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A THESIS REPORT ENTITLED

NEUTROSOPHIC GENERALISED LINEAR MIXED EFFECTS MODELLING OF BASIC SCHOOLS PERFORMANCE

BY

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SUBMITTED IN FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS (STATISTICS)

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DECLARATION

I declare that this thesis is my own work. It is being submitted for the degree of Doctor of Philosophy in Mathematics (Statistics) in the University of Mines and Technology (UMaT), Tarkwa. It has not been submitted for any degree or examination in any other University.

(Signature of Candidate)

10TH day of JULY (year) 2023

ABSTRACT

In this thesis, a six-parameter alternative to the Generalised Linear Model was developed to account for nonnormality, unobserved factors in the form of random effect, interdependence of and heterogeneity among respondents. The techniques used included a novel hybrid of neutrosophic statistics, principal component analysis, and generalized mixed effect modeling. The robustness of the developed model was verified through sample variation, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The Model was tested with Ghana’s educational dataset to explain the achievement gaps among private and public school students in the BECE with 70% used for training and 30% as validation set. The model's estimations found that the random effect parameter had a significant effect on achievement gap, explaining around 9% of variability across people across all factors. According to the magnitude rank of predicted probability of the model, the top five variables influencing the performance gap between students attending private and public schools show that Administrative-Logistic causes account for 80% of the variance. These were listed in order of increasing importance: Daily Quality Supervision of Head teacher and Head teacher supervision by school proprietors, Timely Delivery of Books and Learning Materials by Parents/Stakeholders for Students; Conducive Teaching/Learning Environment; Concern and Parental Support for Students Academic Output (PTA). In conclusion, the proposed modified version in this research is recommended over the existing generalised linear models for educational research due to its robustness. Based on the findings of this research it is recommended for educational stakeholders to consider the major determinants in this research that significantly affect BECE performance in order to maximise teaching and learning in Ghana’s Basic schools.

DEDICATION

This thesis is dedicated two people in my life; to the loving memory of my late father John Osei Wusu – Asibey who predicted when I was seven years old that one day that I will become a doctor.

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CHAPTER 1

INTRODUCTION

1.1 Background to the Study

Recent discussions have brought attention to the importance of the school sector (whether private or public) and its impact on students' academic success. According to conventional opinion and prior studies, private basic schools perform better academically (Adu-Boahen, 2022, National Centre for Education Statistics, 2013; Adediwura *et al.,* 2007; Adamu, 2015). The No Child Left Behind policy model, which serves as the organisational framework for private schools, reflects presumptions regarding the superiority of the private basic school model over the public basic school model. Reforms for private schools are based on the assumption that by making parents the driving force behind the pursuit of excellence, schools will be compelled to raise their game when they come against competition from other institutions with higher test scores (Dangara *et al.*, 2019; Hussain *et al.*, 2018; Endeley, 2017; Adamu, 2015 and Adediwura *et al.*, 2007).

School management entails administration, governance, decision-making, planning, organising, and controlling activities specific to a given school. This when done well, will translate into productive output from teachers, school administrators, and students; with the overarching results of great output from the students in the Basic Education Certificate Examination (BECE). Private schools complement governments' efforts to provide high-quality education for everybody by playing highly useful roles in the educational systems of many developing nations. The number of private schools in Ghana during the 2008–2009 academic year was estimated to be 3% of all Basic school institutions (Dagara *et al.*, 2019; Hussain *et al.*, 2018; Endeley, 2017; Mishra, 2015; Rolleston 2013; McCulloch, 2011; Akaguri, 2011; Lubienski, 2006; Riley, 1997; Raudenbush *et al.*, 1995; Bryk *et al.*, 1993;

Graham *et al.*, 1993).However, according to an International Finance Corporation (2010) Ghana's report, (a division of the World Bank that conducts research to come out with educational reports on both developed and underdeveloped countries) report, private schools only enrolled 18% of Basic school students, with a 26% annual growth rate compared to a 9% annual growth rate for public institutions. This shows that Ghana's private school industry is expanding, and considering how well they are doing overall, particularly at the basic education level, it is anticipated that this trend would continue. It has been reported that an improved monitoring system at the metropolitan, municipal, and district levels of private and public basic education will improve the standard of the schools, how they operate, and how well they do on the Basic Education Certificate Examination (Dagara *et al.*, 2019; Hussain *et al.*, 2018; Endeley, 2017; Mishra, 2015; Rolleston 2013; McCulloch, 2011; Akaguri, 2011; Lubienski, 2006; Riley, 1997; Raudenbush *et al.*, 1995; Bryk *et al.*, 1993; Graham *et al.*, 1993).

Results of the 2000, 2002, 2003, and 2005 assessments for all private schools as well as the three main private school categories (Catholic, Lutheran, and Conservative Christian) when contrasted with those for public schools led to the conclusion that private schools type (i.e. educational philosophy of the religious denomination that established the school; such as Catholic, Lutheran, and Conservative Christian schools) influenced performance of the students, teacher output, administrative efficiency Endeley, 2017; Lubienski *et al.*, 2006a, Lubienski *et al.*, 2006b). In general, it was discovered that children in private schools had higher average reading and mathematics test scores (in external standardised test) than those in public schools (Endeley, 2017; Lubienski *et al.*, 2006a, Lubienski *et al.*, 2006b). The National Assessment of Educational Progress (NAEP) yearly reports (2015, 2019, and 2020) also provide results by school type and by a single student characteristic, such as race/ethnicity, gender, or student-reported parents' highest level of education. The average scores of private school students continued to be higher than those of comparable public school students. Hierarchical linear models were used comparing the performance of public and private pupils on the 2003 NAEP examination. The study's overall finding was that the comparatively high raw scores of private schools are more than adequately explained by demographic disparities between pupils in public and private schools (Endeley, 2017; Lubienski *et al.*, 2006a, Lubienski *et al.*, 2006b).

In order to reduce errors and biases brought on by indeterminacy, the Neutrosophy concept used in this thesis attempted to correctly account for inconsistent, uncertain, imprecise, and ambiguous information from respondents. Neutrosophic approach offers a better way to capture the responses while dealing with real-world circumstances. Despite the numerous proposals that have been made over the years, managing uncertainty within decision-making difficulties remains an extremely difficult research challenge. The application of neutrosophic sets in decision-making processes has become one of the most fascinating research areas in recent years. Fuzzy and intuitionistic fuzzy sets and logic are generalised in neutrosophic sets and logic. Because the information decision-makers receive may be ambiguous and incomplete, it might be difficult for them to come up with answers. Neutrosophy is a notion that is used to correct biases and errors in responses by correcting errors in responses emanating from inconsistency, uncertainty, imprecision and indeterminate information (Aslam, 2018; El-Latif, 2016; Agboola, 2015; Broumi *et al*., 2014).

Popular studies such as NAEP (2015) and IFC (2010) reports that have looked at the performance gaps between private and public schools over time have typically relied on respondents' opinions without taking into account a modeling method that lowers errors caused by inconsistency, ambiguity, imprecision, and ambiguous information. Additionally, they have not taken into account the heterogeneity-related grouping of variables and random effects.

1.2 Statement of the Problem

The assumptions of normality and independence are not tenable for educational research

(Bono *et al.*, 2020; Arnau *et al.*, 2014; Blanca *et al.*, 2013; Bauer and Sterba, 2011; Lei and Lomax, 2005; Micceri, 1989). The Generalised linear models such as Hierarchical Linear model, Logistic regression, probit models and Poisson regression models make assumption of independence and normality which cannot properly model educational dataset which has properties of non-normality and dependence and where linearity and homoscedasticity assumptions of classical regression models are often broken (Bono *et al*., 2020; Arnau *et al.*, 2014; Blanca *et al.*, 2013; Bauer *et al*., 2011; Lei *et al*., 2005; Micceri, 1989). Violations of these assumptions may result in biased standard errors (and hence biased p- values) as well as a reduction in statistical power (Atkins *et al*., 2007; King, 1988).

In spite of the limitations of Generalised Linear models, well known educational research authorities such as the IFC and NAEP, have used hierarchical linear models, regardless, to produce their findings. Studies (McCulloch, 2011; Lubienski, 2006; Raudenbush *et al.*, 1995; Bryk, *et al.*, 1993; Graham *et al.*, 1993; Riley1997; Rolleston 2013; Akaguri 2011) have reported that, the widely used hierarchical linear models do not have the ability to handle heterogeneous and unequal variance situations, unequal sample size, strong correlation situations, unequal numbers of repeats, or the ability to capture random factors.

Although there has been exciting progress (Mammen *et al.*, 2021; Pftzner *et al.*, 2021; Yin *et al.*, 2021; Lil *et al.*, 2020), there is still a significant limitation because existing models cannot handle mixed-effects (i.e., both fixed and random effects) well, which is crucial when analysing non-independent, multilevel/hierarchical, longitudinal, or correlated data. These recent works, have urged for extensions of the Generalised Linear Models to Generalised Linear Mixed Models (GLMMs). Although Mishra (2015) concluded that Parents, students, and supervisors (heads, proprietors) are significant stakeholders that should be the primary sources of information to maximise teaching and learning process, researchers (Pinheiro *et al.* (2015); Kachman *et al.*, (1994) who have proposed Generalised mixed effect model for maximising educational outcome only produced truncated models which lacked major structural parameters outlined in Mishra's (2015) work. There is a need for a more comprehensive Generalised Linear Mixed Model (GLMM) that captures all necessary structural parameters that control effective teaching and learning which will enhance performance in BECE as argued by Mishra (2015). Existing generalised linear mixed-effects models do not account for all possible dependencies among the outcomes (ShunCheng *et al.,* 2022). Therefore, the current study proposed a GLMM with a Neutrosophic treatment level-specific item random effect and also Neutrosophic Principal Component Analysis (PCA) to help maximise dependencies and reduce bias due to mis specifying item random effects.

Since capturing all the required structural components requires measuring responses from stakeholders, there is the need to propose neutrosophic regression technique; an extended form of the classical regression to solve imminent problems of human response errors, which are frequently associated with ambiguous, conflicting, imprecise, indeterminate or uncertainty (Abdel-Basset, 2018; Mustaz *et al.*, 2014). Limitation of proposed GLMMs in recent works is the problem of establishing a random effect structure (ShunCheng *et al.*, 2022). In GLMMs, the fixed effect estimate is baised when the random effect distribution is incorrectly specified (Litière *et al.*, 2007; Verbeke *et al.*, 1997). Therefore, the current study proposed a GLMM with a Neutrosophic treatment level-specific item random effect and also Neutrosophic PCA to help minimise bias and optimise correct specification of random effects. Optimising learning outcomes for public basic school pupils is a challenge if we are to close the achievement gaps between public and private candidates on the Basic School Certificate Examination.

The causes of the differences have been well investigated in the scholarly literature, but

the dynamics of these influences on BECE performance are still unknown. Stakeholders can better influence particular variables to improve learning outcomes when these dynamics have been well investigated and understood. Explaining the dynamics of performance disparity between private and public schools is necessary in order to maximise learning outcomes for public basic schools. Therefore, the goal of the current work is to create a six parameter Generalised Mixed Linear Model with five structural factors that accounts for logistic features of students, teachers, and administrators as well as their interactions and random effects.

1.3 Objectives of Research

The overall goal of this study is to develop an extended Generalised linear model with structural and random effect parameters under Neutrosophic treatment to elucidate the problem of performance gaps between private and public schools.

The specific objectives of the research are to:

i. develop extended six-parameter Generalised Linear Mixed effect model to explain BECE achievement gaps in the case of Ada East and West.

ii. present a modified neutrosophic regression statistics to solve problems of indeterminacy.

iii. use Neutrosophic-Principal Component Analysis approach to analysing the causes of performance gap among private and public school students in the Basic Education Certificate Examination.

iv. Compare the robustness of the developed model with existing models using varying sample sizes on the basis of Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) scores under neutrosophic treatment.

v. Apply the developed model to predict the effects of significant factors on BECE performance in the case of Ada East and West under neutrosophic treatment.

1.4 Research Questions

i. How does the six-parameter Generalised Linear Mixed effect model explain BECE achievement gaps in the case of Ada East and West?

ii. What are the modified neutrosophic regression statistics that solve problems of indeterminacy?

iii. How does Neutrosophic-Principal Component Analysis approach explain the performance gaps between private and public school students in the Basic Education Certificate Examination?

iv. How does the robustness of the developed model compare with existing models using varying sample sizes on the basis of AIC, BIC scores under neutrosophic treatment?

v. What are the estimates of the developed model when used to predict the effects of significant factors on BECE performance in the case of Ada East and West under neutrosophic treatment?

1.5 Methods Used

i. The Generalised Linear model as shown in Equation (1.1) was modified to Generalised Linear Mixed Effect Model:

$$E\left(Y \middle| \boldsymbol{\beta}\right) = g^{-1}\left(\boldsymbol{\beta}X\right) \quad (1.1)$$

ii. The exponential family form as explicated in Equation (1.2) was used to extract the canonical parameters of the Bernoulli response distribution.

$$f(y,\theta,\varphi) = \exp\left[\frac{y\theta - b(\theta)}{a(\varphi)} + c(y,\varphi)\right] \quad (1.2)$$

iii. The classical least square regression statistics was used modified to Neutrosophic statistics to handle indeterminacy problems.

iv. The Maximum Likelihood Estimation was the basis for parameter estimation.

vi. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) as explicited in Equation (1.3) was used to compare models for the study.

$$\left.\begin{aligned} AIC &= -2\, log\left(maximum\ likelihood\right) + 2p \\ BIC &= -2 log\left(maximum\ likelihood\right) + plogn \end{aligned}\right\rangle \quad (1.3)$$

1.6 Facilities / Resources Employed

The facilities employed for this research were;

i. University of Mines and Technology (UMaT) library facility provided extensive resources in the form of books, as well as an outstanding research environment.

ii. Personal Laptop of the researcher increased engagement and learning by enhancing opportunities for interaction with peers, lecturers and supervisors, as well as with course materials.

iii. The good internet connectivity and availability around the UMaT campus and Ada College of Education, made it feasible to obtain more online resources and new material essential for this work without delay, reducing duplication of effort and improving research quality.

iv. The R statistical software version 4.0.5 was used for the analysis.

1.7 Organisation of Thesis

The Thesis was organised into 6 chapters. The background, statement of the problem,

research aims, facilities employed and thesis organisation are all covered in Chapter 1. Chapter 2 reviews important literature on Generalised Linear Modeling, Neutrosophic set, Principal component analysis, and private and public school performance gaps. The review of classical regression models and derivation of pertinent models was the subject of Chapter 3. Chapter 4 covers the formulation of the modified version of the Extended Generalised Linear Mixed Effect model, presentation of the neutrosophic regression approach as well as the assumptions that underpin the new model and its associated statistical properties, such as parameter estimation method are elucidated. The fitting and comparison of the developed model to existing models to predict achievement gaps are also contained in Chapter 4. Chapter 5 covers the model diagnostics, performance testing, validation, and predictions related to the developed model in Chapter 4. The results and discussion of the research are also covered in Chapter 5, along with explanations for various outputs. The research conclusions and recommendations were detailed in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

2.1 The Private and Public Basic School System

The entity that has the duty to shape students into responsible members of society is the school. Here, one is formed into a more admirable individual depending on the type of learning Centre in which one was raised. The diverse concerns of education necessitate collaboration with others to complete set tasks and objectives. These would require committed teachers, cooperative parents, a responsible community, school managers, and creative students. Numerous reform ideas for public schools have looked to the private sector for models to replicate since private schools are more widely believed to be empirically successful in teaching students. Many (Adu-Boahen, 2022, National Centre for Education Statistics, 2013; Adediwura *et al.*, 2007; Adamu, 2015; Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993) have proposed that characteristics associated with private education that are frequently associated with private schools, such as school choice, small schools, and decentralised decision making, may be advantageous to public schools. For parents who are unhappy with the public schools or who have other reasons for wanting their children to attend a private school, private schools offer an option. Parents can select from a variety of religiously linked and non - religious schools in the private sector (as long as they can afford the tuition). While some private schools are quite selective about whom they let in, others are not. The decision between public and private schools is made easier by higher family income. Only parents who have the financial means or financial aid to cover the tuition can legitimately choose a private school because the private schools are fee paying institutions. Parents of students in grades 3 through 12 attending private schools are more likely to be extremely satisfied with their children's overall school performance and with specific aspects like the teachers, academic standards, and discipline than their counterparts in public schools. Again, parents are often happier in the public sector if their children attended a public school of their choosing as opposed to one that was assigned to them (Riley, 1997; Bryk *et al.*, 1993; Graham *et al.,* 1993).

The distinctions between public and private schools' students' demographics are sometimes reflected in those differences. The racial/ethnic and linguistic backgrounds of students are among the background characteristics they bring to school. They also occasionally have personal or family issues that have an impact on their capacity to study. When organising and managing schools, as well as when planning and implementing curriculum and support services, teachers and administrators take these traits into consideration. One can anticipate public and private schools to differ in the same ways as students from public and private schools do. Other student qualities are also taken into consideration, such as attitudes toward learning and behaviour toward teachers; however, because they are influenced by both the school environment and students' backgrounds, they can be referred to as the school climate. In public schools, personal issues that hinder learning are more prevalent. There are more students with limited English proficiency in public schools. More racially and ethnically varied student populations can be found in public schools. Differences between public and private school teachers are a significant factor to consider when comparing public and private schools because of the central role teachers play in the educational process. Overall, there are differences in the racial/ethnic backgrounds, teaching certifications, and salaries of public and private school instructors. Teachers at public schools appear to be better qualified than those in private schools on some metrics. Most instructors and principals are underrepresented in private schools. Teachers in public schools typically earn more money and have greater benefits. The organisation and administration of schools have been the focus of numerous school reform initiatives in an effort to improve school performance. Overall, there are differences between public and private schools' organisational structures regarding factors like school and class size as well as the location of decision-making for a number of crucial policy areas (Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993).

2.1.1 School Size

Public schools typically have bigger student populations. As researchers have looked for the perfect school size, they have investigated the relative merits of various school sizes in great detail. Although larger schools (within reason) are frequently better suited to provide a wider choice of academic programs and support services, smaller schools are typically regarded to be easier to manage and to foster a stronger feeling of community among both students and instructors. The benefits of larger schools apply more to secondary than to elementary institutions. Public schools have larger classes on average. Although research on the relationship between outcomes and class size has not been conclusive, small classes enable teachers to provide students more individualised attention and decrease the teacher's workload, and are therefore generally regarded desirable. Although they may have benefits, small class sizes are also expensive and force other uses of school resources to be sacrificed (Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993).

2.1.2 Decision Making for the School and Classroom

Compared to their colleagues in public schools, Principals (or Heads) of private schools claim to have more control over the curriculum. A crucial component of school management includes making significant decisions about the curriculum, institutional regulations, and instructional strategies. More site-based management and local decision making are widely promoted as a way to increase school effectiveness, even though public schools must inevitably receive guidance from state departments of education, local school boards, and district personnel. Private school teachers and their Principals are more likely than their public counterparts to think they have a lot of influence in a number of policy areas. Teachers in private schools tend to have more freedom in the classroom. Private school teachers tend to be more likely than public school teachers to believe that they had a decent degree of control over student discipline, choosing the subjects, skills, and content to teach, or choosing textbooks and other teaching aids (Riley, 1997; Bryk *et al.,* 1993; Graham *et al.,* 1993).

2.1.3 School Climate

Public schools are much more likely to be exposed to crime or danger. The educational experience of students, instructors, and other staff members as well as parents' happiness with their child's school can all be strongly impacted by the school climate. If a school is dangerous, disrupted by disruptive kids, or if there is a lack of collaboration among instructors or between the school and parents, neither teachers nor students will be able to function at their best. Schools should "provide a disciplined environment conducive to learning and will be devoid of drugs, violence, and the unauthorised presence of firearms and alcohol," according to the National Education Goals for the year 2000. Additionally, the Goals encourage "parental involvement and participation in encouraging the social, emotional, and academic growth of their children". Students must feel secure at school for them to learn properly. Schools where students have to worry about being threatened or being victims of crime may substantially damage the learning environment. Both public and private schools have crime in and around them, but pupils in public schools are significantly more likely to experience it. In both assigned and chosen public schools, the percentages of students in grades 6 through 12 who knew about, saw, or were concerned about being a victim of bullying, physical assault, or robbery were significantly higher than in private schools (Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993).

2.1.4 Some Teacher Beliefs

Teachers in public schools are much more likely to think that certain unfavorable student attitudes and behaviors are major issues in their institutions. Teachers in public schools are also more likely to think that a lack of parental involvement is a major issue at their institution. Within their schools, private school teachers exhibit a stronger feeling of community. A spirit of home-school cooperation is encouraged by communication between parents and school staff, and this cooperation is crucial for students' achievement. Both public and private schools have different communication styles with parents. For instance, parents of students in private schools are more likely to be contacted about development than their counterparts in public schools (Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993).

2.1.5 Public and Private School Disparities

Public school pupils pose more of a challenge to their schools than their peers in private schools, notwithstanding the wide variety within each sector. Public school instructors are more likely than private school teachers to believe that their students and their families are struggling, in addition to the fact that they come from more diverse racial/ethnic and linguistic backgrounds. Overall, instructors at public schools are more likely to possess particular qualities that are regarded to contribute to good teaching than their colleagues in private schools. Increased education, teaching experience, and involvement in professional development activities are a few of them. Teachers at both public and private schools employ comparable teaching methods, nevertheless (Adu-Boahen, 2022, National Centre for Education Statistics, 2013; Adediwura *et al.*, 2007; Adamu, 2015; Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993). Public schools have an edge when attempting to recruit and keep the greatest instructors since, on average; they pay and provide greater benefits to their employees. Private school teachers are generally happier with their professions than public school teachers, despite receiving lower salary. Overall, private schools appear to provide a stronger sense of community, more freedom for teachers in the classroom, and more local control over the curriculum and significant school rules. Additionally, research appears that the environment in private schools is generally more supportive of learning, with higher levels of safety and fewer issues brought on by students' unfavorable attitudes toward learning or interactions with teachers (Adu-Boahen, 2022, National Centre for Education Statistics, 2013; Adediwura *et al.,* 2007; Adamu, 2015; Riley, 1997; Bryk *et al.,* 1993; Graham *et al.,* 1993). Although some systematic distinctions between public and private education have been made here, each sector is rife with diversity. The abilities, attitudes, and issues that students bring to school, the knowledge and experience of their teachers, and the quality of the learning environment are all related in complex ways to how successful students are in school. Therefore, it is the joint responsibility of students, teachers, school administrators, parents, the larger communities in which the schools are located, and policymaking at the local, state, and federal levels to help make students success come to reality (Riley, 1997; Bryk *et al.*, 1993; Graham *et al.*, 1993).

2.2 The State of Basic Education in Ghana

Since the implementation of Free and Compulsory Universal Basic Education (FCUBE) policy in 1995, enrollment in basic schools in Ghana has nearly doubled. Ghana's primary net enrollment rate for 2013 was 86.8%, which is still somewhat lower than the lower-middle income average of 87.3 percent. It’s net secondary enrollment rate, which includes Junior High and Senior High school, is 51%, which is lower than the 58 percent average for lower-middle income nations. In the private, or non-state, education sector, primary enrolment rates have increased by more than three times since 1991, while lower and upper secondary enrollment rates have increased by more than twice between 1999 and 2013

(Quartey *et al.*, 2017). According to Ghana's ministry of education 2014 report, as of 2014, private schools educated 23% of elementary school students, while 16% of secondary school students attended a private Junior High or Senior High school. The high population rise in Ghana from 19 million in 2000 to 25 million in 2012 is strongly tied to the sharp increase in enrollments in private schools. The distribution of students attending private schools reflects the concentration of people in urban areas: the private sector made up 30% and 27%, respectively, of all primary school enrollments in the Greater Accra and Ashanti regions, as opposed to less than 7% in the three regions in the northern part of Ghana (Ghana MOE, 2014).

Although thorough effect analyses of private school performance have not been carried out in Ghana, research indicated that parents use quality (i.e., performance on national examinations, class sizes, and teacher attendance) as a justification for sending their kids to private schools (Heyneman *et al.*,2011). In terms of raw results, pupils in private schools did better in Mathematics and English in grades 3 and 6 of the Ghana National Education Assessment in 2013. In grades 3 and 6, more than 80% of students in private schools demonstrated minimum Mathematics proficiency; in contrast, just around 50% of students in public schools did so. Because poorer districts are less likely to produce high-quality results for students and because poorer households are less likely to send their children to school, some parents opt for private schools even when doing so entails a significant financial burden. Even if impoverished pupils have less access to education, private schools are nonetheless helping the most vulnerable members of society. According to the Ghana Living Standards Survey from 2005, both urban and rural areas of Ghana had private schools with 11% of poor pupils and 5% of extremely poor students enrolled (Akyeampong *et al.*, 2013). But affordability is a problem. According to household surveys conducted in Ghana's Central Region, parents may spend up to 30% of their income to send their kids to private schools (Akaguri, 2011). Ghana's education system is currently under financial stress as a result of low levels of accountability, ineffective resource allocation, and aspirations to increase upper secondary services.

2.3 Generalised Linear Model

A Generalised Linear Model (GLM) is any model in which the variance of the outcome variable is proportional to some function (thus, the variance function) of the mean and the conditional mean of the outcome variable Y is transformable to a linear combination of X-variables (using a link function). A family of models is a collection of generalised linear models that share the same variance function. This term typically refers to a family of distributions, such as the Gaussian, Poisson, or binomial. The user can estimate parameters such as proportions, rates, probabilities, odds, probits, or arithmetic, geometric, harmonic, or algebraic means, as well as their differences or ratios, by selecting the link and variance functions (and/or transforming the outcome variable); Generalised linear models are not appropriate where the range of Y is restricted (e.g. binary, count) and when the variance of Y depends on the mean (Hardin *et al*., 2001; Hilbe. 2001).

2.4 Hierarchical Linear Modelling

Dataset that is nested or hierarchical poses a number of challenges. First, people or creatures that live in hierarchies have a tendency to be more alike than persons chosen at random from the entire community. For instance, third-grade pupils in one classroom are more alike than third-graders randomly picked from the nation's third-grade population or from the school of district as a whole. This is due to the fact that children are often assigned to schools based on their geographic location or other characteristics rather than being randomly assigned to classrooms from the population (e.g., aptitude). Students are definitely more homogeneous when assigned based on similarities in other traits than a random selection of the overall community. Additionally, regardless of how similar or varied their backgrounds are, children in a given classroom have the same teacher, physical setting, curriculum, and experiences, which may lead to an increase in homogeneity over time (Anderson *et al.,* 2013; Snijders *et al.,* 2011; Beaubien *et al.*, 2010; Gill, 2003).

Because studies of student growth usually include repeated observations of people who are themselves nested inside organisational environments, research in education is frequently challenging. For instance, studies on instruction concentrate on how students and teachers engage with particular course materials. These encounters are typically limited to one academic year and take place in a classroom setting.

Multi-level modeling, another name for hierarchical linear modeling, belongs to the category of statistical tests known as "mixed effects modeling" (or, more simply, "mixed models"). When the data's cases are nested, this type of analysis is most frequently applied. Consider the possibility that some of the study's participants may come from a number of classrooms when gathering student data. Students are therefore nested within classrooms in the data. Since the study's participants who attended the same class will have some shared differences as a result of doing so, such cases cannot be viewed as genuinely independent of one another. When analysing nested data, a different type of analysis is necessary because a traditional multiple linear regression analysis presumes that all cases are independent of one another. Nested data can be modeled more effectively with hierarchical linear modeling than with multiple linear regressions (Anderson *et al.*, 2013; Snijders *et al.*, 2011; Beaubien *et al.*, 2010; Gill, 2003).

On the other hand, hierarchical regression addresses the process of choosing and including predictor (independent) variables in the model. Hierarchical regression specifically refers to the procedure of gradually adding or eliminating predictor variables from the regression model. For example, when you are attempting to predict BECE performance (your dependent variable) based on the type of school attended (your independent variable), i.e., private or public, while adjusting for demographic factors (i.e., covariates). You could enter the demographic parameters into the model in the first phase of your analysis, and then the BECE score in the second. This would enable you to observe the additional predictive value of that BECE performance, beyond the demographic components, contributes to your model. In stepwise, reverse, and forward regression, predictors are automatically added to or deleted from the regression model in accordance with statistical procedures. If you have a lot of potential predictor variables and want to figure out (statistically) which factors are more predictive, these types of hierarchical regression are helpful. In other words, hierarchical regression is used to add or delete variables from your model gradually while hierarchical linear modeling is utilised when you have nested data. It can be easier to choose the best analysis for your study if you are aware of the distinctions between these two concepts that appear to be identical (Anderson *et al.*, 2013; Snijders *et al.*, 2011; Beaubien *et al.*, 2010; Gill, 2003).

2.5 Prediction by Contour and 3-D maps

Contour and 3-D nonparametric forecasting estimator have been used to study the dependence pattern between variables in recent works (Solali, 2020; Igwebe *et al.,* 2019; Guegan *et al.,* 2018). Understanding such dependence leads to forecasting of the joint dynamic behaviour between a response and a given predictor variable. Muller (2004) also used Contour and 3-D maps to predict Credit default on AGE and AMOUNT and concluded that approximately 33.3% of credit default is as controlled by AGE and AMOUNT. Details on the used kernel based estimation can be found in the work of Severini *et al.,* (1994) and M¨uller (2001).

The nonparametric contour 3-D stratification allows us in this thesis work to compare performance of public and private school pupils directly with respect to their BECE performance under each element of a given factor at a time. This will help deepen the understanding of the dynamics controlling the performance gap.

2.6 Neutrosophic Modeling

The study of concepts and ideas that are neither real nor incorrect but fall somewhere in between is known as neutrosophy (these concepts have properties such as neutral, indeterminate, unclear, vague, ambiguous, incomplete, contradictory, etc.). Each field has a neutrosophic component, or the portion that is uncertain. The neutrosophic set, neutrosophic probability, neutrosophic statistics, neutrosophic measure, neutrosophic precalculus, neutrosophic calculus, etc. were thus created. There are various kinds of indeterminacies; as a result, neutrosophy can be developed in numerous ways. A model with some indeterminacy is often known as a neutrosophic model (the variables defined in the model have some level of vagueness, unsureness, ambiguity, incompleteness, contradiction, etc.). The Likert scale is the most popular psychometric method for obtaining replies from survey respondents. It is frequently associated with information distortion and information loss issues because to its ordinal structure and restricted format. Typically, responses are inconsistent, vague, and unclear since they depend on the respondents' emotions. Neutrosophy is a concept that is utilised to accurately account for inconsistent, uncertain, imprecise, and ambiguous information, which Likert scale is incapable of doing. Clustering based on respondents' input is an effective technique for classifying respondents and targeting them accordingly. When dealing with real-world situations, indeterminate Likert scaling performs better at collecting the responses. Respondents may be asked to choose from a list of options ranging from strongly agree to strongly disagree in order to express their opinions on the topics being measured by the questionnaire since they must always choose a Likert scale that can convey a range of emotions. Respondents may mark “strongly agree” next to the response that receives the majority of votes, while ignoring any tiny or inconsequential amount of opposition. Respondents could also choose “agree” ,which would provide the least degree of disagreement. As a result, the Likert scale utilised in the questionnaire does not capture the exact perception of the responder expressing the precise level of agreement or disagreement (Broumi *et al.*, 2014a; Broumi *et al.*, 2014b; Broumi *et al.*, 2014c; Kandasamy, *et al.*, 2012). Neutrosophic Likert scaling will reduce the necessity to choose the most well-liked choice or a forced pick, albeit this is not always the case when there is little difference between the two possibilities. The responses from the respondents were mostly influenced by human emotions, which are frequently confusing, contradictory, imprecise, or indeterminate in character and call for neutrosophic analysis to tackle such situations (Abdel-Basset, *et al.*, 2018; Aslam *et al.*, 2018; Mumtaz *et al.*, 2014).

Neutrosophic Statistics refers to a set of data with some degree of indeterminacy in it or a portion of it as well as the techniques used to evaluate the data. The neutrosophic analysis coincides with the set analysis if one employs sets and there is no indeterminacy. Interval analysis and neutrosophic analysis are equivalent if sets are not used and there is no indeterminacy. Neutrosophic analysis is used when there is some indeterminacy, whether utilising only intervals or sets. Although every set can be contained within a closed interval, the result is rougher, coarser, and more inaccurate when working with wider intervals than tight sets. In comparison to interval analysis, the neutrosophic approach is more refined because it uses smaller sets that are included in intervals. The neutrosophic technique also employs open intervals and half-open, half-closed intervals in specific situations (Abdel-Basset, *et al.*, 2018; Aslam *et al.*, 2018; Mumtaz *et al.*, 2014).

The difference between neutrosophic statistics and classical statistics is that all data are determined in classical statistics. Neutrosophic statistics frequently agree with classical statistics when indeterminacy is zero. The neutrosophic metric can be used to quantify the uncertain data. We can interpret and arrange neutrosophic data, which may contain certain ambiguities, using neutrosophic statistical approaches in order to identify underlying patterns. We stress that indeterminacy differs from randomness, just as in neutrosophic probability. In contrast to classical statistics, which only refers to randomness, neutrosophic statistics includes both randomness and, more specifically, indeterminacy. All methods used to enumerate and describe the properties of neutrosophic numerical data are included in neutrosophic descriptive statistics (Abdel-Basset, *et al.*, 2018; Aslam *et al.*, 2018; Mumtaz *et al.*, 2014).

2.6.1 Neutrosophic Principal Component Analysis

Principal component analysis must be used due to the elements included in this study having the following characteristics. First, it is highly likely that some of the many variables we have are measuring the same underlying thing. Again, they probably have a lot in common. In order to effectively reflect the construct, we must include components in our evaluation scale while omitting those that do not. Second, a new measurement scale could be necessary, but we are not even sure if the components we have chosen adequately represent the construct that we are interested in. We must ascertain if the construct of interest "loads" onto all (or just some) of your components in order to decide whether they are sufficiently representational of the construct of interest or if they should be dropped from the new measuring scale. Last but not least, we might want to see if our measurement scale, such as a questionnaire, can be condensed to have fewer items because some of those items may be superfluous (i.e., more than one item may be measuring the same construct), and/or we might want to make a measurement scale that is more likely to be completed (Mawusi-Nugba *et al.*, 2021).

2.7 Summary of Literature Review

In the statistical development of generalized linear models to address recent shortcomings of existing generalized linear models, there appears to be a gap, according to a review of the literature. Evidence from the literature has highlighted existing inadequacies, such as the failure to represent scenarios involving heterogeneity, unequal variation, and random effect. Other issues included mismatched sample sizes, issues with strong correlation scenarios, and issues with indeterminacy (Lil et al., 2020; ShunCheng et al., 2022; Pftzner et al., 2021). A novel development in statistical modeling would be the development of an extended generalized linear model with structural parameters managing explanatory variable elements and a random effect component modeling missing relevant covariates and heterogeneity. The issue of accounting for dependencies and minimizing bias due to misspecifying item random effects persists despite recent efforts to develop an extended version of the generalized linear mixed model. In order to accurately account for the inconsistent, unclear, imprecise, and ambiguous information that is frequently linked with responses from people who are being analyzed in order to make generalizations, the neutrosophy idea is used (Mumtaz et al., 2014; Aslam, 2018).

CHAPTER 3

BASICS OF NEURTROSOPHIC GENERALISED LINEAR MIXED EFFECT MODELLING

3.1 Fundamental Concepts of Neutrosophic Numbers and Generalised Linear Modeling

3.1.1 Neutrosophic Number Notation

Consider that the element x (t,i,f) only partially belongs to the set S, that it does not fully belong to the set, and that the remainder of its appurtenance to the set is uncertain. It is possible that we do not know if an element, y (0,1,0), belongs to the set or not (total indeterminacy). It is also possible that we are unaware of an element's membership in the set. These are not handled by interval analysis or set analysis. Consider the Neutrosophic observations, $[b, c], d\ [e, f]$. $[b, c]$ and $[e, f]$ are unclear as $[b, c]$ means we are uncertain about the exact number that is within the interval $[b, c]$. Same can be explained for $[e, f]$. These two observations represent indeterminacies. Neutrosophic observations can be a subset and not solely a crisp which is either closed, open, half - closed, half - open intervals.

3.1.2 Basics of Neutrosophic Numbers and Statistics

Consider Equation (3) of the form

$$N_t = d_t + U_t \tag{3}$$

WhereN_t represents a Neutrosophic statistical number, d_t denotes the determinate or the certain component of N_t and U_t is the unsure or indeterminate component of N_t

Consider Equation (3) to hold for the interval [a, b], then Equation (3) has other equivalent forms as shown in Equation (3.1)

$$N_t = d_t + U_t; U_t \in [a,b] = \begin{cases} N_1 = (d_{t_1} - 1) + U_1, U_1 \in [a+1, b+1] \\ N_2 = (d_{t_2} - 2) + U_2, U_2 \in [a+2, b+1] \\ N_3 = (d_{t_3} - n) + U_3, U_n \in [a+n, b+n] \end{cases} \tag{3.1}$$

for any real number, n.

Consider Table 1, where some incidences are recorded with corresponding neutrosophic frequencies where, a < b < c < d < e < f < g.

Table 3.1. **Neutrosophic Frequency Distribution**

No. of Incidence	Neutrosophic Frequency
i_1	a
i_2	[b, c]
i_3	[d, e]
i_4	[f, g]

Then the minimum and maximum estimated Neutrosophic frequency is given by Equation (3.2) and Equation (3.3) respectively.

$$Min_n\, f = a + b + d + f \tag{3.2}$$

$$Max_n\, f = a + c + e + g \tag{3.3}$$

From Equation (1), we can generate the Neutrosophic numbers

$d_1 + a_1U, d_2 + a_2U, d_3 + a_3U$ and $d_4 + a_4U$, where U denotes indeterminacy with conditions that $U^2 = U$ and $U = 0$ for the case of products.

The Neutrosophic mean, $\bar{X}_N$ can be computed as Equation (3.4).

$$\bar{X}_N = \frac{(d_1 + a_1U) + ((d_2 + a_2U) + (d_3 + a_3U) + (d_4 + d_4U)}{4} \tag{3.4}$$

$$\bar{X}_N = \frac{(d_1 + d_2 + d_3 + d_4)}{4} + \frac{(a_1 + a_2 + a_3 + a_4)}{4}.U$$

If $\quad \bar{d}_n = \dfrac{(d_1 + d_2 + d_3 + d_4)}{4}$ and $\bar{a}_n = \dfrac{(a_1 + a_2 + a_3 + a_4)}{4}$

Then the simplified form of Equation (3.4) is Equation (3.5)

$$\bar{X}_N = d_n + \bar{a}_n \cdot U \tag{3.5}$$

The Neutrosophic median is computed as Equation (3.6).

$$M_{d_N} = \frac{d_2 + a_2U + d_3 + a_3U}{2} \tag{3.6}$$

$$M_{d_N} = \frac{d_2 + d_3}{2} + \left(\frac{a_2 + d_3}{2}\right)U$$

$$\text{Let } \bar{d}_m = \frac{d_2 + d_3}{2} \text{ and } \bar{a}_m = \frac{a_2 + a_3}{2} \quad .$$

Then the simplified form of Equation (3.6) is Equation (3.7)

$$M_{d_N} = \bar{d}_m + \bar{a}_m U \tag{3.7}$$

The Neutrosophic deviations (D_N) of each Neutrosophic number are respectively expressed in Equations (3.8) to (3.11).

$$D_{N_1} = (d_1 + a_1U) - (\bar{d}_n + a_nU) \tag{3.8}$$

$$D_{N_2} = (d_2 + a_2U) - (\bar{d}_n + a_nU) \tag{3.9}$$

$$D_{N_3} = (d_3 + a_3U) - (\bar{d}_n + a_nU) \tag{3.10}$$

$$D_{N_4} = (d_4 + a_4U) - (\bar{d}_n + a_nU) \tag{3.11}$$

The respective square derivations $D_{N_1}^2, D_{N_2}^2, D_{N_3}^2$ *and* $D_{N_4}^2$ are explicated in Equations (3.12) to (3.15).

$$D_{N_1} = [(d_1 + a_1U) - (\bar{d}_n + a_nU)]^2 \tag{3.12}$$

$$D_{N_2} = [(d_2 + a_2U) - (\bar{d}_n + a_nU)]^2 \tag{3.13}$$

$$D_{N_3} = [(d_3 + a_3U) - (\bar{d}_n + a_nU)]^2 \tag{3.14}$$

$$D_{N_4} = [(d_4 + a_4U) - (\bar{d}_n + a_nU)]^2 \tag{3.15}$$

where $U^2 = U$. Generally, the square deviations can be expressed in Equation (3.16) and (3.17).

$$(\bar{d}_n + a_n U)^2 = \bar{d}_n^2 + 2\bar{d}_n a_n U + a_n^2 U^2 = \bar{d}_n^2 + 2\bar{d}_n a_n U + a_n^2 U \tag{3.16}$$

$$(\bar{d}_n + a_n U)^2 = \bar{d}_n^2 + \left(2\bar{d}_n a_n + a_n^2\right) U \tag{3.17}$$

The Neutrosophic standard deviation (N_s) deduced from Equation (3.17) is expressed in Equation (3.18).

$$N_s = \sqrt{\frac{D_{N_1} + D_{N_2} + D_{N_3} + D_{N_4}}{4}} \tag{3.18}$$

3.2 Neutrosophic Set Operations

The Neutrosophic rules for extended for classical statistics operation where K1 and K_2 are two given sets of numbers are as follows.

$$k_1 + k_2 = \{x_{i1} + x_{i2} | x_{i1} \in k_1 and\ x_{i2} \in k_2\}$$

$$k_1 - k_2 = \{x_{i1} - x_{i2} | x_{i1} \in k_1 and\ x_{i2} \in k_2\}$$

$$k_1 \cdot k_2 = \{x_{i1} \cdot x_{i2} | x_{i1} \in k_1 and\ x_{i2} \in k_2\}$$

$$a \cdot k_1 = k_1 \cdot a = \{a \cdot x_{i1} | x_{i1} \in k_1\}$$

$$a + k_1 = k_1 + a = \{a + x_{i1} | x_{i1} \in k_1\}$$

$$a - k_1 = \{a - x_{i1} | x_{i1} \in k_1\}$$

$$k_1 - a = \{x_{i1} - a | x_{i1} \in k_1\}$$

$$k_1 k_2 = \left\{x_{i1} x_{i2} | x_{i1} \in k_1, x_{i2} \in k_2, x_{i2} \neq 0\right\}$$

$$k_1 n = \left\{ x_{i1} n \middle| x_{i1} \in k_1 \right\}$$

$$k_1 a = \left\{ x_{i1} a \middle| x_{i1} \in k_1, a \neq 0 \right\}$$

$$a k_1 = \left\{ a x_{i1} \middle| x_{i1} \in k_1, x_{i1} \neq 0 \right\}$$

$$\sqrt{k_1 n} = \left\{ \sqrt{x_{i1} n} \middle| x_{i1} \in k_1 \right\}$$

3.3 Neutrosophic Binomial Distribution

The Neutrosophic equivalent of the classical binomial distribution can be deduced. This is an indication that indeterminacy can be proven to be a possible component of probabilistic experiment. Assume that every trial of a probability experiment can have outcomes of Success (T), Failure (F) or a trace of indeterminacy (U). Let x represent the neutrosophic binomial random variable representing the frequency of successes of success after $n \geq 1$ trail. Since each trail contains some indeterminacy, there is indeterminacy in all *n* trails. We wish to determine the case where there is indeterminacy in a number of trials and it's determinate (thus neither success nor failure) remaining trails which is referred to as partially indeterminate and partially determinate trials. Consider that P(T) = likelihood of Success trial; P(F) = likelihood of Failure trial, for both T and F which differs from indeterminacy (U) and P(U) = likelihood of an Indeterminate trail.

Consider $x \,..\{0,1,2,3,\dots,\mathrm{n}\}$, the neutrosophic probability, UP with *x* success among *n* trials so that, $UP = (S_x, U_x, F_x)$

Where:

$$S_x = \frac{n!}{x!} P(T)^x . \sum_{r=0}^{th} \frac{P(U)^r P(F)^{n-x-r}}{r!(n-x-r)!} \tag{3.19}$$

Similarly,

Bernoulli distribution is a special case of binomial can be explicated in Equation (3.20),

$$\left\langle \begin{array}{l} \dot{N} = \left\langle x, S_{\dot{N}} x, U_{\dot{N}} x, F_{\dot{N}} x,\ \ x \in X \right\rangle; x \in X, S_{\dot{N}} x, U_{\dot{N}} x, F_{\dot{N}} x \in [0,1] \\ S_x = \frac{1!}{x!} P(T)^x \sum_{r=0}^{i} \frac{P(U)^r P(F)^{1-x-r}}{r!(1-x-r)!} = \frac{1}{x!} P(S)^x \sum_{r=0}^{i} \frac{P(U)^r P(F)^{1-x-r}}{k!(1-x-r)!} \end{array} \right. \tag{3.20}$$

3.4 Generalised Linear Modeling

The canonical form of the response variables $Y_1,\ldots,Y_N$ resulting from an exponential family of distributions for a generalised linear model is the same provided they share equal distribution from exponential family. The predictor variables $\mathbf{X} = X_1,\ldots,X_N$ with unknown parameters $\boldsymbol{\beta} = \beta_1, \beta_2 \ldots, \beta_p$ to be estimated are defined under such response variables. A monotone and differentiable function g called the Link function models as a linear function of predictor variables the transformation of μ_i.

For a known distribution, the expected value $\mu_i = E[Y_i]$ can be represented as a function of some parameter, θ. The expected value of the predictor variables can then be presented as a linear function of the parameters of the predictor variables, $\eta_i = g(\mu_i) = \mathbf{X\beta}$.

3.5 Fitting the response distribution

The decision between discrete (categorical, count, binary) and continuous distributions, as well as symmetrical and asymmetrical (right and left) distributions and the possibility of detecting extreme values, all depend on the distribution chosen. Despite the fact that generalised linear models are incredibly adaptable, it may occasionally be required to take their extensions into account while still adhering to the generalised linear model framework.

3.5.1 Exponential Family of Distributions

The exponential family is represented by Equation (3.20).

$$f_Y(y;\theta,\delta)=\exp\left\{\frac{y\theta-A(\theta)}{\delta}+B(y,\delta)\right\} \tag{3.20}$$

where A (.) and B (.,.) are well-known functions and Y's range is independent of either θ or δ. The canonical parameter θ and the dispersion parameter δ are referred to as in this formulation. If a function g is used to parameterise the distribution so that θ = g (μ) for some function g, then g (μ) is the canonical link. Equation (3.20) corresponds with the standard definition of the one-parameter exponential family in canonical form if δ is known.

With regard to Y, a random variable whose distribution has the shape of Equation (3.20) has mean and variance represented by Equation (3.21) and (3.22) respectively.

$$\mu\equiv E[Y]=A^{/}(\theta) \tag{3.21}$$

$$Var[Y]=A^{//}(\theta)\delta\equiv V(\mu)\delta \tag{3.22}$$

The variance function is known as V in this context. For exponential families, the variance function is therefore equal to $A^{//}(\theta)$. Since, the property expressed in Equation (3.23) holds,

$$Var[Y]=\delta V(\mu)=(c\delta)\left[V(\mu)/c\right]=\delta^{/}V^{/}(\mu) \tag{3.23}$$

the variance function and the dispersion parameter are only distinct up to a constant; where $\delta^{/}$ is now regarded as the dispersion parameter and $V^{/}(\mu)$ as the variance function, for any constant c. However, since they are both a part of the variance, which is always positive, we would often consider both and V to be positive.

Table 3.2 Distributions and their Parameters

Distribution	Exponential Family Form	Canonical Parameter	Dispersion Parameter	Inverse Link
Binomial : $\binom{n}{y} P^{y}(1-p)^{n-y}$	$exp\left\{\frac{y\,log\left(\frac{p}{1-p}\right)+n\,log(1-p)}{1}+log\binom{n}{y}\right\}$	$log\left(\frac{p}{1-p}\right)$	1	$\frac{1}{1+e^{n}}$
Bernoulli: $\mu^{y}(1-\mu)^{1-\mu}$	$\exp\left(y\log\mu+(1-y)\log(1-\mu)\right)$	$log\left(\frac{p}{1-p}\right)$	1	$\frac{1}{1+e^{n}}$

When only one trial is undertaken, the Bernoulli distribution is a specific case of the binomial distribution (so n would be 1 for such a binomial distribution). Additionally, it is a unique instance of the two-point distribution in which the possible results are not restricted to 0 and 1. Given that the canonical or natural parameters of the Bernoulli and binomial models were equal in the table, both models produced a logit function. They both had logistic functions as their inverse link functions once more. They both also had a dispersion parameter of unity. When discussing the Bernoulli response, the logit link is the most common link, although there are also additional links like the probit and complementary log-log links.

3.6 Logistic Regression

Equation (3.24) describes how logistic regression models the log of the odds, or logit, in terms of explanatory factors.

$$g(\mu)=In\frac{\pi}{1-\pi}=x'\beta\Rightarrow\pi=\frac{e^{x'\beta}}{1+e^{x'\beta}} \tag{3.24}$$

For all β and x, the logit link ensures predictions of π using X in the range (0,1). Logistic regression is defined as using a logit link and Bernoulli response distribution. The Bernoulli and Binomial distributions can only be linked via the logit formula. Measurements are taken of the (0,1) response y and the related values of the explanatory variables x in order to estimate the parameter, β. The general techniques for GLM estimation are used to perform maximum likelihood estimation.

Table 3.3 Distributions and their Link Functions

Model	Link
Probit	$\pi_i = \phi\left(\frac{t_i - \mu}{\sigma}\right) = \phi^{-1}(\pi_i) = x_i^t \beta^*$
Logit	$\log\left(\frac{\pi_i}{1-\pi_i}\right) = x_i^t \beta$
Clog-log	$\log(-\log(1-\pi_i)) = x_i^t \beta$

The log odds are estimated by both the Logit and Tobit models; however, Logit follows a logistic CDF and Tobit a standard normal CDF.

3.7 Construction of Principal Components

From the domain R^m, we consider a random variable X with expected value μ_X and covariance matrix of $\sum X$. The array of Eigen values $\lambda_1 > \lambda_2 > ... > \lambda_m > 0$ for the covariance matrix of $\sum X$ with the ith Eigen value is the largest ith covariance matrix. Consider further the vector of the ith Eigenvector α_i which corresponds to the ith Eigenvalue of the covariance matrix of $\sum X$. We need to maximise $Var\left[\alpha_1^T X\right] = \alpha_1^T \sum_X \alpha_1$ w.r.t

$\alpha_1^T \alpha_1 = 1$ in our quest to derive principal components (PCs) which becomes an optimisation problem to be solved by the Lagrange multiplier approach.

As shown by Equation (3.25), the Lagrange multiplier approach first constructs a function known as the Lagrange function.

$$L(\alpha_1, \phi_1) = \alpha_1^T \Sigma_X \alpha_1 + \phi_1(\alpha_1^T \alpha_1 - 1) \qquad (3.25)$$

Finding the stationary points of the Lagrange function L (α_i, ϕ_1) entails solving the following equations in order to locate the places of local minima of the function subject to equality constraints:

$$\frac{\partial L}{\partial \alpha_1} = 2\Sigma_X \alpha_1 + 2\phi_1 \alpha_1 = 0 \qquad (3.26)$$

Equation (3.26) simplifies to Equation (3.27)

$$\Sigma_X \alpha_1 = -\phi_1 \alpha_1 \Rightarrow \text{var}[\alpha_1^T X] = -\phi_1 \alpha_1^T \alpha_1 = -\phi_1 \qquad (3.27)$$

Where $-\phi_1$ is the eigenvalue of the covariance matrix of Σ_X and α_1 being the normalised eigenvector for which $\text{var}[\alpha_1^T X]$ is considered maximised with initial value taken as α_1 for the eigen vector of Σ_X.

$z_1 = \alpha_1^T X$ is then called the first principal component of X, with α_1 as the vector of coefficients for z_1, for which $\text{var}(z_1) = \lambda_1$.

The second, third and higher principal components can be deduced using similar procedure. The aforementioned findings allow us to draw the conclusion that PCA is the only collection of linear functions of the original data that are uncorrelated and have orthogonal coefficient vectors.

Consider m variables with $m \times m$ covariance matrix containing the sets $\{l_1, l_2, \ldots, l_p\}$ and $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_p\}$ of m eigenvalues and m eigenvectors respectively.

When we take into account the values of the elements of the eigenvalues as the weights of the linear combination, we can construct each PC.

For the kth eigenvector $\mathbf{e}_k = (e_{1k}, e_{2k}, \ldots, e_{pk})$, the Principal Components are produced as shown in Equation (3.28).

$$\left.\begin{aligned} Y_1 &= e_{11}X_1 + e_{21}X_2 + \ldots + e_{m1}X_m \\ Y_2 &= e_{12}X_1 + e_{22}X_2 + \ldots + e_{m2}X_m \\ Y_m &= e_{1m}X_1 + e_{2m}X_2 + \ldots + e_{mm}X_m \end{aligned}\right\rangle \tag{3.28}$$

3.8 Two-Way ANOVA

The four elements of a two-way ANOVA include variability within cells, variability resulting from the interaction of the two factors, variability among the levels of the two factors, and overall variability (error variability). Three distinct statistical tests are used to compare the first three sources of variability—variability due to the first component, variability due to the second factor, and variability due to interaction—to the error variability (based on the F statistic). Each test's resulting p-value aids in our evaluation of the importance of that particular impact. Utilising binary, nominal, or ordinal scales, non-metric data is measured; the distance between the scale values has no real-world relevance. However, metric data can be either discrete or continuous.

$$One-Way\ ANOVA \qquad Two-Way\ ANOVA$$

$$SS_y = SS_x + SS_{error} \qquad SS_y = SS_{x1} + SS_{x2} + SS_{x1x2} + SS_{error}$$

Two-way ANOVA is appropriate based on the model, interaction, and main effect assumptions because our model includes interaction factors and our dataset contains both metric and nonmetric elements.

Alternative Hypothesis H1: It is appropriate to conduct a two-way ANOVA; Ho: It is not appropriate to conduct a two-way ANOVA. The null hypothesis will be rejected if there is a significant P-value ($p = 0.05$), at which point two-way ANOVA will be considered suitable. Relationship Hypothesis: Ho: There is no interaction between the components; H1: There is interaction between the factors If the P-value is significant ($p = 0.05$), the null hypothesis will be rejected, indicating that there is factor interaction.

Main Effect Hypothesis

$$Factor1\ (say, Gender) \qquad Factor\ 2(say, Religion)$$

$$H_o : \mu_1 = \mu_2 = \mu_3 ... = \mu_k \qquad H_o : \mu_1 = \mu_2 = \mu_3 ... = \mu_k$$

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 ... \neq \mu_k \qquad H_1 : \mu_1 \neq \mu_2 \neq \mu_3 ... \neq \mu_k$$

CHAPTER 4

MODEL DEVELOPMENT

4.1 Notation

Let us assume that the response variable, Y_{ij} with $i = 1, 2, ..., m$ is based on the premise of conditionally independence given the random effects under the random components Z_1, Z_2, ..., Z_n. In addition to including additional exponentially distributed distributions, generalised linear models also provide a link function g(.) that connects the mean, or, put another way, the estimated fitted values E(y), to the linear predictor Xβ, is commonly represented with the symbol η.

4.2 Model Development

The general form of Generalised linear model is thus expressed in Equation (4.0).

$$g(\mu) = \beta X; g(\mu) = \eta; E(y) = \mu = g^{-1}(\eta) \qquad (4.1)$$

In the inverse link function of the Generalised Linear Mixed Model (GLMM), a linear mixed model is present and this component is known as linear predictor as indicated by Equation (4.2).

$$E(Y|\rho, \varpi) = g^{-1}(\rho X_0 + \varpi Z) \qquad (4.2)$$

Z symbolises (r × 1) vector of random effects whereas (n × 1) vector of observed dataset. The structural components of the observed covariate is the X_0.

Equation (4.3) depicts the structural component, which is divided into three parts that each quantify the effects of the teacher, the students, and the administrative-logistic elements.

$$E(Y|\varpi) = g^{-1}(\rho(X_{0T} + X_{0P} + X_{0A}) + \varpi Z) \qquad (4.3)$$

Due to a potential interaction between student and teacher components, Equation (4.4) adds an interaction component.

$$E\left(Y\middle|\varpi\right)=g^{-1}\left(\rho_{_{0T}}X_{0T}+\rho_{_{0P}}X_{0P}+\rho_{_{0A}}X_{0A}+\rho_{_{PT}}X_{0T}\cdot X_{0P}+\varpi Z\right) \tag{4.4}$$

Equation (4.5) is a more advanced version of Equation (4.4) that includes the teacher's interaction and the administrative-logistic components.

$$E\left(Y\middle|\varpi\right)=g^{-1}\left(\rho_{_{0T}}X_{0T}+\rho_{_{0P}}X_{0P}+\rho_{_{0A}}X_{0A}+\rho_{_{PT}}X_{0T}\cdot X_{0P}+\rho_{_{AT}}X_{0T}.X_{0A}+\varpi Z\right) \tag{4.5}$$

g (·) is the inverse of g $^{-1}$ (·) where g (·) is a link function with property of being differentiable and monotonic.

The random variable's design matrix Z is a (n × r) matrix, while the matrix X is a (n × p) matrix of rank k. The two interaction terms in Equation (4.5) were added because of the underlying stakeholder relationship, which tends to support the overall goal of education.

For the purposes of this study, it is assumed that our response variable, Y, follows the Bernoulli distribution; as a result, the probability mass function (PMF) has the following form of Equation (4.6).

$$f(y,\theta,\varphi)=\exp\left[\frac{y\theta-b(\theta)}{a(\varphi)}+c(y,\varphi)\right] \tag{4.6}$$

Equation (4.7) represent a PMF on (0, ∞) defines Bernoulli response variable in the range where y is $\{0,1\}$.

$$f(y)=P(Y=y)=\mu^{y}(1-\mu)^{1-\mu}=\begin{cases}\mu \quad if & y=1\\ 1-\mu \quad if & y=0\end{cases} \tag{4.7}$$

Equation (4.8) is the extension when Equation (4.7) is written in the form of Equation (4.6).

$$f(y|\mu)=\exp\left(y\log\mu+(1-y)\log(1-\mu)\right) \tag{4.8}$$

Taking the logarithm of Equation (4.8) yields Equation (4.9),

$$\log\left(p\left(y|\mu\right)\right)=\log\left(1-\mu\right)+y\log\left(\frac{\mu}{1-\mu}\right), 0<p<1, y=0,1. \tag{4.9}$$

As a result, Equation (4.9) provides the logit link, which is the canonical link function (also known as the natural parameter).

$$\theta=\eta=g\left(\mu\right)=\log\left(\frac{\mu}{1-\mu}\right) \tag{4.10}$$

The odds ratio, $\frac{\mu}{1-\mu}$, is in the range of (0, ∞), and g is the odds ratio's logarithm, sometimes known as "log odds,". When $\mu=P\left[y=1\right]$, the students are from a private school.

The inverse link is expressed in Equation (4.11).

$$\mu\left(\eta\right)=g^{-1}\left(\eta\right)=\frac{e^{\eta}}{1+e^{\eta}}=\frac{1}{1+e^{-\eta}} \tag{4.11}$$

The canonical link function is the derivative of its inverse is the variance of the response shown in Equation (4.12).

$$\frac{d\mu}{d\eta}=\frac{e^{\eta}}{\left(1+e^{\eta}\right)^{2}}=\frac{1}{1+e^{-\eta}}.\frac{e^{-\eta}}{1+e^{-\eta}}=\mu\left(1-\mu\right)=Var\left(y\right) \tag{4.12}$$

For the purpose of interpretation, $\boldsymbol{\mu}_i$ and p_i are linked to covariates through the log-linear and logistic link functions. The log-odds is another name for the logit of "success." Equation (4.13) provides the binary response model for the longitudinal and clustered data.

$$\ln(\mu_i)=\rho_{0T}X_{0T}+\rho_{0P}X_{0P}+\rho_{0A}X_{0A}+\rho_{PT}X_{0T}\cdot X_{0P}+\rho_{AT}X_{0T}.X_{0A}+\varpi Z \tag{4.13}$$

Equation (4.14) is our modified generalised mixed effect model with five parameters that is predicated to follow the Bernoulli exponential family, with five structural component parameters and one random effect parameter predicated to be conditionally independent.

$$\operatorname{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \rho_{0T} X_{0T} + \rho_{0P} X_{0P} + \rho_{0A} X_{0A} + \rho_{PT} X_{0T} \cdot X_{0P} + \rho_{AT} X_{0T} . X_{0A} + \varpi Z \qquad (4.14)$$

4.3 Assumptions of the Modified Model

1. The constructed model's response distribution was predicated on the Bernoulli exponential family, according to the first assumption.

2. The model's structural component was believed to be made up of the teacher factors, student factors, and administrative-logistics factors, with the unobserved elements incorporated into the random effect component.

3. Through the use of a link function, the Generalised Linear Mixed Model was used to transform the nonlinear component into an outcome that was non-normal.

4. Due to the nature of our dataset, we used our GLMM rather than the GLM to handle correlated data and unequal variances.

4.4 Maximum Likelihood Parameter Estimation of the Six-Parameter GLMM

Equation (4.15) makes it clear that the conditional distribution of the response variable Y for the modified GLMM of the ith unit with the jth cluster and the random effect z_i is independent and conforms to exponential family distributions.

$$L(\theta) = \prod_{i=i}^{k} f\left(y_{ij}\right) = \prod_{i=1}^{k} \int \prod_{j=1}^{l_i} f\left(y_{ij} \middle| z_i\right) f\left(z_i\right) dz_i \qquad (4.15)$$

To maximise $g\left(\rho; Y_1, Y_2, Y_3, Y_4, Y_5\right)$, ρ is used as the argument of $g\left(\rho; Y_1, Y_2, Y_3, Y_4, Y_5\right)$ and at the same time Y_1, Y_2, Y_3, Y_4 and Y_5 are considered as respective parameters leading to Equation (4.16).

$$g\left(\rho;Y_1,Y_2,Y_3,Y_4,Y_5\right)=\left(e^{-\left(Y_1-\rho_T X_1\right)^2/2\sigma^2}\right)\left(e^{-\left(Y_2-\rho_p X_2\right)^2/2\sigma^2}\right)\left(e^{-\left(Y_3-\rho_A X_3\right)^2/2\sigma^2}\right)\left(e^{-\left(Y_4-\rho_{AT} X_4\right)^2/2\sigma^2}\right)\left(e^{-\left(Y_4-\rho_{pT} X_4\right)^2/2\sigma^2}\right)$$

$$L\left(\sigma\right)=\ln g\left(\rho;Y_1,Y_2,Y_3,Y_4,Y_5\right)=\sum_{i=1}^{5} ln\left(e^{-\left(Y_i-\rho X_i\right)^2/2\sigma^2}\right)-\ln\left(\sqrt{2\sigma^2\pi}\right)^5=\sum_{i=1}^{5}\ln\left(^{-\left(Y_i-\rho X_i\right)^2/2\sigma^2}\right)-\ln\left(\sqrt{2\sigma^2\pi}\right)^5$$

The maximum likelihood estimator for ρ, ρ^{mle} is obtained by taking the derivative of L w.r.t ρ, $\frac{dL}{d\rho}$ while setting it to zero such that ;

$$\frac{dL}{d\rho}=\sum_{i=1}^{5}\left(-2X_i\left(Y_i-\rho^{mle}X_i/2\sigma^2\right)\right)=0$$

$$thus \quad \sum_{i=1}^{5}\left(-X_i\left(Y_i-\rho^{mle}X_i/\sigma^2\right)\right)=0$$

$$and \ \sum_{i=1}^{5}\left(XY_{ii}-\rho^{mle}X_i^{\ 2}\right)=0$$

$$\sum_{i=1}^{5}XY_{ii}-\sum_{i=1}^{5}\rho^{mle}X_i^{\ 2}=0$$

$$\rho^{mle}=\frac{\sum_{i=1}^{5}XY_{ii}}{\sum_{i=1}^{5}X_i^{\ 2}} \tag{4.16}$$

The 5 parameters of ρ^{mle} can then be estimated from Equation (4.16).

4.5 Analysis of the Indeterminacy Component By Neutrosophic Regression

4.5.1 The Modified Least Square Neutrosophic Regression Statistics

The computations used to determine the Neutrosophic Least-Squares Lines, which roughly approximate neutrosophic bivariate data (x_1, y_1), (x_2, y_2), ..., (x_n, y_n), follow the same formula as in classical statistics. In order to reflect the Neutrosophic Least-Squares statistics, the traditional statistics are adjusted, as illustrated in Table 4.1.

Table 4.1 Adjusted Neutrosophic Statistics

Neutr. Obs.	x	y	x^2	xy	y^2
i	a	[a,b]	a^2	(a^2,ab)	$[a^2,b^2]$
ii	(a,b)	(c,d)	(a^2, b^2)	(ad,bc)	(c^2,d^2)
iii	k	{m,n}	k^2	{km,kn}	$\{m^2,n^2\}$
Sum	$\sum x$	$\sum y$	$\sum x^2$	$\sum xy$	$\sum y^2$

The following is how the sums of X and Y are calculated.

$$\sum x = (a+a+k, a+b+k) = (2a+k, a+b+k)$$

$$\begin{aligned}\sum y = (a+c, b+d) + \{m,n\} &= \{(a+c, b+d)+m, (a+c, b+d)+n\}\\ &= \{(a+c+m, b+d+m), (a+c+n, b+d+n)\}\\ &= \{(\mathbf{a+c+m}, b+d+m), (a+c+n, \mathbf{b+d+n})\}\\ &= (\mathbf{a+c+m, b+d+n})\end{aligned}$$

$$\sum x^2 = (a^2+a^2+k^2, a^2+b^2+k^2) = (2a^2+k^2, a^2+b^2+k^2)$$

The following formula is used to calculate the product's total.

$$\sum xy = (a^2+ad, ab+bc) + \{km, kn\} = \{(a^2+ad, ab+bc)+km, (a^2+ad, ab+bc)+kn\}$$

$$= \{(\mathbf{a^2+ad+km}, ab+bc+km), (a^2+ad+kn, \mathbf{ab+bc+kn})\}$$

$$= (\mathbf{a^2+ad+km, ab+bc+kn})$$

These are X and Y's modified Neutrosophic means.

The modified Neutrosophic means of X and Y are respectively

$$\bar{x}_N = \frac{\sum x_N}{n_N} = \frac{(2a+k, a+b+k)}{n_N} \quad \text{and} \quad \bar{y}_N = \frac{\sum y_N}{n_N} = \frac{(\mathbf{a+c+m, b+d+n})}{n_N}$$

With the neutrosophic line modified as $a_N = \bar{y}_N - b_N \bar{x}_N$ with gradient modified as

$$b_N = \frac{\sum x_N y_N - \left[\left(\sum x_N \right) \left(\sum y_N \right) / n_N \right]}{\sum x_N{}^2 - \left[\left(\sum x_N \right)^2 / n_N \right]}.$$

The predicted neutrosophic value of y_N, is $\hat{y}_N = a_N + b_N y_N$.

The modified Residual Neutrosophic Sum of Squares, denoted by NSSResid, is

$$NSS\text{Resid} = \sum \left(y_N - \hat{y}_N \right)^2 = \sum y_N{}^2 - a_N \sum y_N - b_N \sum x_N y_N$$

And the Neutrosophic modified Total Sum of Squares is

$$NSS\text{To} = \sum \left(y_N - \bar{y}_N \right)^2 = \sum y_N{}^2 - \frac{\sum \left(y_N \right)^2}{n_N}.$$

The Neutrosophic Coefficient of Determination is

$$r_N{}^2 = 1 - \frac{NSS\,\text{Re}\,sid}{NSSTo} = 1 - \frac{\sum y_N{}^2 - a_N \sum y_N - b_N \sum x_N y_N}{\sum y_N{}^2 - \frac{\sum \left(y_N \right)^2}{n_N}}$$

and represents the proportion of variation in y_N, when considering a linear relationship between variables x_N and y_N

CHAPTER 5

DATA ANALYSIS, RESULTS AND DISCUSSION

5. 1 Data Scope

The data for this thesis work came from one hundred and ninety-eight (198) respondents who attended randomly selected Private and Public Basic schools in Ada East and West District of Ghana. The required sample size was computed using Cochran's sample size estimator. The growing performance gap between the Private and Public Basic Schools was investigated using the closed- and open-ended responses from students regarding factors and practices in schools that are thought to foster academic accomplishment. Due to the fact that the items in the External BECE have already been standardised, the items' reliability and validity were soaring. Focus group discussions involving experts from various departments of the Quality Assurance Units in four selected Colleges of Education were used to generate questions and create the questionnaire. Step-by-step procedures were followed for pilot testing, face validity, and content validity. After gathering information from one hundred and ninety-eight (198) study participants, internal consistency was calculated to estimate reliability. Data were gathered via a self-administered questionnaire on paper, with responses on a five-point Likert scale for each item. For reliability analysis, Cronbach's alpha was utilised. Using Cochran's sample size estimator, the size n was determined in order to compute the required sample size with confidence interval 95% (Z = 1.960), proportion of 50% (for unknown population), and 6.965% margin of error as shown in Equation (5.1).

$$n = \frac{z^2 \times \hat{p}(1-\hat{p})}{\varepsilon^2} = \frac{1.96 \times \frac{50}{100} \times \left(1 - \frac{50}{100}\right)}{0.06965^2} \approx 198. \tag{5.1}$$

5.2 Checking Model Assumptions

Mixed Generalised Linear Effect Model assumptions have been evaluated, including the degree of correlation between variables and concerns with nonlinearity, outliers, and homoscedasticity.

Table 5.1. Dependent Variable: Type of School

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	11.448[a]	28	.409	2.245	.001
Intercept	14.663	1	14.663	80.526	.000
PupilGd* TeacherGd	.931	1	.931	5.115	**.025**
TeacherGd* Subjtot	2.022	2	1.011	5.551	**.005**
TeacherGd* RelDeno	3.005	3	1.002	5.500	**.001**

The two-way ANOVA results in Table 5.1 met the normality requirements of the Levene's test for residual homogeneity of variance. The two-way ANOVA results show that several factors, including the interaction between student-gender and instructor-gender, have a significant impact ($p<0.05$) on BECE performance levels in both public and private institutions. This shows that some students believe that taking specific subject lessons from teachers of a particular gender will improve their performance. The significant interaction impact between the teacher's gender and the subject(s) he or she teaches on the BECE performance of students from both private and public schools ($p<0.05$) lends support to this.

5.2.1 Assessing issues of Nonlinearity, Outliers and Homoscedasticity

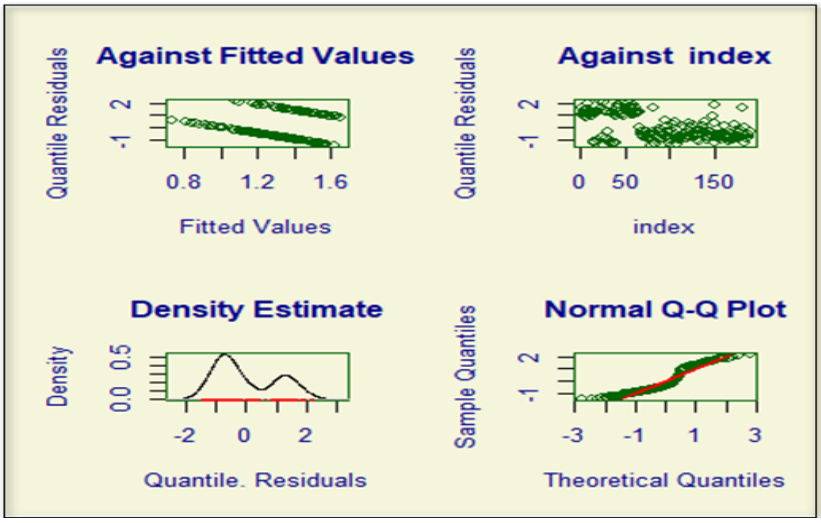

Figure 5.1. Assessing issues of Nonlinearity, Outliers and Homoscedasticity

We reject the null hypothesis that the data come from a population with a normal distribution because, as shown in Figure 5.1, the Filliben Correlation coefficient of 0.9468 at a 5% significance level is less than 0.9927. We therefore use the GLMM for outcomes of this kind that are non-normal. Through transformation with a link function, this will modify the nonlinear component. In contrast, there is no linear relationship between the percentiles in the sample and the theoretical percentiles. The criterion for the distribution of the error terms to be regular is not met. Regarding outliers, the quantile residual plot against the index shows no discernible issues. There is no outlier problem because there are no isolated residuals that appear to have deviated from the typical random distribution of residuals.

5.2.2 Correlation Check

In order to create a heat map with the Spearman method-calculated coefficient of correlation, we change the factor level type to a numeric format. The entire heat map appears dark, indicating a significant problem with collinearity in our data as shown in Figure 5.2. The correlated data and unequal variances will be handled using MIXED models rather than GLM, according to this indication. The bigger the number and darker the color, the stronger the correlation between any two being compared. Strongly correlated data, like those in our example, are typically seen when test subjects or survey respondents are repeatedly measured. MIXED expands the GLM repeated measurements models to support an uneven number of repetitions. Additionally, it can handle more sophisticated situations when experimental units are layered in a hierarchy.

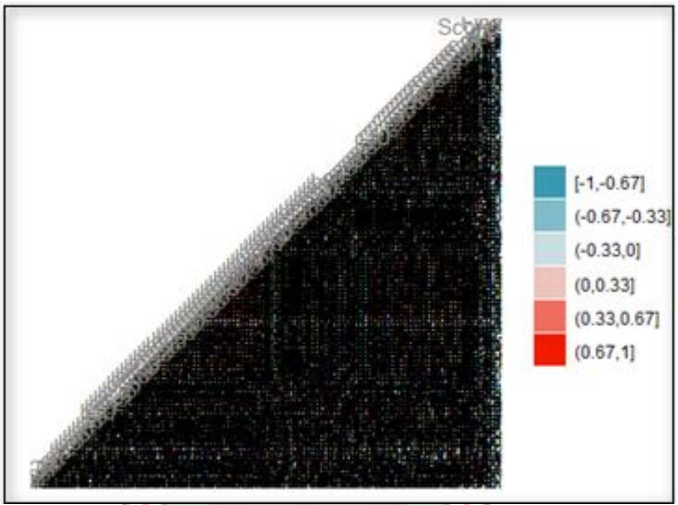

Figure 5.2. Heat map Correlation Check

5.3 Model Selection Criteria

AIC, BIC, Delta AIC, and sample variation strength were used to select the most robust model.

Table 5.2. Model Selection Criteria by AIC, BIC

Sample Variation	**Model**	**BIC**	**AIC**	**Delta (AIC)**
100%	Modified GMM	**323.7**	**213.6**	0
	Probit GMM	339.4	219.6	6
	Clog-log GMM	344.8	221.8	8.2
	Modified GMM	**258.32**	**248.61**	0

70%	Probit GMM	263.46	250.51	1.9
	Clog-log GMM	260.51	250.8	2.19
50%	Modified GMM	**267.43**	**248.02**	0
	Probit GMM	268.69	252.5	4.48
	Clog-log GMM	271.04	251.63	3.61
30%	Modified GMM	**269.71**	**250.3**	0
	Probit GMM	270.95	254.76	4.46
	Clog-log GMM	274.94	252.29	1.99

The model selection criteria and robustness assessment are shown in Table 5.2. The results demonstrate that our modified GLMM was reliable across sample sizes, remaining the top model with the lowest AIC and BIC values. Since the AIC difference between the chosen model and each of the other two candidate models is typically greater than two units, the chosen model is considerably robust. Regarding varying of sample sizes, the GLMM was the most reliable.

5.4 Parameter Estimation of chosen Model

Table 5.3 displays the parameter estimates for the condensed model, which accounts for fifteen (15) significant factors (out of the 70 components included) that contribute to the differences in BECE performance between students in private and public basic schools. The variance of the random effect was **0.**933. According to conclusions drawn from Table 5.3, the top five factors that affect differences in BECE performance between public and private schools are daily quality supervision of the head teacher and teachers by superiors, timely provision of books and learning materials by parents and stakeholders for students, a conducive teaching environment, concern for and parental support of their children's academic output (PTA), and influence of private home tutor for the learner. Additionally, it may be concluded that administrative features account for the majority of the differences in BECE performance between public and private elementary schools. According to probability estimates, a private school teacher who receives daily quality supervision from superiors is about four times more effective in the classroom than their public school counterparts. The BECE performance of students in private schools is improved four times more than that of their public school peers when parents and other stakeholders provide books and other educational resources on time. When compared to a public school, a private school's collaborative learning atmosphere speeds up students' academic progress by nearly three times.

Table 5.3. Reduced Model Estimations

Factors	**Parameter Estimates (E)**	**P-value**	**Standard Error**	**Exp(E)**
Teacher Factors				
Identifies and remediates learners' difficulties or misconceptions (B21)	0.87447	0.00902	0.1989	2.4
meaningfully communicates progress clearly to parents and learners (B24)	0.9036	0.00221	0.33489	2.5
Enhanced parent-teacher relationship (B18)	0.6094	0.01442	0.2491	1.8
Highly supervised to achieve set targets (B1)	0.63720	0.042448	0.31403	1.9
Reflects to modify outputs (B3)	0.8141	0.00571	0.2946	2.3
Pupil Factors				

Influence of Private Home Tutor (B6)	1.09589	0.01798	0.46317	3
Engages in Holiday Classes (B9)	0.4303	0.0305	0.1989	1.5
Class competition (B20)	1.06365	0.048147	0.53827	2.9
Well organised, disciplined with time management due to school culture that translates into academic life (B23)	0.70675	0.044826	0.35227	2
Prep Time Culture (C1)	0.6593187	0.03842	0.3184649	1.9
Pupil well managed and supervised at home by parents to focus on academics as a continuity from school (B28)	0.6775728	0.04168	0.3326766	2
Adm. Log. Factors				
Daily Quality Supervision of Head teacher and Teacher by superiors (B10)	1.42558	0.000875	0.42836	4.2
Timely Provision of Books and learning materials by Parents /Stakeholders for Pupils (B12)	1.4006656	0.00136	0.4374075	4.0
CondusiveTeaching /Learning environment (B15)	1.2121599	0.03617	0.5785873	3.4
Interaction Factors				
Concern and parents' support parents towards their Pupils' Academic Output (PTA) (C2)	1.1848393	0.04422	0.5888820	3.3

Random Effects: Variance = 0.933.

Table 5.3 displays the parameter estimates for the condensed model, which accounts for fifteen (15) significant factors (out of the 70 components included) that contribute to the differences in BECE performance between students in private and public basic schools. The variance of the random effect was **0.**933.

5.5 Predictions based on Stratification by Contour and 3D Maps

With the red area indicating performance in public schools and the yellow region reflecting performance in private schools, contour graphs have been used to predict BECE performance stratified by a specific covariate level. A clearer image of the projections in terms of percentage levels is provided by the adjacent 3D map.

5.5.1 Factor B21: Identifies and remediates learners' difficulties or misconception

The results of Table 5.3 show that teachers in private schools are twice as likely to identify and address misconceptions or learning challenges in their pupils (B21 factor), which leads to better BECE performance than teachers in public schools (B =0.87447, $p < 0.05$, se = 0.1989). Private school pupils are hence twice as likely to get encouraging feedback about their misconceptions or learning challenges. Figure 5.3 shows a contour graph of the BECE performance area that is anticipated to be impacted by this factor. The red area corresponds to performance in public schools, while the yellow area corresponds to performance in private schools. A clearer picture is given by the 3D figure, which demonstrates that private candidates outperform public candidates in the BECE due to the B21 factor by 34%.

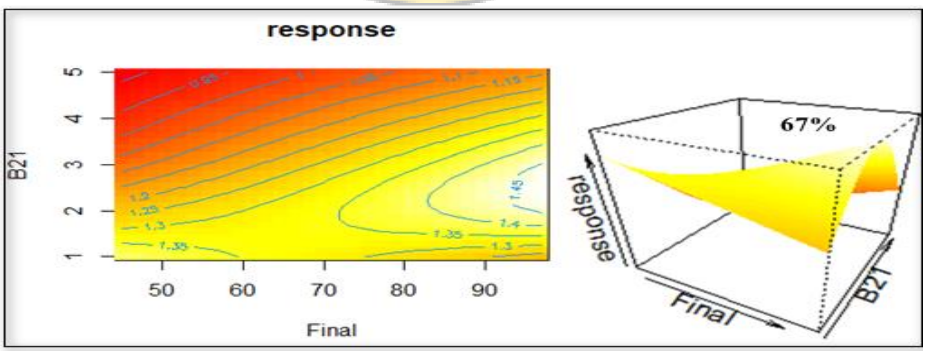

Figure 5.3. Graph Predicting Performance with respect to Factor "Identifies and Remediates Learners 'difficulties or Misconceptions"

5.5.2 Factor B24: Predicting Performance with respect to Factor "Meaningfully Communicates Progress Clearly to Parents and Learners"

According to Table 5.3, “teachers in private schools typically are three times more effective than those in public schools at significantly and plainly communicating progress to students and parents.” Figure 5.4 displays a contour graph of the BECE performance region that is predicted to be impacted by this factor. The 3D figure, which shows that private candidates perform 8% better than public candidates in the BECE was due to the B24 factor, provides a clearer picture.

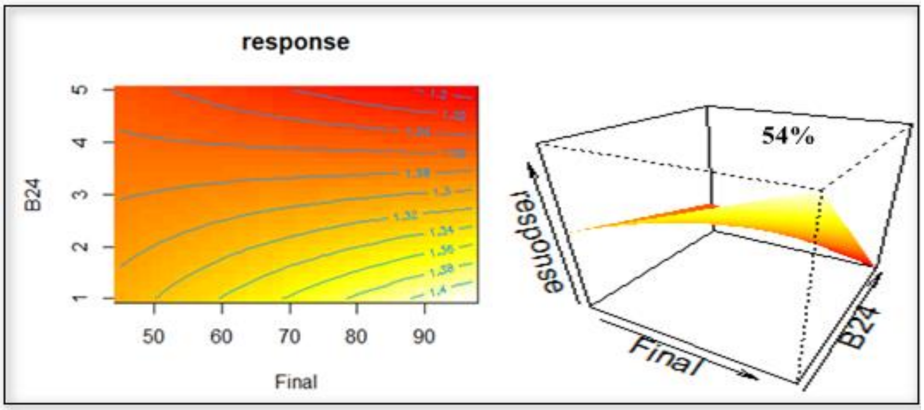

Figure 5.4. Graph Predicting Performance with respect to Factor “Meaningfully Communicates Progress Clearly to Parents and Learners”

5.5.3 Factor B18: Predicting Performance with respect to Factor “Enhanced parent-teacher relationship”

In private schools, improved teacher-parent relationships (B18) have a two-fold stronger beneficial impact on BECE performance than in public schools (B = 0.6094, $p<0.05$, se = 0.2491). The contour graph in Figure 5.5 shows that students in private schools perform 56% higher on the BECE due to improved parent-teacher relationships, which allow parents to provide timely interventions for their children.

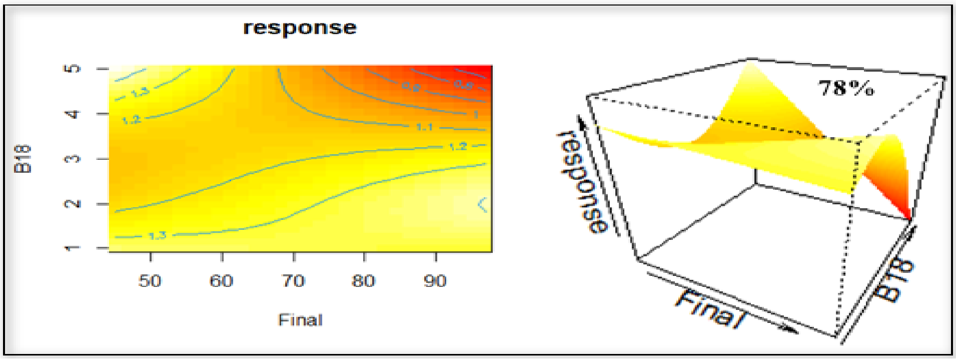

Figure 5.5. Graph Predicting Performance with respect to Factor "Enhanced Parent-Teacher relationship"

5.5.4 Factor B1: Predicting Performance with respect to Factor "Highly Supervised to Achieve Set Targets"

Private teachers are twice as likely to demonstrate numerous high-quality composure and constant alertness to achieve short- to long-term academic goals due to the frequent monitoring they receive from their superiors and the eventual instillation of the culture of alertness and vigilance to improve students' output (B = 0.63720, $p<0.05$, se = 0.31403). The BECE scores of students in private schools were 18% higher than those of students in public schools as a result of private teachers' greater capacity to respond to Factor B1. This is seen in Figure 5.6.

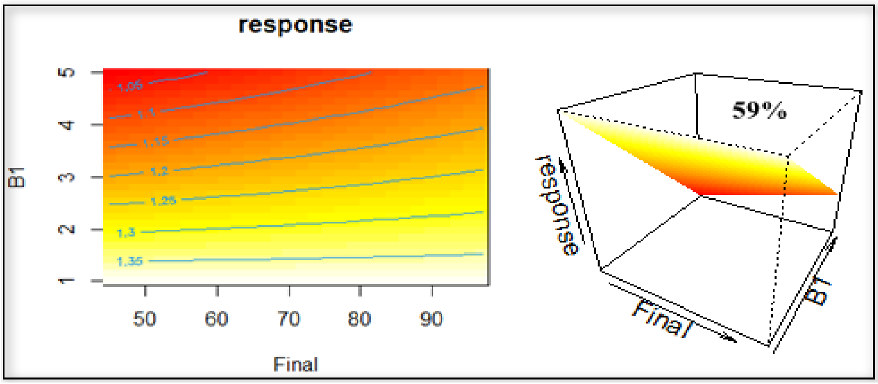

Figure 5.6. Graph Predicting Performance with respect to Factor "Highly Supervised to Achieve Set Targets"

5.5.5 Factor B3: Reflects to Modify Outputs

Teachers in private schools are significantly more competent to assess their own work and recognise when it has to be adjusted in order to produce effective results (B = 0.63720, $p < 0.05$, se = 0.31403). The outcome shown in Figure 5.7 indicates that Factor B3's influence is the reason why students in private schools performed 10% better on the BECE than students in public schools.

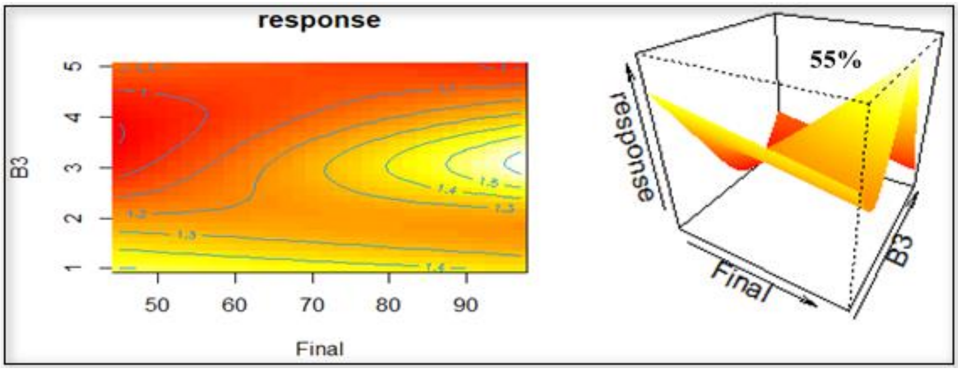

Figure 5.7. Graph Predicting Reflection to Modify Outputs

5.5.6 Factor B6: "Predicting Performance with Respect to Factor "Influence of Private Home Tutor"

When compared to public school students who do not benefit from a private home tutor, the academic achievement of private school students is three times better (B = 1.09589, $p<0.05$, se = 0.46317). According to Figure 5.8, Factor B6 is responsible for a 28% increase in BECE performance for pupils attending private schools as opposed to public ones.

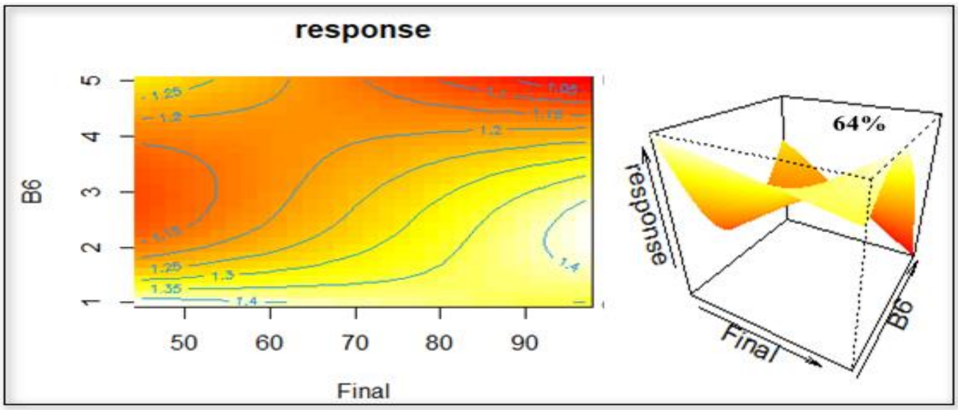

Figure 5.8. Graph Predicting Performance with Respect to Factor "Influence of Private Home Tutor"

5.5.7 Factor B9: "Predicting performance with respect to factor "engages in holiday classes"

Private students frequently take Extra or Holiday classes, which increases their scholastic performance by a factor of two compared to students in public institutions (B = 0.4303, p< 0.05, se = 0.1989). The 12% increase in BECE scores for students in private institutions over those in public schools can be attributed, according to Figure 5.9, to Factor B9.

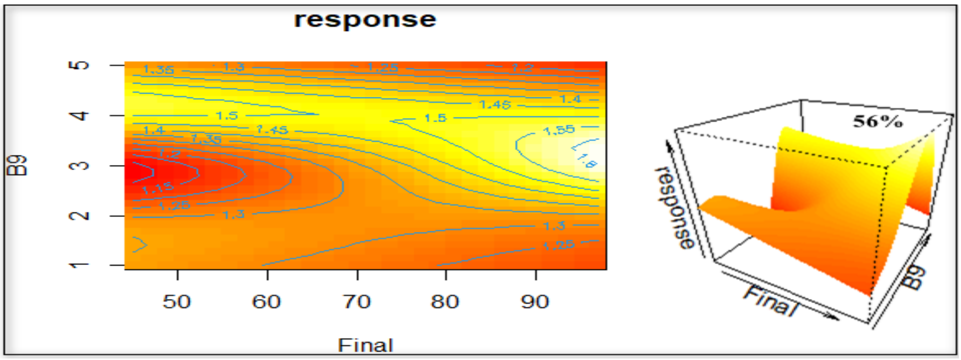

Figure 5.9. Graph "Predicting performance with respect to factor "engages in holiday classes"

5.5.8 Factor B20: Predicting performance with respect to factor "class competition"

Due to the intense competition among students, who strive for academic excellence to raise their class performance rank, students in private schools have a three times greater propensity to outperform those in public schools on the BECE (B = 1.06365, p<0.05, se = 0.53827). According to Figure 5.10, Factor B20 is to blame for the 60% increase in BECE scores between students in public and private institutions.

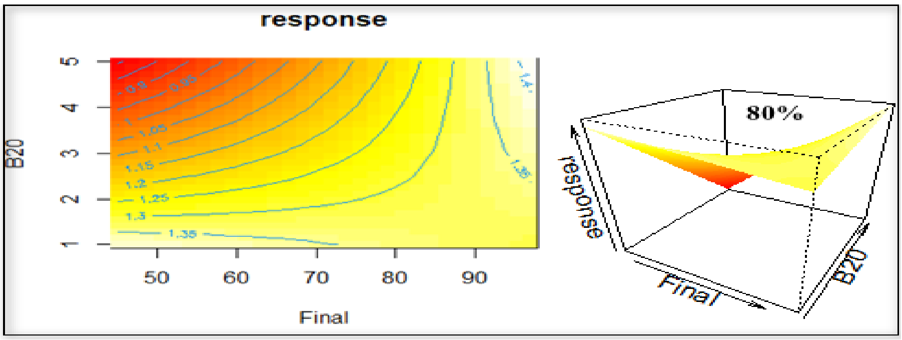

Figure 5.10. Graph Predicting performance with respect to factor" Class Competition"

5.5.9 Factor B23: Predicting performance with respect to factor "well organised, disciplined with time management due to school culture that translates into academic life"

Private school students typically demonstrate higher levels of organisation, discipline, and time management. These characteristics and qualities, which are developed from appropriate school culture and translate into their academic life, give them a double advantage over their public school counterparts in BECE success (B = 0.70675, $p<0.05$, se = 0.35227). According to Figure 5.11, Factor B23 is in charge of the 50% rise in BECE scores between students in private schools and those in public schools.

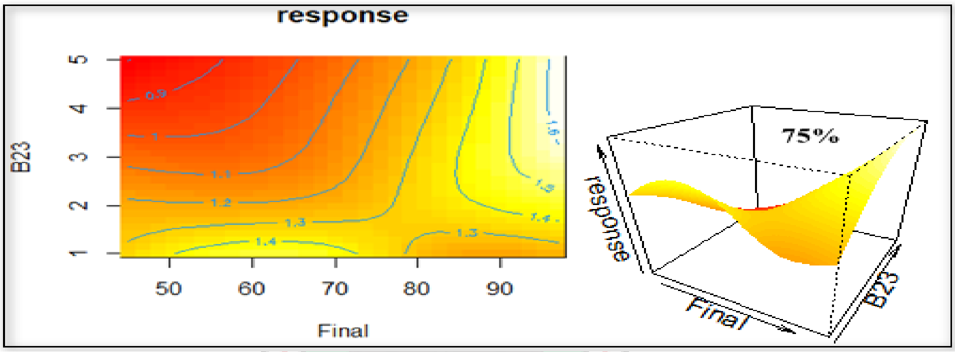

Figure 5.11. Graph Predicting performance with respect to factor "well organised, disciplined with time management due to school culture that translates into academic life"

5.5.10 Factor C1: Predicting performance with respect to factor "Prep Time Culture"

With a 2-fold higher likelihood of passing the BECE than candidates from public institutions, students in private schools are more likely to adhere to the Prep Time Culture (B = 0.6593187, $p < 0.05$, se = 0.3184649). According to Figure 5.12, Factor C1 is in charge of the 32% rise in BECE scores between students in private institutions and those in public schools.

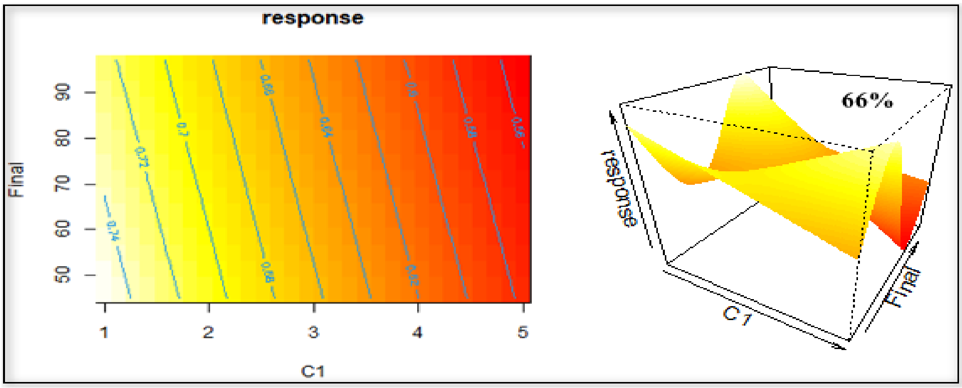

Figure 5.12. Graph Predicting performance with respect to factor "Prep Time Culture"

5.5.11 Factor B28: Predicting performance with respect to factor "Pupil well managed and supervised at home by parents to focus on academics as a continuity from school"

Private school students are usually well-managed and closely watched at home by their parents, so they have a two-fold higher chance of passing the BECE exam than students from public schools (B = 0.6775728, $p<0.05$, se = 0.3326766). According to Figure 5.13, Factor B28 is to blame for the 50% increase in BECE scores between students in public and private institutions.

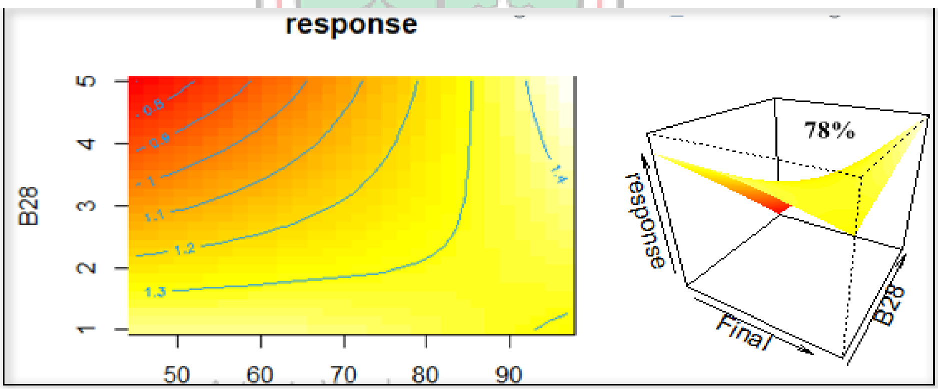

Figure 5.13. **Graph** Predicting with respect to factor "Pupil well managed and supervised at home by parents to focus on academics as a continuity from school"

5.5.12 Factor B10: Predicting performance with respect to factor "Daily quality supervision of head teacher and teacher by superior"

Excellent teaching and learning are the results of daily high-quality teacher monitoring by school owners and teacher supervision by school heads. As a consequence, private school students are 4-times more likely than students in public schools to succeed in the BECE (B = 1.42558, p<0.05, se = 0.42836). According to Figure 5.14, Factor B10 is in charge of the 74% rise in BECE scores between students in private schools and those in public schools.

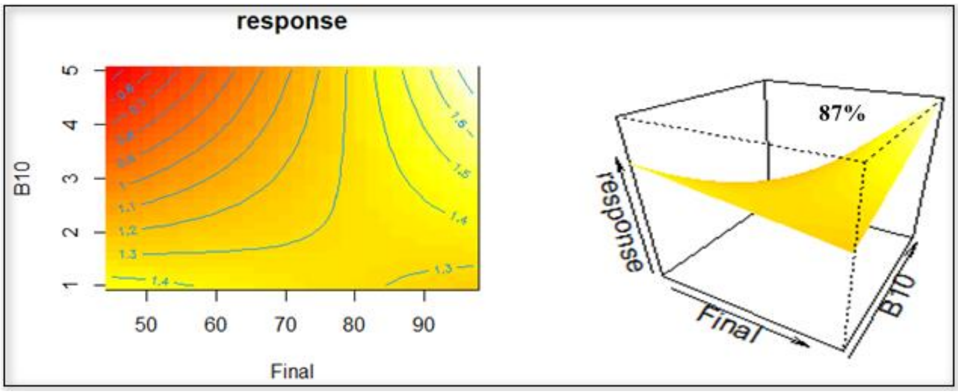

Figure5.14. **Graph** Predicting performance with respect to factor "Daily quality supervision of head teacher and teacher by superior"

5.5.13 Factor B12: Predicting performance with respect to factor "Conducive teaching /learning environment"

As their parents are more likely to provide the necessary books and materials on time as instructed by the school, private school students are four times more likely than public school students to achieve quality grades in the BECE (B = 1.4006656, p<0.05, se = 0.4374075). According to Figure 5.15, Factor B12 is the cause of the 58% improvement in BECE performance between students in private institutions and those in public schools.

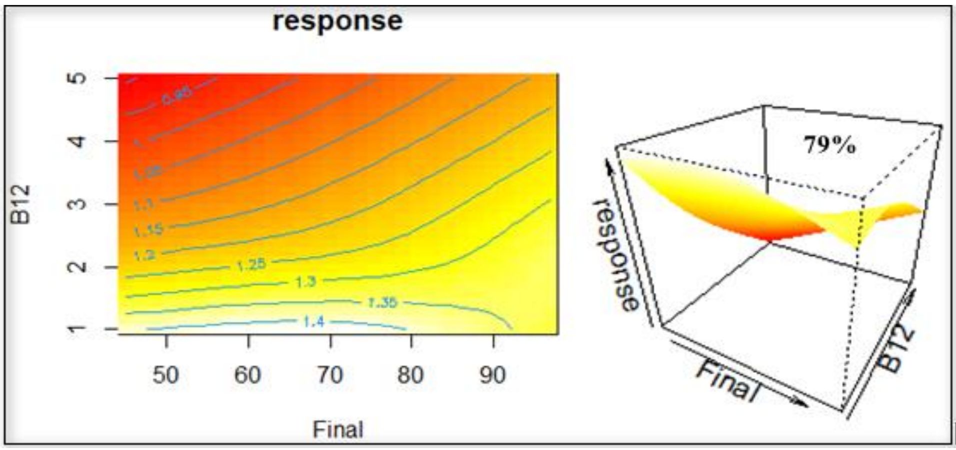

Figure 5.15. **Graph** Predicting performance with respect to factor "Conducive teaching /learning environment"

5.5.14 Factor B15: Predicting performance with respect to factor "Conducive teaching /learning environment"

The BECE scores of students attending private schools are usually higher due to the favorable teaching and learning environment. This element makes private school students three times more likely than public school students to perform better on the BECE (B = 1.2121599, p <0.05, se = 1.2121599). According to Figure 5.16, Factor B15 is in charge of the 64% rise in BECE scores between students in private schools and those in public schools.

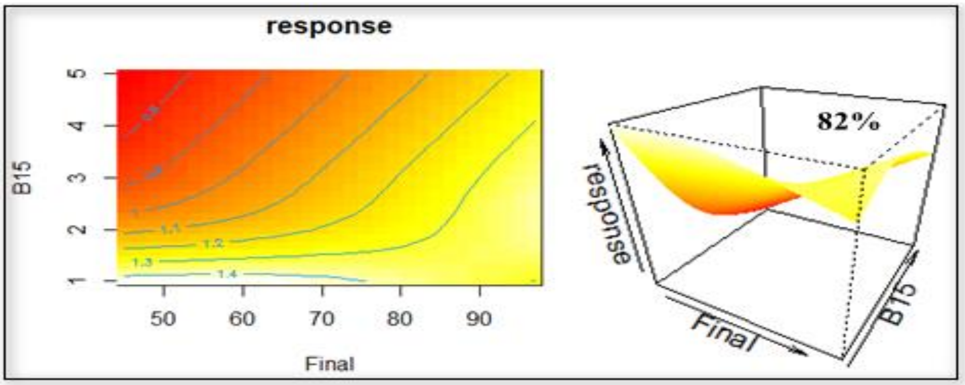

Figure 5.16. Graph Predicting performances with respect to factor "Conducive teaching /learning environment"

5.5.15 Factor C2: Predicting performance with respect to factor "Concern and parents' support parents towards their Pupils' Academic Output (PTA)"

Due to the high level of parental involvement and desire to support the goals of the private schools their children attend, students who attend private schools typically perform three times better in the BECE than students who attend public schools (B = 1.1848393, p<0.05, se = 0.5888820). According to Figure 14, Factor C2 is in charge of the 50% increase in BECE performance between students in private and those in public school.

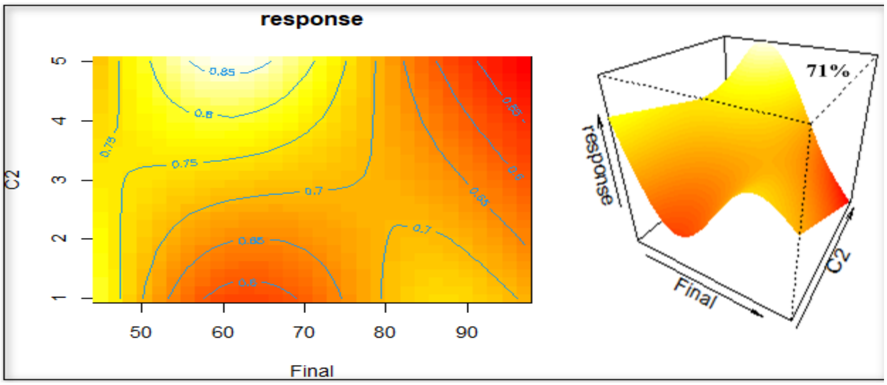

Figure 5.17. Graph Predicting performance with respect to factor "Concern and parents' support parents towards their Pupils' Academic Output (PTA)"

5.6 Analysis of the Indeterminacy Component By Neutrosophic Analysis

We will now use a portion of the dataset in Table 5.4 to illustrate how to use the Neutrosophic set technique.

Table 5.4 The Neutrosophic Set Technique

Neutr. Obs.	x_N	y_N	x^2_N	xy_N	$y_N^{\ 2}$	Neutrosophic Predicted value
1	2	[10,16]	4	[20,32]	[100,256]	(28.6248, -88.283)
2	[5,6]	5	[25,36]	[25,30]	25	(37.3904, -27.683)
3	1	7	1	7	49	(26.6834, -108.483)
4	(7,8)	(12,14)	(49,64)	(84,112)	(144,196)	(40.2732, 12.717)
5	9	{15,20}	81	{135,180}	{225,400}	(42.2146, 53.117)
6	4	6	16	24	36	(32.5076, -47.883)
7	(20,26)	(120,190)	(400,676)	(2400,4940)	(14400,36100)	(75.2184, 275.317)

The corresponding sums of X and y are presented in Equations 5.2 and 5.3 respectively.

$$\sum x = 2+[5,6]+1+(7,8)+9+4+(20,26)$$
$$=16+[5,6]+(7+26,8+20)$$
$$=16+[5,6]+(33,30)=16+(5+33,30+6)$$
$$\sum x = 16+(38,36)=(38+16,36+16)=(54,52) \quad (5.2)$$
$$\sum y = [10,16]+(12,14)+(120,190)+\{15,20\}+18$$
$$=[10,16]+(12+190,14+120)+18+\{15,20\}$$
$$=[10,16]+(202,134)+18+\{15,20\}$$
$$=(202+10,134+16)+18+\{15,20\}$$
$$=(212,150)+18+\{15,20\}=(230,168)+\{15,20\}$$
$$=(230,168)+\{15,20\}=\{(230,168)+15,(230,168)+20\}$$
$$\sum y = \{(245,183),(250,188)\}=(245,188) \quad (5.3)$$

The sums of square X, square Y and the sums of the product XY are presented in Equations 5.4, 5.5 and 5.6 respectively.

$$\sum x^2 = 102+[25,36]+(49,64)+(400,676)$$
$$=102+[25,36]+(725,464)$$
$$=102+(750,500)$$
$$\sum x^2 \quad =(852,602) \quad (5.4)$$
$$\sum y^2 = 110+(36344,14852)+\{225,400\}$$
$$=\{(36454,14962)+225,(36454,14962)+400\}=\{(36679,15187),(36854,15362)\}$$
$$\sum y^2 \quad =(36679,15362) \quad (5.5)$$

$$\sum xy = 31+[20,32]+[25,30]+(84,112)+\{135,180\}+(2400,4940)$$
$$=31+(5069,2574)+\{135,180\}=(5100,2605)+\{135,180\}$$
$$=\{(5100,2605)+135,(5100,2605)+180\}=\{(5235,27401),(5280,2785)\}$$
$$\sum xy = \{(5235,27401),(5280,2785)\}=(5235,2785) \quad (5.6)$$

$$b_N = \frac{\sum xy_N - \left[\left(\sum x_N\right)\left(\sum y_N\right)/n_N\right]}{\sum x_N{}^2 - \left[\left(\sum x_N\right)^2/n_N\right]} = \frac{(5235,2785)-[(54,52)(245,188)/7]}{(852,602)-\left[(54,52)^2/7\right]}$$

$$= \frac{(5235,2785)-\left[\left(13230,9776\right)/7\right]}{\left(576,878\right)-\left[\left(2916,2704\right)/7\right]} \approx \frac{(5235,2785)-\left(1890,1397\right)}{\left(576,878\right)-\left(417,386\right)}$$

$$\approx \frac{\left(3838,895\right)}{\left(190,461\right)} \approx \left(\frac{3838}{190},\frac{895}{461}\right) \approx \left(20.2,1.9414\right)$$

The Neutrosophic mean values results in finding the neutrosophic least-squares line Equation as shown in Equation 5.7.

$$\bar{x}_N = \left(\frac{\sum x_N}{n_N}\right) = \frac{(54,52)}{7} = \left(7.7,7.43\right), \qquad \bar{y}_N = \left(\frac{\sum y_N}{n_N}\right) = \frac{\left(245,188\right)}{7} = \left(35,26.857\right)$$

Where $a = \bar{y} - b\bar{x}$

$$a_N = \left(35,26.857\right)-\left(20.2,1.9414\right)\left(7.7,7.43\right) = \left(32.857,26.857\right)-\left(155.54,14.4246\right) = \left(24.742,-128.683\right)$$

thus, the neutrosophic least-squares line is: $\hat{y}_N = a_N + b_N x_N$

$$\hat{y}_N = \left(24.742,-128.683\right)+\left(20.2,1.9414\right)\cdot x_N \tag{5.7}$$

The Neutrosophic Predicted Values can be computed using Equation 5.7.

$$\hat{y}_{N_1} = \left(24.742,-128.683\right)+\left(20.2,1.9414\right)\cdot x_{N1}$$
$$\hat{y}_{N_1} = \left(24.742,-128.683\right)+\left(20.2,1.9414\right)\cdot 2 = \left(28.6248,-88.283\right)$$
$$\hat{y}_{N_2} = \left(24.742,-31.723\right)+\left(20.2,2.1931\right)\cdot x_{N_2}$$
$$\hat{y}_{N_2} = \left(24.742,-128.683\right)+\left(20.2,1.9414\right)\cdot\left[5,6\right]$$
$$\hat{y}_{N_2} = \left(37.3904,-27.683\right)$$

$$\hat{y}_{N_3} = (24.742, -128.683) + (20.2, 1.9414) \cdot x_{N_3}$$
$$= (24.742, -128.683) + (20.2, 1.9414) \cdot 1$$
$$= (26.6834, -108.483)$$
$$\hat{y}_{N_4} = (24.742, -128.683) + (20.2, 1.9414) \cdot x_{N_4}$$
$$= (24.742, -128.683) + (20.2, 1.9414) \cdot (7, 8)$$
$$= (40.2732, 12.717)$$
$$\hat{y}_{N_5} = (24.742, -128.683) + (20.2, 1.9414) \cdot x_{N_5}$$
$$\hat{y}_{N_5} = (24.742, -128.683) + (20.2, 1.9414) \cdot 9$$
$$= (42.2146, 53.117)$$
$$\hat{y}_{N_6} = (24.742, -128.683) + (20.2, 1.9414) \cdot x_{N_6}$$
$$\hat{y}_{N_6} = (24.742, -128.683) + (20.2, 1.9414) \cdot 4$$
$$= (32.5076, -47.883)$$
$$\hat{y}_{N_7} = (24.742, -128.683) + (20.2, 1.9414) \cdot x_{N_6}$$
$$\hat{y}_{N_7} = (24.742, -128.683) + (20.2, 1.9414) \cdot (20, 26)$$
$$= (75.2184, 275.317)$$

The modified Neutrosophic residuals can now be computed as follows.

$$y_1 - \hat{y}_1 = [10, 16] - (28.6248, -88.283) = (-18.6248, 104.283)$$
$$y_2 - \hat{y}_2 = 5 - (37.3904, -27.683) = (-32.3904, 32.683)$$
$$y_3 - \hat{y}_3 = 7 - (26.6834, -108.483) = (-19.6834, 105.483)$$
$$y_4 - \hat{y}_4 = (12, 14) - (40.2732, 12.717) = (0.717, -28.2868)$$
$$y_5 - \hat{y}_5 = (15, 20) - (42.2146, 53.117) = (-22.2146, -24.4799)$$
$$y_6 - \hat{y}_6 = 6 - (32.5076, -47.883) = (-26.5076, 53.883)$$
$$y_7 - \hat{y}_7 = (120, 190) - (75.2184, 275.317) = (-155.317, 114.7816)$$

5.7 Deneutrosophication Procedure

Now that the indeterminacy issue has been resolved, we perform Deneutrosophications, which will transform the neutrosophic dataset into a classical dataset by choosing the midpoint of each set. The Table 5.5 outlines this.

Table 5.5 Deneutrosophication Values

Midpoint of Neutrosophic Predicted Value	Midpoint of Neutrosophic Residual
-18.9026	-5.9026
53.5888	-48.5888
7.70605	-0.47255
75.9819	-62.9819
97.62495	-80.12495
41.2957	-35.2957
227.0198	-72.0198

This method also transforms the neutrosophic least-squares line into a classical least-squares line by replacing the set representations of the coefficients "a" and "b" with their corresponding midpoints.

The Neutrosophic Sum of Squares (RNSS), residuals of Neutrosophic Sum of Squares (RNSS), Neutrosophic Total Sum of Squares and Neutrosophic Coefficient of Determination (NCD) can then be computed.

The equation $\hat{y}_N = (24.742, -31.723) + (20.2, 2.1931) \cdot x_N$ can now be modified as $\hat{y}_N = -3.5 + 11.19655 \cdot x_N$.

$$\mathbf{RNSS} = \sum\left(y_N - \bar{y}_N\right) = \sum y_N{}^2 - a_N \sum y_N - b_N = (42.8291)^2 + \ldots + (-20.2677)^2 = 5007.996396$$

$$NTSS = \sum\left(y_N - \bar{y}_N\right)^2 = \sum y_N - \frac{\left(\sum y_N\right)^2}{n_N} = (36679, 15362) - \frac{(245,188)^2}{7} = (35957.6939, 14137)$$

$$r^2{}_{NCD} = 1 - \frac{\mathbf{RNSS}}{NTSS} = 1 - \frac{5007.996396}{(35957.6939, 14137)} = 1 - \left(\frac{5007.996396}{35957.6939}, \frac{5007.996396}{14137}\right) = (0.8607, 0.6458)$$

As a result, between 86% and 65% of the sample variation is explained by the neutrosophic approximate linear connection between x and y.

Following deneutrosophication, we discover a flawless linear connection between variable and response, demonstrating that the data's indeterminacy has been eliminated. This guarantees that the data is now consistent with traditional modeling, which will reduce estimation bias shown in Figure 5.18.

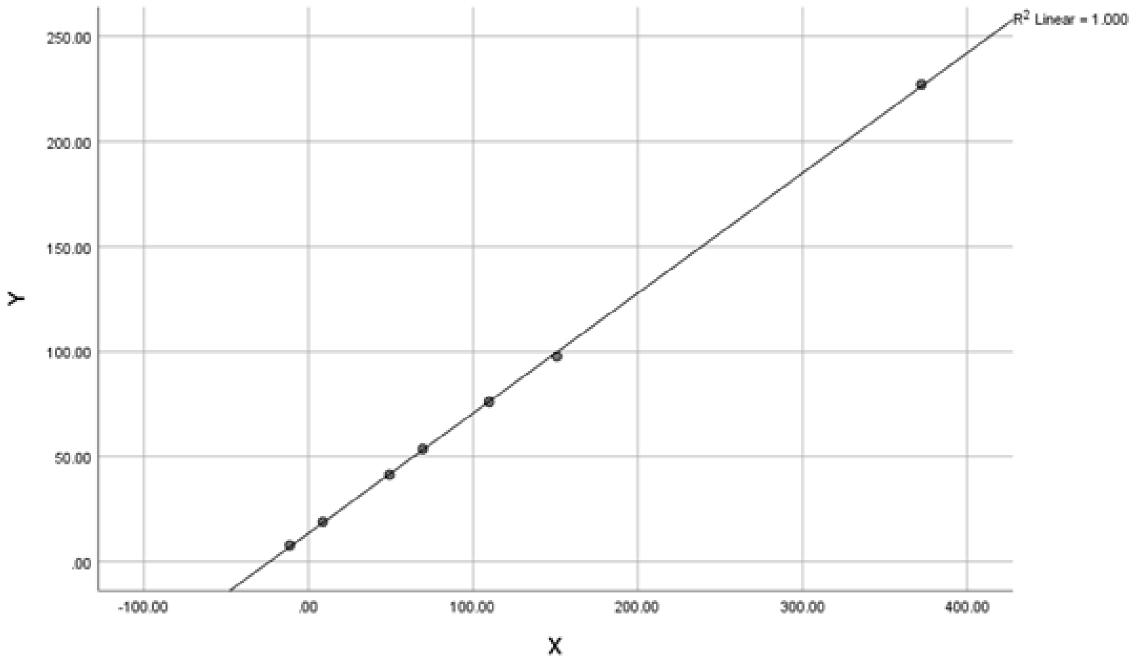

Figure 5.18. Graph of Deneutrosophicated Connection between X and Y

5.8 Principal Component Analysis following Deneutrosophication

The level of the factors' interdependence is established using Bartlett's Test of Sphericity, as shown in Table 5.6. It is presumed that the population matrix is an identity matrix because the variables in the null hypothesis are uncorrelated. In the data above, the Bartlett's Test resulted in a Chi Square value of 1741.892, DF of 406, and significance level of 0.00000. Therefore, the correlation matrix cannot be an identity matrix because the null hypothesis is denied. The significance supports the notion that the matrix should be regarded as factorable by showing how much our correlation matrix for our measured variables deviates from an identity matrix. This demonstrates that the information currently accessible is more than sufficient for the Bartlett's sphericity test.

Table 5.6. KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy	Bartlett's Test of Sphericity		
	Approx. Chi-Square	df	Sig.
0.806	1741.892	406	.000

Table 5.7. Comparison of Parallel Analysis (Monte Carlo PA Output) and Kaiser's Eigenvalue > 1 Rule

Factor	Random order from parallel analysis	Eigenvalue from PCA	Decision
1	2.461511	20.265	Accept
2	2.333521	6.506	Accept
3	2.235278	5.913	Accept
4	2.154729	5.226	Accept
5	2.077648	4.752	Accept
6	2.012439	4.473	Accept
7	1.945982	3.757	Accept
8	1.889200	3.476	Accept
9	1.830679	3.363	Accept
10	1.783094	3.093	Accept
11	1.729416	2.851	Accept
12	1.684089	2.708	Accept

13	1.633839	2.586	Accept
14	1.589818	2.328	Accept
15	1.547742	2.270	Accept
16	1.506302	2.099	Accept
17	1.465871	1.856	Accept
18	1.429441	1.795	Accept
19	1.395169	1.719	Accept
20	1.354644	1.657	Accept
21	1.316386	1.551	Accept
22	1.283549	1.467	Accept
23	1.247764	1.420	Accept
24	1.215083	1.184	Reject
25	1.183768	1.170	Reject

The Scree plot outcome in Figure 2 shows that the associated Eigen values led to a 25-factor divergence from linearity. According to the results of this exam, 25 criteria should be taken into account when analysing the data. However, this method is famous for including a subjective element. The Kaiser's eigenvalue > 1 criterion states that factors can only be preserved if their eigenvalues are higher than 1. Kaiser's Eigenvalue method proposed 17 components to accomplish that. A better decision is made when these methods are compared to the parallel analysis strategy.

Parallel analysis was performed using 189 observations and 70 components indicator factors. 100 correlation matrices were created with 95 as the percentile Eigen value's default setting. The correlation matrices of the parallel analysis that were generated at random were compared with the Eigen values that were extracted from the dataset. The Eigen values (from the data set) above those of the Monte Carlo PA Output were the factors that met the requirement for retention. To achieve that, 23 factors—which are listed in Tables 5.7 and 5.8 were accepted and maintained.

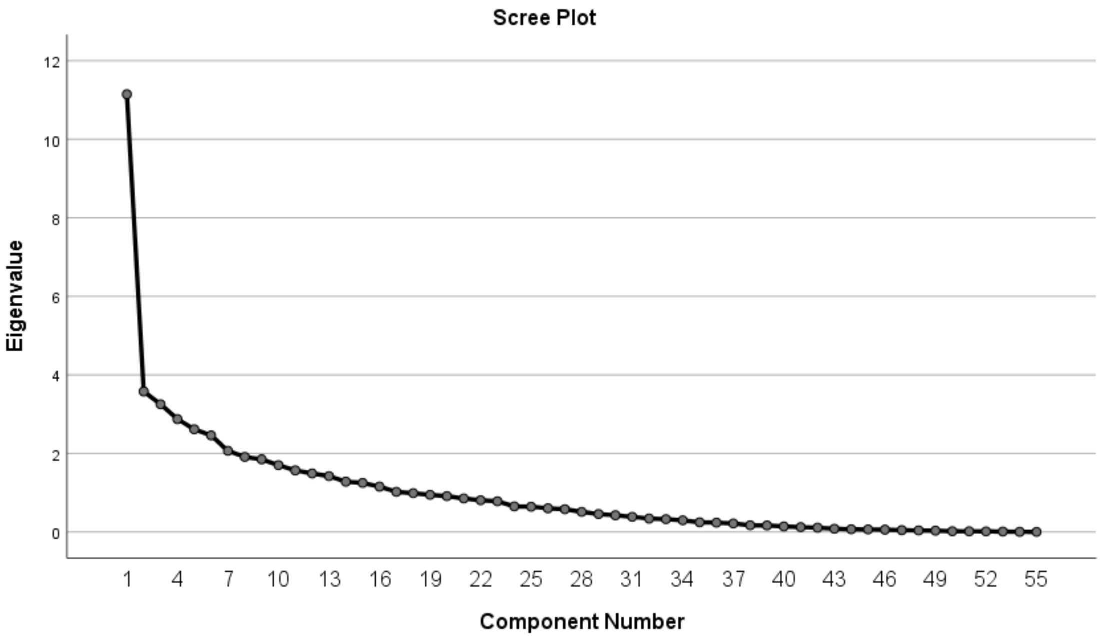

Figure 5.19. Scree Plot Test

Table 5.8. Total Variance Explained after using Multiple Extraction Approaches

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	11.146	20.265	20.265	11.146	20.265	20.265
2	3.579	6.506	26.771	3.579	6.506	26.771
3	3.252	5.913	32.684	3.252	5.913	32.684
4	2.874	5.226	37.910	2.874	5.226	37.910
5	2.614	4.752	42.662	2.614	4.752	42.662
6	2.460	4.473	47.135	2.460	4.473	47.135
7	2.066	3.757	50.892	2.066	3.757	50.892
8	1.912	3.476	54.367	1.912	3.476	54.367
9	1.850	3.363	57.730	1.850	3.363	57.730
10	1.701	3.093	60.823	1.701	3.093	60.823
11	1.568	2.851	63.673	1.568	2.851	63.673
12	1.489	2.708	66.381	1.489	2.708	66.381
13	1.422	2.586	68.968	1.422	2.586	68.968
14	1.281	2.328	71.296	1.281	2.328	71.296
15	1.249	2.270	73.566	1.249	2.270	73.566
16	1.154	2.099	75.665	1.154	2.099	75.665
17	1.021	1.856	77.521	1.021	1.856	77.521
18	.988	1.795	79.317	.988	1.795	79.317
19	.946	1.719	81.036	.946	1.719	81.036

20	.912	1.657	82.693	.912	1.657	82.693
21	.853	1.551	84.244	.853	1.551	84.244
22	.807	1.467	85.711	.807	1.467	85.711
23	.781	1.420	87.132	.781	1.420	87.132

The Scree Test, Kaiser Criterion, and parallel analysis were a few of the extraction methods used to avoid over- and under-extraction issues. When used alone, the Scree Test and Kaiser's Eigen value larger than 1 rule advised retaining 17 and 25 factors, respectively. However, the parallel analysis technique, which bases its decisions on the Kaiser's Eigen value, suggested keeping 23 elements. Studies that compared the three methods for calculating the amount of elements to the results of the parallel analysis were found to be more accurate than the Scree test and Kaiser's Eigen value. Therefore, PCA with 23 components was enforced.

Rotation was used to maximise high item loadings and minimise low item loadings, resulting in a solution that is more understandable, efficient, and economical. The most widely used rotation technique, orthogonal varimax, was used to produce uncorrelated factor structures. It attempts to lessen the complexity of the components by enlarging the large loadings and shrinking the minor loadings inside each component. The first component is responsible for explaining about 20.3% of the total variation. The second factor also explains 6.5% of the variance caused by the other 21 variables. As shown in Table 5.8, the 23 factors explained approximately 87.1% of the variance.

Table 5.9. Rotated Component Matrix

	Component		
	1	2	3
Influence of Private Home Tutor	0.888		
Well organised, disciplined with time management due to school culture that translates into academic life	0.895		
I always want to be in class	0.864		

My teacher pays attention to everyone especially pupils with special needs	0.930		
Prep Time Culture	0.851		
Class competition	0.893		
Engages in Holiday Classes	0.610		
Pupil well managed and supervised at home by parents to focus on academics as a continuity from school	0.881		
Religious denomination		0.943	
My teacher provides me with enough learning activities		0.678	
Enhanced parent-teacher relationship		0.701	
I do perform well in school because my teacher teaches well		0.662	
My teacher treats everybody equally in the class		0.844	
My teacher is always regular in school		0.695	
My teacher gives me prompt feedback for my class exercises		0.784	
Sex of Class Teacher		0.852	
meaningfully communicates progress clearly to parents and learners		0.885	
Identifies and remediates learners' difficulties or misconceptions		0.876	
CondusiveTeaching /Learning environment			0.922
Daily Quality Supervision of Head teacher and Teacher by superiors			0.956
Timely Provision of Books and learning materials by Parents/Stakeholders for Pupils			0.940
Type of School			0.804
Concern and parents' support parents towards their Pupils' Academic Output			0.885

The Rotated Component Matrix findings from Table 5.9 show strong loadings for variables that explain the significant variance that separates the performance of elementary students between private and public schools. Eight pupil factors were discovered to control the pupil-characteristic variations component of this mismatch. To put it another way, positively affecting these factors can place students in a situation where they can perform well and react appropriately. If handled properly, ten distinct but controlled teacher characteristics can raise BECE scores. However, it was also found that five variables could affect administrative and organisational aspects.

Table 5.10. Variance Explained by Pupil Characteristics

Pupil Factor	**Variance Explained (%)**
My teacher pays attention to everyone especially pupils with special needs	4.211901
Well organised, disciplined with time management due to school culture that translates into academic life	4.053388
Class competition	4.044331
Influence of Private Home Tutor	4.021686
Pupil well managed and supervised at home by parents to focus on academics as a continuity from school	3.989983
I always want to be in class	3.912992
Prep Time Culture	3.854116
Engages in Holiday Classes	2.762645
Total Variance Explained	**30.85104**

According to Table 5.10, differences in performance between elementary students in private and public schools can be attributed to student characteristics in about 31% of cases. The table shows that teachers must give students with a variety of academic skills adequate attention because this factor has the highest variance score of all the pupil factors.

According to the second leading variance score, students are in the best possible position to make individual decisions that improve their academic lives when they are well-organised, disciplined with time management due to school culture that translates into their academic lives. Thirdly, the impact of home tutors makes it significantly more likely for elementary school students to perform better on the BECE test.

Table 5.11. Variance Explained by Teacher Characteristics

Teacher Factor	**Variance Explained (%)**
Religious denomination	4.27077686
meaningfully communicates progress clearly to parents and learners	4.008099174
My teacher provides me with enough learning activities	3.07061157
I do perform well in school because my teacher teaches well	2.99814876
Identifies and remediates learners' difficulties or misconceptions	3.967338843
Sex of Class Teacher	3.858644628
My teacher treats everybody equally in the class	3.822413223
My teacher gives me prompt feedback for my class exercises	3.550677686
Enhanced parent-teacher relationship	3.17477686
My teacher is always regular in school	3.147603306
Total Variance Explained	**35.86909**

According to Table 5.11, there are 10 teacher characteristics that contribute to the BECE achievement gaps between students in public and private institutions. The total of these traits makes up about 36% of the overall variable, making it the most significant of the three major factors chosen to control the variations. The primary factor adding to the unpredictable nature of the teacher-factor is the teacher's religious affiliation. It suggests that how teachers perform their responsibilities may be affected by religious discipline. The second significant teacher-related characteristic is that when teachers truly update parents and students on progress, parents are more likely to take action to improve their children's academic performance. Thirdly, when instructors recognise and correct students' problems or misunderstandings, it can improve their BECE performance.

Table 5.12. Variance Explained by Administrative-Logistic Characteristics

Administrative Logistic Factor	Variance Explained (%)
Concern and parents' support parents towards their Pupils' Academic Output	4.329653
Timely Provision of Books and learning materials by Parents/Stakeholders for Pupils	4.25719
CondusiveTeaching /Learning environment	4.175669
Daily Quality Supervision of Head teacher and Teacher by superiors	4.008099
Type of School	3.641256
Total Variance Explained	**20.41186777**

According to Table 5.12's inferences, five administrative-logistical variables account for about 20% of the overall variance governing achievement disparities. The timely provision of books and other educational materials for pupils by parents and other stakeholders is the most crucial element of this component. The favorable teaching and learning atmosphere or infrastructure is the second most significant element of this element. The third crucial element of this component is the daily quality supervision of the head teacher and teachers by superiors, which, if handled correctly, can improve performance levels in BECE.

Table 5.13 Reduced Two-Way Deneutrosophic PCA Model for Between-Subject Effects

Dependent Variable	Type III Sum of Squares	Mean Square	F	P-Value
Teacher Factor	1.956	2.616	12.373	.001
Pupil Factor	2.925	2.249	10.637	.000
Administrative-Logistic Factor	1.396	1.073	5.076	.007
Int. Teacher-Pupil	2.106	0.692	3.275	.013
Int. Teacher-Admi	1.696	0.556	2.632	.036

	Df	SS	MS
Residuals	**185**	**39.11**	**0.2114**

According to Table 5.13, the Teacher factor, Pupil factor, Administrative-Logistic component, Teacher-Pupil interaction effects, and Teacher-Administration interaction effects were all statistically significant ($p < 0.05$).

5.9 Discussion on Neutrosophic PCA

The study's overall conclusion was that the three components—student factors, teacher factors, and administrative-logistic factors—are what lead to variations in BECE performance between pupils in public and private basic schools. Approximately 36% of the total variance was explained by the teacher factor, while 31% and 20%, respectively, were explained by the student and administrative-logistic factors. Together, these three variables accounted for about 87% of the total variance.

The research concluded that random factors were to blame for a difference of about 13% in performance disparity. This unexplained variation may in part be the result of interactions between variables that the neutrosophic-PCA technique did not take into account. We will be able to understand the true variation resulting from the random effect, which is not directly determined by the factors taken into account in the data scope of this investigation, with the aid of a follow-up modeling approach like the generalised linear mixed effect model in a future work. A two-way neutrosophic ANOVA test was performed to determine whether the independent factors, which included the Teacher factor, Pupil factor, Administrative-Logistic factor, and two interacting effects, had any discernible influence. The results showed that all independent variables had statistically significant relationships with pupils' BECE performance ($p < 0.05$).

This study aimed to use the Neutrosophic-Principal Component combined method to pinpoint and explain the causes of the current achievement gaps, which have not yet been completely explained in the scientific literature. The achievement gaps in BECE performance between students in public and private basic schools were explained by student factors, instructor factors, and administrative-logistic factors, which together accounted for 36%, 31%, and 20% of the overall 87% variability. The remaining 13% variability that was ascribed to random effects by the Neutrosophic PCA approach is part of a two-way Neutrosophic ANOVA test additional interaction factors such as Teacher-Pupil and Teacher-Administrative Logistic factors are part of the remaining 13% variability.

CHAPTER 6

CONCLUSIONS, CONTRIBUTION TO KNOWLEDGE, RECOMMENDATIONS AND FUTURE WORK

6.1 Conclusions

To account for the differences in academic performance between students in private and public basic schools during the BECE examinations, a novel extended version of the generalised linear model with five structural parameters and a random effect parameter has been developed. The developed model has been found to be robust irrespective of small or large sample size variation. On the premise of AIC and BIC values, the proposed model outperforms existing models and has demonstrated significance. The criterion of significant Delta AIC values and sample variation highlighted the robustness of the proposed model. Contour and 3D nonparametric forecasting estimator have also been used to study the dependence pattern between variables. The proposed model resolves generalised linear models' inability to handle non-normality and dependence assumptions in educational research, making it by far the most comprehensive educational model to explain differences in BECE performance between Private and Public Basic Schools. The proposed model takes into consideration both observed and unobserved variables that could affect the differences in students' performance between public and private basic schools. The current study proposed a GLMM with a Neutrosophic treatment level-specific item random effect and also Neutrosophic PCA to help minimise bias and optimise correct specification of random effects. Biased specification of random effect structure is a major limitation of proposed GLMMs in recent works (ShunCheng *et al.*, 2022), which causes biased specification of random effect structure.

According to the magnitude rank of predicted probability of the model, the top five variables influencing the performance gap between students attending private and public schools show that Administrative-Logistic causes account for 80% of the variance. These were listed in order of increasing importance: Daily Quality Supervision of Head teacher and Head teacher supervision by school proprietors, Timely Delivery of Books and Learning Materials by Parents/Stakeholders for Students; Conducive Teaching/Learning Environment; Concern and Parental Support for Pupils' Academic Output (PTA). Pupil factor which is Competition within a class was the fifth element. Private school students are 4-times more likely than students in public schools to succeed in the BECE ($B = 1.42558$, $p<0.05$, $se = 0.42836$). private school students are four times more likely than public school students to achieve quality grades in the BECE ($B = 1.4006656$, $p<0.05$, $se = 0.4374075$). Conducive teaching /learning environment element makes private school students three times more likely than public school students to perform better on the BECE ($B = 1.2121599$, $p <0.05$, $se = 1.2121599$). Due to the intense competition among students, who strive for academic excellence to raise their class performance rank, students in private schools have a three times greater propensity to outperform those in public schools on the BECE ($B = 1.06365$, $p<0.05$, $se = 0.53827$).

6.2 Contributions to Knowledge

This thesis work has proposed by far the most comprehensive educational model that handles nonnormality and dependency difficulties while also taking into consideration variable variability. The suggested model is an extension of generalised linear models because these models cannot account for instances of nonnormality, interdependence between variables, and heterogeneous cases, which lead to biased standard errors and diminished statistical power. Examples of these models include the hierarchical linear model, logistic regression, probit models, and poisson regression models.

Neutrosophic treatment by level-specific item random effect and Neutrosophic-Principal Component Analysis to help minimise bias and optimise correct specification of random effects have been presented as novel extensions of the classical least square regression to reduce noise from indeterminacy and cluster effects.

The long-standing discrepancy between Private and Public Basic Schools' BECE performance has been explained, allowing stakeholders to now know which variables to control in order to maximise BECE performance.

6.3 Recommendations

Based on the findings of this study, it is advised that educational stakeholders consider the major determinants in this research that significantly affect BECE performance in order to maximise teaching and learning.

When compared to current models, the new model has shown to maintain its robustness under a variety of sample sizes. It is advised that scholars use this paradigm in fields of study other than educational research.

Finally, it is recommended that problems with indeterminacy in academic research be dealt with using neutrosophic PCA and neutrosophic treatment.

6.4 Further Research

Further research may consider developing Neutrosophic-Bayesian model to compare with the developed model.

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APPENDICES

APPENDIX A

ATTITUDES AND BELIEFS OF TEACHERS ON PUPILS' PERPERFORMANCE IN BASIC SCHOOLS

INTERVIEW GUIDE FOR PUPILS

You are kindly requested to provide answers to each of the following questions orstatement possible. The responses will be treated as confidential.

Section A: Background Information

Please tick (√) where appropriate

1. Gender: Male [ ] Female [ ]

2. Age: 10 and below [ ] 11 and 12 [ ] 13 and 14 [ ] 15 and 16 [ ] 17 and above [ ]

3. Level: Lower Primary [ ] Upper Primary [ ] Junior High School [

4. My class teacher is: Male [ ] Female [ ]

5. Type of school: Public [ ] Private [ ]

6. Religious Denomination: Catholic [ ] Methodist [ ] Presbyterian [ ] Pentecost [ ]

Islamic [ ] Others [ ]

7. Subject taught by teacher:English [ ] Mathematics [ ] Integrated Science []

Section B: Classroom Attitudes and Beliefs of Teachers on pupil' performance

Please, indicate the extent to which you agree with each of the following statements usingthe five - point Likert scale provided.

1	2	3	4	5
Strongly Agree (SA)	Agree (A)	Neutral (N)	Disagree (D)	Strongly Disagree (SD)

Please tick (√) your choice in the appropriate box

1. I feel comfortable to share my learning problems with my teacher
Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

2. My teacher provides me with enough learning activities
Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

3. I don't feel lonely in my classroom

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

4. My teacher always corrects me when I go wrong

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

5. My teacher is very observant

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

6. I do not feel bored with the lessons

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

7. I do perform well in school because my teacher teaches well

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

8. I always want to be in class

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

9. My teacher always uses varying TLMs in teaching concepts

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

10. My teacher treats everybody equally in the class

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

11.My teacher pays attention to everyone especially pupils with special needs

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

12. I am not discriminated against **stigmatized** in any way

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

13. I feel comfortable interacting with my teacher

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

14. My teacher is always regular in school

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

15. I feel comfortable interacting with classmates

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

16. I feel happy that I am part of my class

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [

17. My teacher always comes to school early
Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

18. My teacher is always ready to help me

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

19. My teacher always teaches till school closes

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

20. My teacher respects my religion

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

21. My teacher gives me prompt feedback for my class exercises

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

22. My teacher listens to my problems/complaints

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

23. My teacher gives me prompt feedback for my home work

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

24. My teacher encourages my parents to buy textbooks and other materials for me

Strongly Agree [ ]Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

25. My teacher helps me solve my academic problems

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

26. My teacher assists me solve my financial problems

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

27. My teacher teaches us how to dress well
Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

28. There is good pupil-teacher relationship in my class

Strongly Agree [ ] Agree [ ] Neutral [ ] Disagree [ ] Strongly Disagree [ ]

29. My teacher does not allow us to cheat in examination

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

30. I am comfortable with how my teacher discipline us in my class

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

31. My teacher checks on pupils who have been absent from class for a longtime

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

32. My teacher gives pupils progress report to their parents

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

Section C: Non-classroom Attitudes and Beliefs of teachers on pupils' performance

1. My school organizes play activities for us in the school

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

2. My school is concerned about my health

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

3. My school organizes weekly school worship programme for us S

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

4. I am counseled in this school when I have problem

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

5. My teacher sometimes pays unannounced visits to our home

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

6. My teacher does not smoke

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

7. My teacher does not drink alcohol

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

8. My teacher is very much respected by my parents and the community

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

9. My teacher dresses well to school

Strongly Agree [ ]   Agree [ ]   Neutral [ ]   Disagree [ ]   Strongly Disagree [ ]

SECTION D: Pupil's performance in class

Please, indicate the extent to which you have performed in English, Mathematics and Integrated Science.

1. How many tests have you done in English for this term?

   A. 1 – 2 ☐

   B. 3 - 4 ☐

   C. 5 and above ☐

2. How many tests have you done in Mathematics in this term?

   A. 1 – 2 ☐

   B. 3 - 4 ☐

   C. 5 and above ☐

3. How many tests have you done in Integrated Science in this term?

   A. 1 – 2 ☐

   B. 3 - 4 ☐

   C. 5 and above ☐

4. State the marks you obtained in English in this term.

Highest ………………… Lowest ………………

5. Are you happy with your performance in English in this term?

   A. Yes ☐

   No ☐

6. State the marks you obtained in Mathematics in this term.

Highest ………………… Lowest ………………

7. Are you satisfied with your performance in Mathematics in this term?

   A. Yes ☐

B. No ☐

8. What was your performance in Science in this term? …………………

9. Are you satisfied with your performance in Science in this term?

A. Yes ☐

B. No ☐

10. Which of the following subjects do you like most?

A. English ☐

B. Mathematics ☐

C. Integrated Science ☐

11. Give **one** reason for your answer in question 10 above?

……………………………………………………………………………………………

……………………………………………………………………………………………

12. What other subject do you like apart from English, Mathematics and Science?

…………………………………………………………………………………

13. Give **one** reason for your answer in question 12 above?

……………………………………………………………………………………………
…

14. Which of the following elective courses would you like to pursue in SHS?

A. Science ☐

B. Business ☐

C. General Arts ☐

15. Give **one** reason for your answer in question 14 above?

……………………………………………………………………………………………
.

16. What would you like to be in future?

……………………………………………………………………………………

APPENDIX B

ATTITUDES AND BELIEFS OF TEACHERS ON PUPILS' PERFORMANCE INBASIC SCHOOLS

QUESTIONNAIRE FOR TEACHERS

You are kindly requested to provide answers to each of the following statements in thisquestionnaire. Be as objective as possible. Your responses will be treated as confidential. **Section A: Background Information**

Please tick [√] where appropriate or write the required information in the spaces provided.

1. Gender: Male [ ] Female [ ]

2. Age: Below 26 [ ] 26-30 [ ] 31-35 [ ] 36-40 [ ] 41 and above [ ]

3. Highest level of education [ ]

Diploma

[ ] First Degree

[ ] Post graduate

Diploma [ ]

Masters

[ ] Others: Please, specify …………………………………………………...

4. How many years have you been teaching?

1	2	3	4	5
Below 5 [ ]	5-9 [ ]	10-14 [ ]	15-19 [ ]	20 and above [ ]

5. What category of a teacher are you?

Subject [ ] Please, specify the subject ………………….

Class teacher [ ] Please, specify the class ………………….

Section B: Classroom Attitudes and Beliefs of teachers on pupils' performance

Please, indicate to what extend you agree to each of the following statements using thefive-point Likert scale provided.

1 Strongly Agree (SA) 2 Agree (A) 3 Neutral (N) 4 Disagree (D) 5 Strongly Disagree (SD)

Please tick (√) your choice in the appropriate box

SN	Statement	SA 1	A 2	N 3	D 4	SD 5
1	I believe that each child has certain unique characteristics					
2	I use individual tutoring technique in my lesson					
3	I distribute my questions evenly in my class					
4	I respect gender equity when teaching					
5	I organize remedial lessons for low achievers					
6	I respect the contributions of both low and high achievers					
7	I use humour during lessons					
8	I correct students who break classroom rules					
9	I use various useful methods during lesson delivery					
10	I evaluate pupils' understanding of lesson at each teaching stage					
11	I use child learner centered approach in lesson delivery					
12	I engage pupils in lesson delivery					
13	I believe that the competency of the teacher is to improve pupils' performance					
14	I believe the teacher's use of varying motivational techniques during instruction can improve pupils' performance					
15	I do not feel bored with lessons					
16	I provide opportunities for pupils to question and criticize					

17	I provide Students with learning activities					
18	I am prepared to teach a special needs child in my classroom					
19	I have been trained to teach all kinds of pupils					
20	I like pupils with disabilities					
21	I understand pupils' peculiar problems					
22	My class has pupils with varying cultural backgrounds					
23	I provide social and academic counseling to my class pupils					
24	I make my class lively and interesting					
25	I give prompt feedback to pupils on their assessment					
26	I encourage pupils to learn always					
27	I do organize educational trip for my pupils					
28	I ensure that my pupils do not cheat in examination					

Section C: Non-classroom Attitudes and Beliefs of teachers on pupils' performance

SN	**Statement**	**SA 1**	**A 2**	**N 3**	**D 4**	**SD 5**
1	I am aware of various capacity building programmes					
2	I avail myself for professional development programmes					
3	I value cultural diversity among my pupils					
4	I embrace freedom of religion among my students					
5	I encourage my students to develop their talents					
6	Where possible, I visit parents/guardian of my students to get first-hand information					
7	I encourage my students to get involved in the various co-curricular activities					
8	I am concerned about the health of my pupils					
9	I take part in the weekly school worship programme					

10	I counsel my pupils who have peculiar problems					
11	I do not engage myself in partisan politics					
12	I treat pupils equally irrespective of their religion or religious belief					
13	I do not discriminate against pupils because of their tribe					
14	I give equal treatment to pupils irrespective of their parents' social status					

Section D: Pupils' performance in your subject area

Please, indicate the extent to which your pupils performed in your subject area.

1. How many tests have you conducted in your subject area for this term?

A. 1 – 2 ☐

B. 3 - 4 ☐

C. 5 and above ☐

2. What was the marks your pupils obtained.

Highest ………………… Lowest ………………

3. How many pupils obtained the following categories of marks in your subject area?

A. 0% – 25% ……………..

B. 26% – 50% …………

C. 51% – 75% …………..

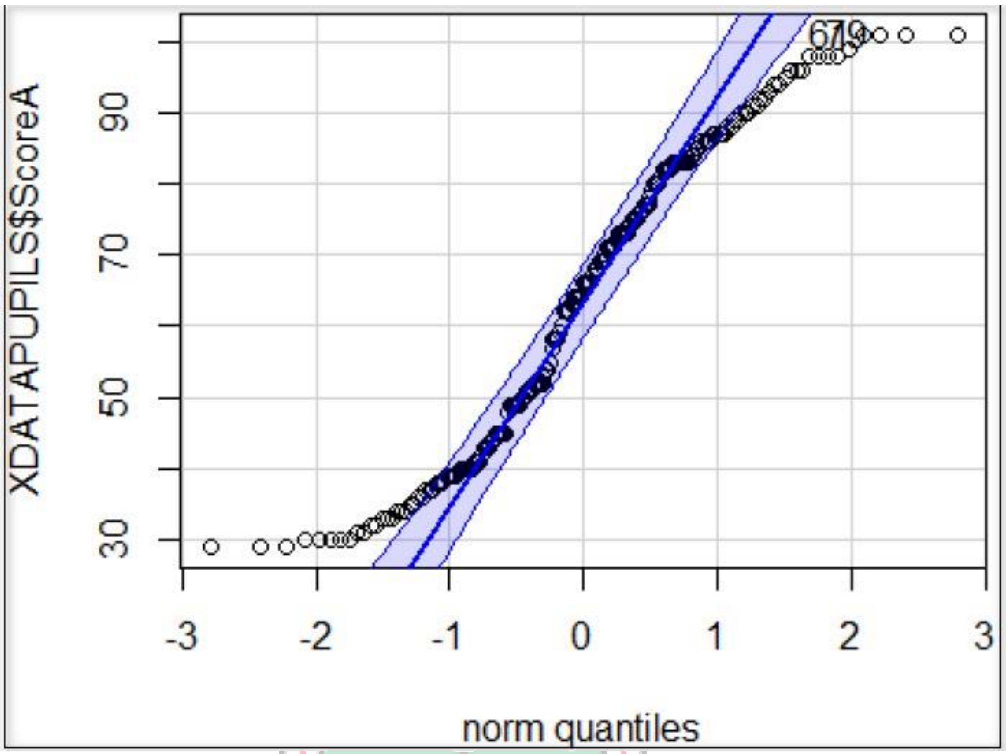
D. 76% – 100% ………

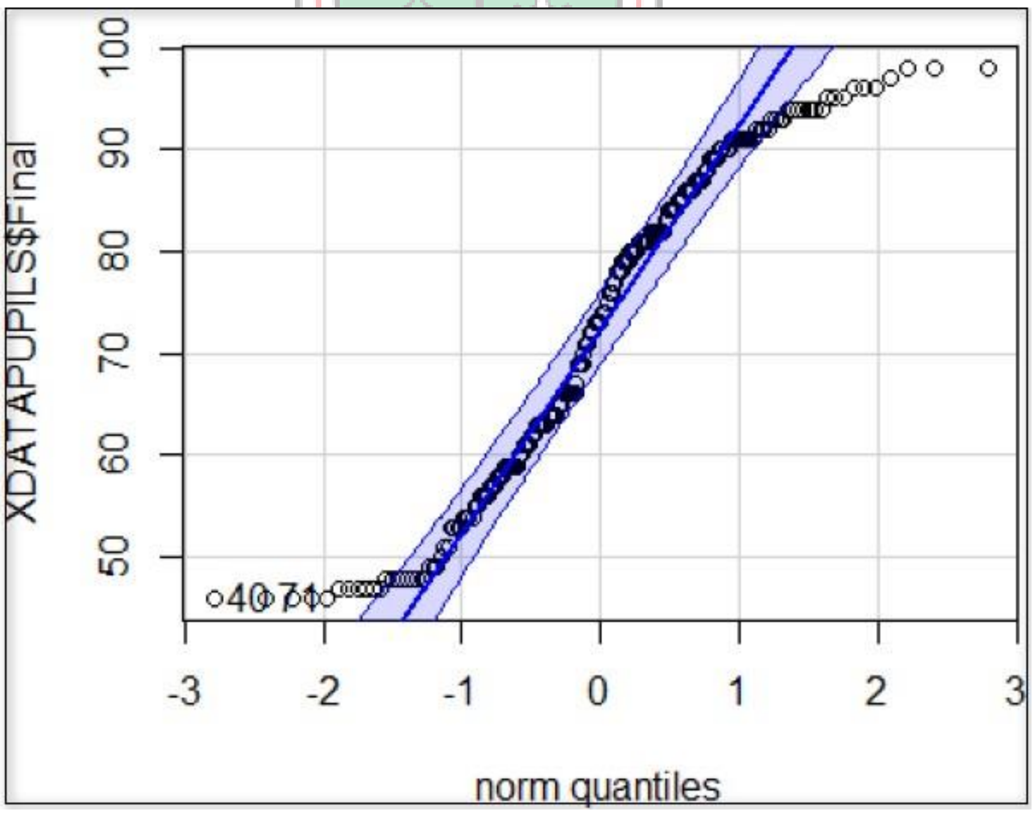
4. Which gender performs better in your subject area?

Male ☐ Female ☐

APPENDIX C

RANDOMLY DISTRIBUTED RANDOM EFFECTS ASSUMPTION FULFILLED

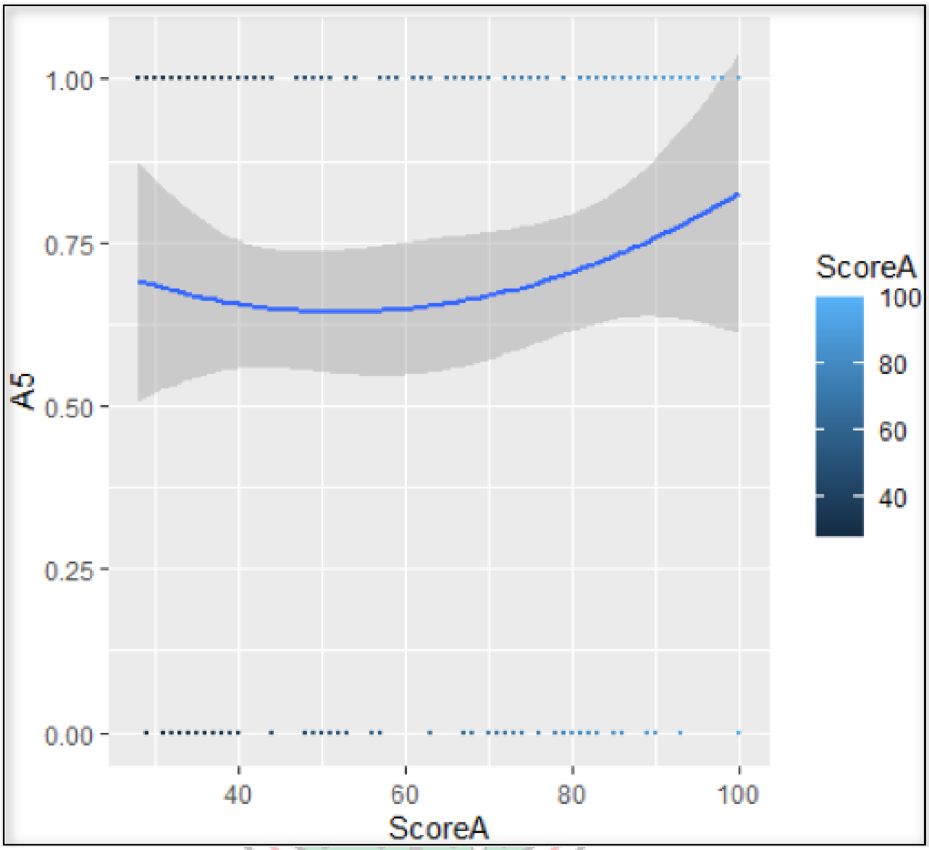


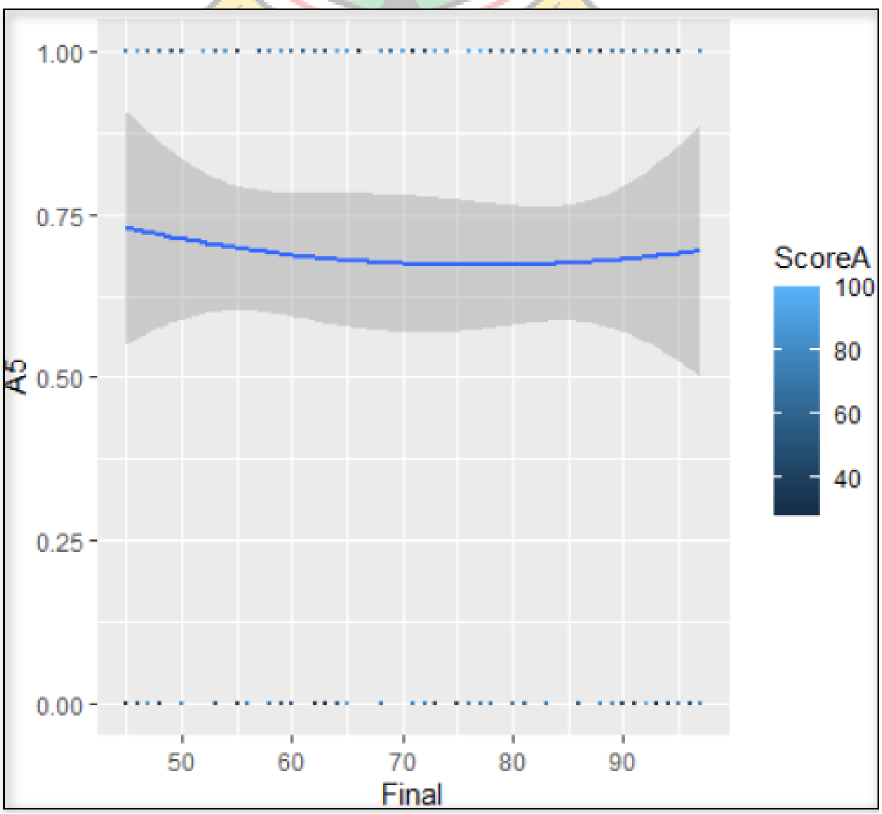


APPENDIX D

NONLINEARI

TY TEST





APPENDIX E

R CODES USED IN SIMULATION

ANOVA IN R

```
>Group1<-c(A1,A2,A3)

> Group2<-c(A4,A5,A6)

> Group3<-c(A7,B1,B2)

> Combined_Groups<-data.frame(cbind(Group1,Group2,Group3))

> Stacked_Groups<-stack(Combined_Groups)

> Stacked_Groups

> Anova_Results<-aov(values~ind,data=Stacked_Groups)

> summary(Anova_Results)

>plot(PUPILS_DATA_Maa_U_PhD$A3,PUPILS_DATA_Maa_U_PhD$A7,type
="n",xla b="Level",ylab="subject matter")

>text(PUPILS_DATA_Maa_U_PhD$A3,PUPILS_DATA_Maa_U_PhD$A7,sub
string(as.character(PUPILS_DATA_Maa_U_PhD$A4),1,1))

> g2<-lm(A5~B1*B2,PUPILS_DATA_Maa_U_PhD)

> summary(g2)

> g<-lm(A5~B1+B2,PUPILS_DATA_Maa_U_PhD)

> summary(g)

> anova(g2,g)
```

PURELY ANOVA

> Anova_Results<-aov(B1~A5,PUPILS_DATA_Maa_U_PhD)

> summary(Anova_Results)

Glm

>library (tidyverse)

>library(sjPlot)

>library(Ime4)

> library(haven)

> PUPILS_DATA_Maa_U_PhD <- read_sav("C:/Users/SENYEFIA BOSSON/Desktop/PUPILS DATA - Maa U PhD.sav")

> View(PUPILS_DATA_Maa_U_PhD)

> head(PUPILS_DATA_Maa_U_PhD)

> m=lm(y¬x1+x2. Data=df)

> n=glm(y¬x1+x2. Data=df, family=binomial)

Correlation

> library(GGally)

> corr <- data.frame(lapply(XXDATAPUPILS, as.integer)) ggcorr(corr,method = c("pairwise", "spearman"),nbreaks = 6,hjust = 0.8,label =TRUE,label_size = 3,color = "grey50")

Build the model

formula <- A5~.

```
> logit <- glm(formula, data = data_train, family = 'binomial')

<summary(logit)

> XDATAPUPILS$ScoreA<-XDATAPUPILS$ScoreA +1

> qqp(XDATAPUPILS$ScoreA

,"norm")[1] 61 79

> XDATAPUPILS$Final<-XDATAPUPILS$Final +1

> qqp(XDATAPUPILS$Final,"norm")

> str(XDATAPUPILS)

>head(XDATAPUPILS)

> library(lme4)

> lmm <- lmer(A1 ~ Final+(1 | ScoreA), data = XDATAPUPILS,REML = FALSE)

> summary(lmm)

> lmm1 <- lmer(A1 ~ Final+A2+(1 | ScoreA), data = XDATAPUPILS,REML = FALSE)

> summary(lmm1

> PQL  <-  glmmPQL(A5~  Final+  A1,  ~1  |  ScoreA/A5,  family  =
gaussian(link ="log"),data = XDATAPUPILS, verbose = FALSE)

> summary(PQL)

> PQL8  <-  glmmPQL(A5~  Final+  B2,  ~1  |  ScoreA/A5,  family  =
binomial(link ="logit"),data = XXDATAPUPILS, verbose = FALSE)
```

> summar

y(PQL8)

CODES

LEVEL 2

>mod6 = glmer(cbind(A5, Final) ~ ScoreA+A1+A3+A7+(1|A5), data = XXDATAPUPILS,family = binomial(link = "logit"))

> summary(mod6)

Model Comparison

> anova(mo

d1,mod2)

CODES 2

> XXDATAPUPILS <- read_sav("G:/XXDATAPUPILS.sav")

> View(XXDATAPUPILS)

> library(dplyr)

> glimpse(XXDATAPUPILS)

> continuous <-select_if(XXDATAPUPILS, is.numeric)

> summary(continuous)

> library(ggplot2)

> ggplot(continuous, aes(x = A5)) +geom_density(alpha = .2, fill = "#FF6666")

The Model

> **Model =1mer(A5** ~ fixed 1 +fixed 2 +fixed3 +fixed 4 +fixed 5 +(1|random),data=DATAPUPILS)

> family = binomial(link = "logit")

> Modelx= glmer(cbind(A5, Final) ~ fixed 1 +fixed 2 +fixed3 +fixed 4 +fixed 5+(1|random), data = XXDATAPUPILS,family = binomial(link = "logit"

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An Extended Six-parameter Generalised Linear Mixed Effects Model for Achievement Gaps among Public and Private Basic Schools of Ghana

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Original Research Article

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Abstract

The mixed generalized linear model extension has distinct advantages over generalised linear models and hierarchical linear models by reducing estimation and precision errors, and increasing power. This paper aimed at building a six parameter Generalised Mixed linear Model with five structural parameters that explain students, teachers, and administrative-logistic characteristics and interaction effects as well as random effect parameter. Our model, which has Bernoulli response, and Logit link function outperformed existing models and was robust across varied sample sizes on the basis of AIC and BIC. Parameters of the model were

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Neutrosophic-principal Component Analysis of Causes of Performance Gap among Private and Public School Students in the Basic Education Certificate Examination

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Original Research Article

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Abstract

The decade - old academic achievement discrepancies at the basic school level that are widely emphasized are still being worked on heavily in current research. In this study, we present a novel Neutrosophic-Principal Component Analysis and Two-way Neutrosophic ANOVA to analysing the causes of Performance Gap among private and public school students in the Basic Education Certificate Examination. One-hundred and eighty-nine respondents from the Ada East and West Districts of Ghana were involved in the study. We present a modified Neutrosophic regression equivalence of the classical least squares to solve problems of indeterminacy. The results showed that of the total 87% variability, three main components—student characteristics, instructor characteristics, and administrative-logistic features—accounted for 36%, 31%, and 20%, respectively. The remaining 13% of the variability, which was attributed to random effects by the Neutrosophic PCA technique, was tested using a two-way Neutrosophic ANOVA where the results identified that interaction between the identified factors was a contributing factor that needed to be explained. Our results reveal that Neutrosophic-PCA is a potential method to lessen human response errors, which are frequently tainted with ambiguous, conflicting, imprecise, indeterminate or uncertainty. When there is just a slight difference between the two options, the forced selection caused by the traditional Likert method will be

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