODE unemployment model has revealed that in equilibrium, the number of unemployed persons decreases as the parameter rate of movement of the unemployed class to the employed class (a_2) increases. Thus, at equilibrium point, the rate of movement of unemployed persons to the employed class grows with a decrease in the number of unemployed persons. For employment, we observe an increase when the rate of movement of the unemployed class into employment (a_2) increases. This increase in the employed population is a result of a decrease in the population of the unemployed class.

Further analysis on the parameter α indicates that an increase in the rate at which new vacancies are created in favour of the unemployed causes a drop in the number of unemployed people, but an increase in the number of people in employment. As more vacancies are created, it is anticipated that the unemployment rate will decline as more positions become available and people hired by both government and private sectors in the agriculture, industry and service sectors. Varying the rate of transition of individuals from the unemployment to the employment class in the agriculture sector has a significant impact on unemployment in agriculture over the years, however, the impacts were more pronounced after 10 years of unemployment in the agricultural sector. There is an indication that when the rate of transition of persons from the unemployment to the employment class in the agriculture sectors goes higher, the number of unemployed persons reduces. However, as the rate of transition of persons from the unemployment to the employment class in agriculture sector increases, the number of unemployed persons in the agriculture sector decreases with time. Varying the rate of transition of persons from the unemployment to the employment class in agriculture sector with regard to employment does not have a significant impact on total employment for the initial thirty years. Conversely, the gains were more pronounced after 50 years of employment in agriculture. Generally, the result shows that higher rates of newly created vacancies reduce unemployment in the agriculture sector. Varying the rate at which new vacancies are created in favour of the unemployment in agriculture has had a significant impact on unemployment in agriculture over the years; nonetheless, the impacts were more pronounced after 10 years of unemployment in the agriculture sector. It is evident that if the rate of newly created vacancies in favour of the unemployed increases, the amount of unemployment in the agricultural sector decreases with time. This is an indication that the variation in the rate of newly created vacancies in favour of the employment has a significant impact on total employment in agriculture generally; however, the impacts were more pronounced after 30 years of employment in the agriculture sector. Notably, the gains were more pronounced after 50 years of employment in agriculture. Largely, unemployment reduces when the parameter (a_3) value is at the highest rate of 0.05. Remarkably, at different rates of transitions of persons from the unemployment to the employment class in the industry sector, the simulation indicated that, at the initial years between 0-10 years, the rate of transition of individuals from the unemployment class to the employment class in the industry sector (a_3) does not have a significant impact on unemployment in the industry, but the impacts were more pronounced after 15 years in the industry sector. The effect of unemployment reduction was evidently clear as the rate of transition of individuals from the unemployment class to the employment class in the industry sector became established as an inverse determinant of the number of unemployed persons. Unemployment increases only when the rate is at the smallest value of 0.00001 in respect of the transitional rate of unemployed to the employed in industry. Unemployment reduces when the rate of transition of persons from the unemployment to the employment class in the services sector (a_4) is higher. The initial results show that there were no significant impacts on unemployment in the services over the first ten years at different parameter values; however, the impacts were more pronounced after 20 years. Total unemployment at a rate $a_4 = 0.00001$ increases at an increasing rate till it attains its maximum at year 24, declines at a decreasing rate and remains constant after year 60. It is evident that if the rate of transition of persons from the unemployment class to the employment class in the services sector increases, then the number of total unemployed persons in the services sector decreases.

The sensitivity analysis of employment, unemployment and newly created vacancies in respect of Ghana's GDP growth was carried out as a measure of the employment intensity of growth using the respective elasticities of these parameters. It is clear that both unemployed and newly created vacancies by all three sectors, on average, have the highest elasticities for the unemployed than the number of employed persons in all three sectors, which recorded relatively lower values and negative elasticities. These negative elasticities indicate that in view of employment, the rates of movement of unemployed persons joining the employed in all three main economic sectors are inversely linked to economic growth with lower rate of sensitivities as compared to their positive and strong elasticities in view of unemployment which is an indication of their strong connections with growth. Hence, in combating the issue of unemployment in Ghana, it is necessary for policy makers take into consideration the crucial role of Ghana's economic growth in addressing the problem of unemployment.

6.2 Recommendations

Based on the findings, it is recommended that:

i. In future research, the model should be adjusted with unemployment population growth and applied to real time data to study certain coordinate agencies for accurate predictions.
ii. Finally, it is also recommended that further modification of the model should integrate intersectoral movement of labour force and a control parameter into the model.

6.3 Contribution to Knowledge

The study has made relevant contributions to knowledge that can have a significant impact on the entire academic and economic communities. In this study, a system of nonlinear ODE model, a realistic representation of the dynamics of unemployment, was proposed and developed. Symbolic parameters, which are the key determinants of unemployment control or management, were integrated into the model.

i. Specifically, the agriculture, industry, and the services sectors constitute essential variables of the model. In the literature, these variables appear to be overlooked in the mathematics-based unemployment models. Hypothetically, their relevance has been explained, but their quantitative dynamics, in respect of featuring sectoral contribution, have not been tested. Therefore, incorporating sectoral contribution makes it possible to quantify the role of GDP through sectoral expansion in unemployment management and control. Additionally, it can be applied to guide both government and private sectors in the creation of new vacancies in the three major sectors of the economy.

ii. Other important parameters integrated include the rate at which unemployed persons join the employed in agriculture, industry and services sectors; the significant roles of these parameters have not been handled within the quantitative context in the broader field of unemployment dynamics and control.

iii. In general, the unemployment model has been enhanced by integrating the complementing roles of death and retirement rates, rate of employed persons getting unemployed, rate of resignation of employed persons out of a sector, rate at which new vacancies are created in favour of the unemployed, rate at which new vacancies are created in favour of the employed and the rate of decadence (diminution) of newly created vacancies by all sectors. More importantly, the rate at which new vacancies are created in favour of the employed persons (δ) which connects the employed class to available vacancies, thereby enabling the employed persons to contribute to the vacancy creation within all three sectors of the economy is also, a new contribution to knowledge.

In general, the outcome of the study can be used for policy formulations in unemployment control and economic growth. Hence, the study has made essential contributions to knowledge, and can assist to address goal 8 of the SDGs challenges in Ghana.

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APPENDICES

APPENDIX A: SUMMARY OF IMPLEMENTATION CODES IN MATLAB FOR a_2, a_3 AND a_4

Codes for the Nonlinear Model for Unemployment in Agriculture Model

```
clear;
clc
a_list=[0.001,0.0001,0.00001];
figure
hold on
for i= 1:numel(a_list)
    a2=a_list(i);
%for a2=[0.001 0.01 0.1]

dt=1/10000;
T=1:dt:100;
a1=30000;
a4= 0.0001;
%a1=[500000;750000;950000];
%a2=0.00001;
 a5= 0.07;
a3=0.05;
a6= 0.05;
alpha=0.02; a7= 0.0056;
delta= 0.2;
beta= 0.62;
Ua=zeros(size(T));
Ui=zeros(size(T));
Us=zeros(size(T));
Ea=zeros(size(T));
Ei=zeros(size(T));
Es=zeros(size(T));
C=zeros(size(T));
A=ones(size(T))*100;
I=ones(size(T))*100;
S=ones(size(T))*100;
%Initial conditions
Ua(1)=2659; %20959;
Ui(1)= 3098;%10733;
Us(1)=  1650;%30240;
Ea(1)= 13187;%50040;
Ei(1)= 21100;%30267;
Es(1)= 5470;%16060;
C(1)= 100;
A(1)=200;
I(1)=450;
S(1)= 454;


for idx= 1:length(T)-1
```

```
    Ua(idx+1)= Ua(idx)+dt*(-a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ua(idx)+a6*Ea(idx)+a1);
    Ui(idx+1)= Ui(idx) + dt*(-a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ui(idx)+a6*Ei(idx)+a1);
    Us(idx+1)= Us(idx)+ dt*(-a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Us(idx)+a6*Es(idx)+a1);
    Ea(idx+1)= Ea(idx)+ dt*(a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ea(idx)-a6*Ea(idx)-a7*Ea(idx));
    Ei(idx+1)= Ei(idx) +dt*(a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ei(idx)-a6*Ei(idx)-a7*Ei(idx));
    Es(idx+1)= Es(idx) +dt*(a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Es(idx)-a6*Es(idx)-a7*Es(idx));
    C(idx+1)= C(idx)
+dt*(alpha*(Ua(idx)+Ui(idx)+Us(idx))+delta*(Ea(idx)+Ei(idx)+
Es(idx))-C(idx)*beta);
end
plot(T, Ua, '--', 'DisplayName', ['a2'
num2str(a2)],'LineWidth',1.6)
xlabel('Time (year)');
ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%legends{i}=sprintf('multiple value a1%#d',i);
end
legend("a2= "+ a_list)
l=legend("a2= "+ a_list)
l.FontSize=20;
%legend(legends)
hold off
%ylim([0 1000000])
%xlim([1 10])
%set(gca, 'ylim',[0 1000000]);
%set(gca, 'ytick',0:dt: 1000000);
%set(gca,'xlim',[1 10]);
%set(gca,'xtick',1:1:10);

%plot(T, Ua, '-', 'DisplayName', ['a2' num2str(a2)])
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%figure
%plot(T,Ua,'LineWidth',1.6)
%grid on;
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ua');
%grid on
title('Unemployment in Agriculture');
```

Codes for the Nonlinear Model of Unemployment in the Industry Sector

```
clear;

clc
a_list=[0.05,0.0001,0.00001];
figure
hold on
for i= 1:numel(a_list)
    a3=a_list(i);
%for a2=[0.001 0.01 0.1]

dt=1/10000;
T=1:dt:100;
a1=30000;
a4= 0.0001;
%a1=[500000;750000;950000];
a2=0.00001;
 a5= 0.07;
%a3=0.05;
a6= 0.05;
alpha=0.02; a7= 0.0056;
delta= 0.2;
beta= 0.6;

Ua=zeros(size(T));
Ui=zeros(size(T));
Us=zeros(size(T));
Ea=zeros(size(T));
Ei=zeros(size(T));
Es=zeros(size(T));
C=zeros(size(T));
A=ones(size(T))*100;
I=ones(size(T))*100;
S=ones(size(T))*100;
Ua(1)=2659; %20959;
Ui(1)= 3098;%10733;
Us(1)=  1650;%30240;
Ea(1)= 13187;%50040;
Ei(1)= 21100;%30267;
Es(1)= 5470;%16060;
C(1)= 100;
A(1)=200;
I(1)=450;
S(1)= 454;

for idx= 1:length(T)-1
    Ua(idx+1)= Ua(idx)+dt*(-a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ua(idx)+a6*Ea(idx)+a1);
    Ui(idx+1)= Ui(idx)  + dt*(-a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ui(idx)+a6*Ei(idx)+a1);
```

```
    Us(idx+1)= Us(idx)+ dt*(-a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Us(idx)+a6*Es(idx)+a1);
    Ea(idx+1)= Ea(idx)+ dt*(a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ea(idx)-a6*Ea(idx)-a7*Ea(idx));
    Ei(idx+1)= Ei(idx) +dt*(a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ei(idx)-a6*Ei(idx)-a7*Ei(idx));
    Es(idx+1)= Es(idx) +dt*(a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Es(idx)-a6*Es(idx)-a7*Es(idx));
    C(idx+1)= C(idx)
+dt*(alpha*(Ua(idx)+Ui(idx)+Us(idx))+delta*(Ea(idx)+Ei(idx)+
Es(idx))-C(idx)*beta);
end
plot(T, Ui, '--', 'DisplayName', ['a3'
num2str(a3)],'LineWidth',1.6)
xlabel('Time (year)');
ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%legends{i}=sprintf('multiple value a1%#d',i);
end
legend("a3= "+ a_list)
l=legend("a3= "+ a_list)
l.FontSize=20;
%legend(legends)
hold off
%ylim([0 1000000])
%xlim([1 10])
%set(gca, 'ylim',[0 1000000]);
%set(gca, 'ytick',0:dt: 1000000);
%set(gca,'xlim',[1 10]);
%set(gca,'xtick',1:1:10);

%plot(T, Ua, '-', 'DisplayName', ['a2' num2str(a2)])
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%figure
%plot(T,Ua,'LineWidth',1.6)
%grid on;
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ua');
%grid on
title('Unemployment in Industry');
```

Codes for the Nonlinear Model of Unemployment in the Services Sector

```
clear;
clc
```

```
a_list=[0.0001,0.00001,0.000001];
figure
hold on
for i= 1:numel(a_list)
    a4=a_list(i);
%for a2=[0.001 0.01 0.1]

dt=1/10000;
T=1:dt:100;
a1=30000;
%a4= 0.0001;
%a1=[500000;750000;950000];
a2=0.00001;
 a5= 0.07;
a3=0.05;
a6= 0.05;
alpha=0.02; a7= 0.0056;
delta= 0.2;
beta= 0.62;

Ua=zeros(size(T));
Ui=zeros(size(T));
Us=zeros(size(T));
Ea=zeros(size(T));
Ei=zeros(size(T));
Es=zeros(size(T));
C=zeros(size(T));
A=ones(size(T))*100;
I=ones(size(T))*100;
S=ones(size(T))*100;
Ua(1)=2659; %20959;
Ui(1)= 3098;%10733;
Us(1)=  1650;%30240;
Ea(1)= 13187;%50040;
Ei(1)= 21100;%30267;
Es(1)= 5470;%16060;
C(1)= 100;
A(1)=200;
I(1)=450;
S(1)= 454;

for idx= 1:length(T)-1
    Ua(idx+1)= Ua(idx)+dt*(-a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ua(idx)+a6*Ea(idx)+a1);
    Ui(idx+1)= Ui(idx)  + dt*(-a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ui(idx)+a6*Ei(idx)+a1);
    Us(idx+1)= Us(idx)+ dt*(-a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Us(idx)+a6*Es(idx)+a1);
    Ea(idx+1)= Ea(idx)+ dt*(a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ea(idx)-a6*Ea(idx)-a7*Ea(idx));
```

```
    Ei(idx+1)= Ei(idx) +dt*(a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ei(idx)-a6*Ei(idx)-a7*Ei(idx));
    Es(idx+1)= Es(idx) +dt*(a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Es(idx)-a6*Es(idx)-a7*Es(idx));
    C(idx+1)= C(idx)
+dt*(alpha*(Ua(idx)+Ui(idx)+Us(idx))+delta*(Ea(idx)+Ei(idx)+
Es(idx))-C(idx)*beta);
end
plot(T, Us, '--', 'DisplayName', ['a4'
num2str(a4)],'LineWidth',1.6)
xlabel('Time (year)');
ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%legends{i}=sprintf('multiple value a1%#d',i);
end
legend("a4= "+ a_list)
l=legend("a4= "+ a_list)
l.FontSize=20;
hold off
%ylim([0 1000000])
%xlim([1 10])
%set(gca, 'ylim',[0 1000000]);
%set(gca, 'ytick',0:dt: 1000000);
%set(gca,'xlim',[1 10]);
%set(gca,'xtick',1:1:10);

%plot(T, Ua, '-', 'DisplayName', ['a2' num2str(a2)])
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%figure
%plot(T,Ua,'LineWidth',1.6)
%grid on;
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ua');
%grid on
title('Unemployment in Service');
```

APPENDIX B: SUMMARY OF IMPLEMENTATION CODES IN MATLAB FOR α AND δ

Implementation Codes for the Newly Created Vacancy for Unemployed in the Agriculture Sector

```
clear;
clc
a_list=[0.02,0.04,0.08];
figure
hold on
for i= 1:numel(a_list)
    alpha=a_list(i);
%for a2=[0.001 0.01 0.1]

dt=1/10000;
T=1:dt:100;
a1=30000;
a4= 0.0001;
%a1=[500000;750000;950000];
a2=0.00001;
 a5= 0.07;
a3=0.05;
a6= 0.05;
%alpha=0.02;
a7= 0.0056;
delta= 0.2;
beta= 0.62;
Ua=zeros(size(T));
Ui=zeros(size(T));
Us=zeros(size(T));
Ea=zeros(size(T));
Ei=zeros(size(T));
Es=zeros(size(T));
C=zeros(size(T));
A=ones(size(T))*100;
I=ones(size(T))*100;
S=ones(size(T))*100;
%Initial conditions
Ua(1)=2659; %20959;
Ui(1)= 3098;%10733;
Us(1)=  1650;%30240;
Ea(1)= 13187;%50040;
Ei(1)= 21100;%30267;
Es(1)= 5470;%16060;
C(1)= 100;
A(1)=200;
I(1)=450;
S(1)= 454;

for idx= 1:length(T)-1
```

```
    Ua(idx+1)= Ua(idx)+dt*(-a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ua(idx)+a6*Ea(idx)+a1);
    Ui(idx+1)= Ui(idx)  + dt*(-a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ui(idx)+a6*Ei(idx)+a1);
    Us(idx+1)= Us(idx)+ dt*(-a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Us(idx)+a6*Es(idx)+a1);
    Ea(idx+1)= Ea(idx)+ dt*(a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ea(idx)-a6*Ea(idx)-a7*Ea(idx));
    Ei(idx+1)= Ei(idx)  +dt*(a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ei(idx)-a6*Ei(idx)-a7*Ei(idx));
    Es(idx+1)= Es(idx)  +dt*(a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Es(idx)-a6*Es(idx)-a7*Es(idx));
    C(idx+1)= C(idx)
+dt*(alpha*(Ua(idx)+Ui(idx)+Us(idx))+delta*(Ea(idx)+Ei(idx)+
Es(idx))-C(idx)*beta);
end
plot(T, Ua, '--', 'DisplayName', ['alpha'
num2str(alpha)],'LineWidth',1.6)
xlabel('Time (year)');
ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%legends{i}=sprintf('multiple value a1%#d',i);
end
legend("\alpha= "+ a_list)
l=legend ("\alpha= "+ a_list)
l.FontSize=20;
%legend(legends)
hold off
%ylim([0 1000000])
%xlim([1 10])
%set(gca, 'ylim',[0 1000000]);
%set(gca, 'ytick',0:dt: 1000000);
%set(gca,'xlim',[1 10]);
%set(gca,'xtick',1:1:10);

%plot(T, Ua, '-', 'DisplayName', ['a2' num2str(a2)])
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%figure
%plot(T,Ua,'LineWidth',1.6)
%grid on;
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ua');
%grid on
title('Newly created vacancies in Agric to unemployed ');
```

Implementation Codes for the Newly Created Vacancy for the Unemployed in the Industry Sector

```
clear;
clc
a_list=[0.02,0.04,0.08];
figure
hold on
for i= 1:numel(a_list)
    alpha=a_list(i);
%for a2=[0.001 0.01 0.1]

dt=1/10000;
T=1:dt:100;
a1=30000;
a4= 0.0001;
%a1=[500000;750000;950000];
a2=0.00001;
 a5= 0.07;
a3=0.05;
a6= 0.05;
%alpha=0.02;
a7= 0.0056;
delta= 0.2;
beta= 0.62;
Ua=zeros(size(T));
Ui=zeros(size(T));
Us=zeros(size(T));
Ea=zeros(size(T));
Ei=zeros(size(T));
Es=zeros(size(T));
C=zeros(size(T));
A=ones(size(T))*100;
I=ones(size(T))*100;
S=ones(size(T))*100;
%Initial conditions
Ua(1)=2659; %20959;
Ui(1)= 3098;%10733;
Us(1)=  1650;%30240;
Ea(1)= 13187;%50040;
Ei(1)= 21100;%30267;
Es(1)= 5470;%16060;
C(1)= 100;
A(1)=200;
I(1)=450;
S(1)= 454;

for idx= 1:length(T)-1
    Ua(idx+1)= Ua(idx)+dt*(-a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ua(idx)+a6*Ea(idx)+a1);
```

```
    Ui(idx+1)= Ui(idx)  + dt*(-a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ui(idx)+a6*Ei(idx)+a1);
    Us(idx+1)= Us(idx)+ dt*(-a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Us(idx)+a6*Es(idx)+a1);
    Ea(idx+1)= Ea(idx)+ dt*(a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ea(idx)-a6*Ea(idx)-a7*Ea(idx));
    Ei(idx+1)= Ei(idx)  +dt*(a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ei(idx)-a6*Ei(idx)-a7*Ei(idx));
    Es(idx+1)= Es(idx)  +dt*(a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Es(idx)-a6*Es(idx)-a7*Es(idx));
    C(idx+1)= C(idx)
+dt*(alpha*(Ua(idx)+Ui(idx)+Us(idx))+delta*(Ea(idx)+Ei(idx)+
Es(idx))-C(idx)*beta);
end
plot(T, Ui, '--', 'DisplayName', ['alpha'
num2str(alpha)],'LineWidth',1.6)
xlabel('Time (year)');
ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%legends{i}=sprintf('multiple value a1%#d',i);
end
legend("\alpha= "+ a_list) l=legend ("\alpha= "+ a_list)
l.FontSize=14;
%legend(legends)
hold off
%ylim([0 1000000])
%xlim([1 10])
%set(gca, 'ylim',[0 1000000]);
%set(gca, 'ytick',0:dt: 1000000);
%set(gca,'xlim',[1 10]);
%set(gca,'xtick',1:1:10);

%plot(T, Ua, '-', 'DisplayName', ['a2' num2str(a2)])
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%figure
%plot(T,Ua,'LineWidth',1.6)
%grid on;
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ua');
%grid on
title('Newly created vacancies in Industry to unemployed ');
```

Implementation Codes for the Newly Created Vacancy in Favour of the Unemployed in the Services Sector

```
clear;
clc
a_list=[0.02,0.04,0.08];
figure
hold on
for i= 1:numel(a_list)
    alpha=a_list(i);
%for a2=[0.001 0.01 0.1]

dt=1/10000;
T=1:dt:100;
a1=30000;
a4= 0.0001;
%a1=[500000;750000;950000];
a2=0.00001;
 a5= 0.07;
a3=0.05;
a6= 0.05;
%alpha=0.02;
a7= 0.0056;
delta= 0.2;
beta= 0.62;

Ua=zeros(size(T));
Ui=zeros(size(T));
Us=zeros(size(T));
Ea=zeros(size(T));
Ei=zeros(size(T));
Es=zeros(size(T));
C=zeros(size(T));
A=ones(size(T))*100;
I=ones(size(T))*100;
S=ones(size(T))*100;
%Initial conditions
Ua(1)=2659; %20959;
Ui(1)= 3098;%10733;
Us(1)=  1650;%30240;
Ea(1)= 13187;%50040;
Ei(1)= 21100;%30267;
Es(1)= 5470;%16060;
C(1)= 100;
A(1)=200;
I(1)=450;
S(1)= 454;

for idx= 1:length(T)-1
    Ua(idx+1)= Ua(idx)+dt*(-a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ua(idx)+a6*Ea(idx)+a1);
```

```
    Ui(idx+1)= Ui(idx)  + dt*(-a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ui(idx)+a6*Ei(idx)+a1);
    Us(idx+1)= Us(idx)+ dt*(-a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Us(idx)+a6*Es(idx)+a1);
    Ea(idx+1)= Ea(idx)+ dt*(a2*Ua(idx)*(A(idx)+C(idx)-
Ea(idx))-a5*Ea(idx)-a6*Ea(idx)-a7*Ea(idx));
    Ei(idx+1)= Ei(idx)  +dt*(a3*Ui(idx)*(I(idx)+C(idx)-
Ei(idx))-a5*Ei(idx)-a6*Ei(idx)-a7*Ei(idx));
    Es(idx+1)= Es(idx)  +dt*(a4*Us(idx)*(S(idx)+C(idx)-
Es(idx))-a5*Es(idx)-a6*Es(idx)-a7*Es(idx));
    C(idx+1)= C(idx)
+dt*(alpha*(Ua(idx)+Ui(idx)+Us(idx))+delta*(Ea(idx)+Ei(idx)+
Es(idx))-C(idx)*beta);
end
plot(T, Us, '--', 'DisplayName', ['alpha'
num2str(alpha)],'LineWidth',1.6)
xlabel('Time (year)');
ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%legends{i}=sprintf('multiple value a1%#d',i);
end
legend("\alpha= "+ a_list) l=legend ("\alpha= "+ a_list)
l.FontSize=16;
%legend(legends)
hold off
%ylim([0 1000000])
%xlim([1 10])
%set(gca, 'ylim',[0 1000000]);
%set(gca, 'ytick',0:dt: 1000000);
%set(gca,'xlim',[1 10]);
%set(gca,'xtick',1:1:10);

%plot(T, Ua, '-', 'DisplayName', ['a2' num2str(a2)])
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ea','Es','Ei');
%legend(compose('a2=%.5f',a2),"Location","best");
%figure
%plot(T,Ua,'LineWidth',1.6)
%grid on;
%xlabel('yrs');
%ylabel('Unemployment');
%legend('Ua');
%grid on
title('Newly created vacancies in Services to unemployed ');
```

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