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A THESIS REPORT ENTITLED

A HYBRID B.SPLINE-GARCH (BSGARCH) MODEL FOR STOCK MARKET VOLATILITY

BY

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DECLARATION

I declare that this thesis is my own work. It is being submitted for the degree of Doctor of Philosophy in the University of Mines and Technology (UMaT), Tarkwa. It has not been submitted for any degree or examination in any other University.

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(Signature of candidate)

.....24...... day of ....October........... (year) ....2023....

ABSTRACT

This study aimed to propose the BSGARCH (1,1) model, a hybrid Basis Spline GARCH-type model suitable for modelling the volatility of financial time series. The proposed model was compared with other classical GARCH-type models such as GARCH, EGARCH, GJR-GARCH, and APARCH models, and it was found to outperform them in terms of predictive accuracy, as measured by RMSE, MAPE, TIC, and QLIKE. The results showed that the BSGARCH (1,1) model had a superior predictive ability compared to the other models. The study also compared the performance of the BSGARCH (1,1) model with the Spline-GARCH model of Engle and Rangel, which used the exponential quadratic spline to model the non-stationary part of volatility. In this comparison, the Spline- GARCH model slightly outperformed the proposed BSGARCH (1,1) model. However, the difference in performance was negligible, and thus, the proposed BSGARCH (1,1) model can be considered a good alternative to the Spline-GARCH model. The study demonstrates that the proposed BSGARCH (1,1) model is a useful tool for modelling non-linear and non-stationary financial time series data, with superior predictive ability compared to other classical GARCH-type models.

DEDICATION

To Marjorie

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I would like to express my deep appreciation to my supervisors Assoc Nana Kena Frempong and Dr Eric Neebo Wiah and all the lecturers in the Department of Mathematical Sciences, for their irreplaceable guidance and support during my thesis work. Their wealth of knowledge, expertise, and insight has been a inspired me greatly, and their unwavering commitment to my success has been an immense encouragement to me. Precisely, I would like to thank Dr B. Odoi and Dr P. Boye for their continuous advice and encouragement, which have been crucial to my success.

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CHAPTER 1

INTRODUCTION

1.1 Overview

This chapter gives an overview of the study by presenting the background information, stating the problem, defining the objectives, and outlining the structure of the thesis. It serves as a foundation for the subsequent chapters and provides the reader with a clear understanding of the context and scope of the research.

1.2 Background of study

The stock market serves as a public platform where stocks are issued, bought, and sold, either through a stock exchange or over-the-counter. Stocks, also known as equities, represent partial ownership in a company, and investors engage in trading these assets within the stock market (Gregoriou, 2009). The efficient functioning of the stock market is crucial for economic development, as it enables companies to access capital from the public quickly. Over time, the occurrence of various market crashes and crises has prompted practitioners, researchers, and regulators to shift their focus from traditional financial economics, which primarily examined the mean of stock market returns, to exploring the volatility levels and stationarity of stock prices. This shift has led to the development of econometric tools that better capture and model stock market volatility (Matei, 2012).

Stock market volatility, which refers to the rate of price fluctuations in the stock market over a given time period, has been a subject of intensive research in time series econometrics for several decades. This is due to its significant role in various activities, ranging from portfolio allocation to risk management's density forecasting (Brooks, 2008). The concept of volatility has been defined in different ways, reflecting its relevance in various contexts. In a broader sense, volatility can be described as a series of fluctuations that characterize a phenomenon over a specific period of time. However, in the realm of financial economics, volatility is more narrowly defined. According to Andersen *et al.* (2010), it represents the instantaneous standard deviation of a random component driven by Wiener process in a continuous-time diffusion model.

For all investors concerned with risk-adjusted returns, volatility modelling and forecasting has significant consequences, especially for those who use asset allocation, risk parity, and volatility targeting techniques (Dufitinema, 2021; Henriksen, 2011). Quantifying the possible loss of assets is a key component of risk management, asset allocation, and trading on financial markets. It is a crucial component of many investment choices and portfolio constructions. Assessing investment risk can be started with a good estimate of the volatility of asset prices across the investment period (Dufitinema, 2021).

Since the first Basel Accord was founded in 1996, financial risk management has been a key component of investment. This effectively turns volatility forecasting into a requirement for risk management for many financial institutions globally (Chang *et al.*, 2011). Volatility in the financial markets can have significant effects on a nation's economy. Recent financial reporting scandals in the US have wreaked havoc on global financial markets and had a detrimental effect on the global economy. This demonstrates the critical connection between financial market volatility and public confidence. As a result, policymakers frequently utilize market estimates of volatility as a measure of the vulnerability of the economy and financial markets (Ajao and Wemambu, 2012).

There are various statistical and econometric techniques that can be used to calculate stock market volatility. The Random Walk model is the most basic model based on historical price. The Historical Average (HA), Simple Moving Average (MA), Exponential Smoothing and Exponentially Weighted Moving Average (EWMA) methods are extensions of the Random Walk model. These estimators and models assume that volatility is constant. Traditional estimates of unconditional volatility include the standard deviation, close-to-close volatility, extreme value estimator of Parkinson (1980) and Garman and Klass (1980) but these models do not take into account the financial market's time-varying features.

There are other complimentary time series models such as those belonging to the ARCH family, that are vastly used by researchers. Many academics and practitioners have utilized the ARCH and GARCH models developed by (Engle, 1982) and (Bollerslev, 1986) respectively to estimate the stock market's volatility. Conditional volatility, which integrates the time-varying characteristics of the financial market, was developed as a result of these investigations on the ARCH model. These models have helped in understanding the empirical characteristics of many financial time series' volatility, including fat tails, leverage effects, and clustering (Cont, 2001 and Gregoriou, 2009). Forecasting stock market volatility is a challenging task, and despite the existence of numerous models and techniques, their performance can vary across different stock markets (Bhowmik and Wang, 2020). This complexity in market returns and volatility forecasting poses a significant challenge for researchers and financial analysts. The dynamic and unpredictable nature of stock markets, influenced by various factors such as economic conditions, investor sentiment, and geopolitical events, adds to the difficulty of accurately forecasting volatility. As a result, ongoing research and advancements in econometric modelling and statistical techniques are necessary to improve the accuracy and reliability of volatility forecasts in different stock markets. Consequently, a flexible model based on the hybrid of GARCH(1,1) and B-spline expansion is proposed in this thesis for stock market volatility.

The use of B-splines for function approximation has been mathematically validated. See, for instance, de Boor (2001). In fact, B-splines represent a piece-wise polynomial function of interest, making the model simple to interpret. The admissibility of B-splines stems from their ability to capture complex patterns, maintain smoothness, and provide computational efficiency. These properties make B-splines a popular choice in various fields. The suggested model has the advantage of being computationally practical, despite the fact that there may be a vast number of parameters to be considered. Real and simulated data are used to confirm the goodness of the model in terms of accuracy in predicting volatility. The proposed BSGARCH (1,1) model is compared with APARCH(1,1), EGARCH(1,1), GARCH (1,1), and the GJR-GARCH (1,1) model.

1.3 Problem Statement

Stock market volatility modelling and forecasting has attracted a lot of attention from scholars and researchers in financial markets because of the crucial function it has in most financial applications in real life (Audrino and Bühlmann, 2009). The forecast of expected returns and asset volatility heavily influences investment choices in the stock market (Dufitinema, 2021; Henriksen, 2011). Volatility is a measure of price fluctuations in relation to an expected value for a financial security (Dash and Dash, 2016).

Various time series models have been developed for financial data that has time varying volatility, however, all these models still have many flaws (Bhowmik and Wang, 2020). The first breakthrough in volatility modelling was made by Engle (1982) with the development of the ARCH model. This was a significant advancement as it provided a framework for considering the volatility clustering observed in financial time series.

Following Engle's work, numerous studies recognized the potential of the ARCH model and applied it to model various financial time series. However, it was noted by (Rydberg, 2000) that ARCH models often require large lag values, leading to a higher number of parameters. This can pose challenges in estimation and interpretation, especially when dealing with complex and high-frequency data. Despite the need for many parameters, the ARCH model's ability to capture time-varying volatility patterns made it a valuable tool in financial econometrics. Subsequent research has built upon Engle's original work to develop more sophisticated and efficient models that address the shortcoming of the ARCH model while still capturing the essential characteristics of financial volatility.

In their independent studies, Taylor (1986) and Bollerslev (1986) developed the GARCH model as an extension to the ARCH model. The GARCH model incorporates an Autoregressive Moving Average (ARMA) formulation to achieve greater simplicity in modelling. It models the conditional variance as a function of its lagged values and the squared lagged values of the innovation term. Since its inception, the GARCH family of models have proven to be effective in considering various stylized facts observed in financial time series such that Hansen and Lunde (2005) noted that it is difficult to obtain a model that outperforms the GARCH (1,1) model in volatility forecasting. The capacity of the GARCH models and its extensions to capture the stylized facts such as volatility clustering and leverage effect has led to their widespread application in the field of financial econometrics and provide valuable insights into the dynamics of financial markets. They are instrumental in risk management, derivative pricing, and forecasting volatility, aiding investors, traders, and financial institutions in making informed decisions (Atoi, 2014; Koima *et al.*, 2015; Hong and Lee, 2017 and Korkpoe and Junior, 2018). The GARCH model has shown to be effective at determining the symmetric effect of volatility, but it has several drawbacks, such as a violation of the non-negativity requirements placed on the parameters to be evaluated and its inability to capture the asymmetry in volatility (Bildirici and Ersin, 2014).

To address these limitations, several extensions to the standard GARCH models have been suggested, such as the EGARCH by Nelson (1991), threshold GARCH

(TGARCH) by Zakoian (1994) and GJR-GARCH by Glosten *et al.* (1993) to accommodate the asymmetric nature of volatility. The TGARCH and GJR-GARCH are closely related.

The EGARCH model was introduced to overcome three key deficiencies of the GARCH model. These deficiencies include parameter restrictions that guarantee positive conditional variance, limited sensitivity to the asymmetric response of volatility to shocks, and challenges in evaluating persistence in strongly stationary series Ekong and Onye (2017). Malmsten and Teräsvirta (2010) argued that the first-order EGARCH model with normal errors may not have sufficient flexibility to capture the autocorrelation and kurtosis present in stock returns. To address this limitation, they proposed improving the GARCH model by changing the normal error distribution with a fat-tailed distribution. By raising the kurtosis of the error distribution, the improved GARCH model would be better equipped to model the low autocorrelation and kurtosis observed in stock returns. Nelson (1991) noted that using a student-t distribution could result in infinite unconditional variance for the errors. Therefore, employing an error distribution with heavier tails than the normal distribution can effectively increase kurtosis and reduce autocorrelation in squared observations. For their proposed model, Malmsten and Teräsvirta (2010) recommended using a Generalized Error Distribution (GED) in the EGARCH model. They found that if the innovation follows a GED, the EGARCH model becomes stationary.

Zakoian (1994) specified the TGARCH model by allowing the conditional standard deviation to depend on sign of lagged innovation. The specification does not show parameter restrictions to guarantee the positivity of the conditional variance. However, to ensure stationarity of the TGARCH model, the parameters of the model must be restricted and the choice of error distribution account for the stationarity. Thus in the classical GARCH-type models, the choice of error distribution play a significant role in their ability to forecast volatility.

Numerous studies have attempted to addressed the limitations of classical GARCH models by proposing alternative approaches. For instance, Catania and Proietti (2020) introduced a measurement model that incorporates time-varying connection between asset returns and realized volatility using a bivariate framework. This model captures key features such as heavy-tailed return distributions, long-term memory of volatility, and negative dependence between daily market returns and volatility. Another approach to address persistence in volatility is to assume a "smoothly" non-stationary volatility process and model it appropriately (Amado and Teräsvirta, 2013). Dahlhaus and Rao (2006) proposed a time-varying ARCH process to model non-stationary volatility, while Engle and Rangel (2008) decomposed the variance of a financial time series into non-stationary and stationary parts using exponential quadratic splines within a multiplicative decomposition structure. Similarly, Mishra *et al.* (2010) employed a multiplicative decomposition framework to fix potential misspecification in a parametric GARCH model by incorporating a smooth non-parametric component. Additionally, Amado and Teräsvirta (2013) proposed two non-stationary GARCH models to handle situations where volatility are non-stationary, including a multiplicative decomposition approach and an additive time-varying parameter model (with a focus on the former).

In addressing the limitations of individual models, the proposal of hybrid models has emerged as an efficient alternative for modelling and forecasting stock market volatility (Dash and Dash, 2016). Hybrid models integrate first principle-based models with data-based models, integrating their strengths to enhance model quality, robustness, and interpretability (Kurz *et al.*, 2022). In this context, a hybrid model that integrate the B-Spline approach with the GARCH model is proposed. B-Splines, consisting of connected polynomial curves known as knots, represent piecewise polynomial functions capable of approximating any function of interest. The piecewise nature of

B-Splines allows for the interpretation of threshold-regime functions, where different regions of the predictor correspond to different regimes. By leveraging the strengths of B-Splines and GARCH, the proposed hybrid model offers a promising solution for accurately modelling and forecasting stock market volatility.

1.4 Reseaerch Objectives

The objectives of the research are to:

i. conduct a review of existing GARCH-type models, including APARCH, GJR-GARCH GARCH, and EGARCH under different error distributions.

ii. propose a hybrid BSGARCH model to forecast the volatilities of financial time series.

iii. evaluate the performance of the hybrid BSGARCH model on simulated time series and compare it with the performance of the other classical GARCH-type models in (i) using RMSE, MAPE, TIC and QLIKE performance metric.

iv. assess the performance of the hybrid BSGARCH model on real time series and compare it with the performance of the other classical GARCH-type models in (i) above using RMSE, MAPE, TIC and QLIKE performance metric.

1.5 Facilities and Resources Used for the Research

These facilities were used for this thesis:

i. the library and internet facilities at the University of Mines and Technology (UMaT), Tarkwa;

ii. the R-software and Microsoft Excel

iii. Federal Reserve Economic Data (FRED) data repository

iv. the website of the Ghana Stock Exchange

1.6 Contribution To Science and Knowledge

This study adds to the body of literature on financial time series modelling by proposing a hybrid model called BSGARCH(1,1), which integrates the GARCH (1,1) and the basis spline (B-Spline). The integration of the GARCH model and B-Spline makes the model flexible enough to capture volatility clustering and asymmetry very well which serves as an improvement on the GARCH model. The model can be used to analyse and forecast the volatility of financial asset returns, which is a critical component of option pricing, risk management and portfolio management. Additionally, this study adds to the literature on volatility modelling by offering a comparison of the BSGARCH(1,1) model's forecasting performance with other widely used models in the literature.

1.7 Structure of Thesis

The thesis is structured into six chapters, with each chapter contributing to the research and analysis of the proposed model. Chapter 1 serves as the introductory chapter where the problem statement and research objectives are presented. In Chapter 2, the existing literature on the topic is reviewed to provide a comprehensive understanding of the research area. Chapter 3 provides the theoretical background of the methods used, along with some preliminary information. Chapter 4 is dedicated to the formulation of the proposed model. The application of the proposed model on both real and simulated data is presented in Chapter 5, and a comparison with other models is made. Finally, in Chapter 6, conclusions are made from the findings and recommendations are given for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter presents an extensive review of the theoretical and empirical literature on volatility modelling and forecasting. The literature review is essential in providing a background for the current study by highlighting the main developments, limitations, and gaps in existing studies. The chapter begins with a general overview of the concept of volatility, its importance in financial modelling, and the various models developed to capture its dynamics.

2.2 Stock Market

It is a vital component of any economy, serving as a barometer of economic performance and a source of financing for businesses. In general, a stock market is a type of financial market where buyers and sellers exchange business stocks and other listed securities (Omar, 2012). Stock markets, also known as share markets or equity markets, represent a collective platform where buyers and sellers come together to trade stocks. Unlike being a physical location or discrete entity, stock markets serve as an aggregation of individuals and institutions engaged in the selling and buying of stocks. They are essential in facilitating the exchange of shares, bonds, and other financial instruments.

A stock exchange, also referred to as a securities exchange or bourse, is the formal infrastructure that enables the buying and selling of shares, bonds, and other financial instruments Musonera and Safari (2008). It provides a regulated environment and necessary mechanisms for investors to engage in trading activities. To be traded on a stock exchange, a security must be listed on a major exchange, meeting specific listing requirements and adhering to regulatory standards.

The system of stock markets comprises multiple stock exchanges and their interrelationships. These exchanges, operating within a country or across international borders, form a network that facilitates the trading of securities. The interconnectivity and cooperation among different stock exchanges contribute to the overall functioning and efficiency of the stock market system (Rjumohan, 2019).

Stock markets are crucial and an indispensable part of the economy of a country. They serve as a vital source of capital for companies, enabling them to raise funds through the issuance of stocks or bonds. Stock markets provide investment opportunities for individuals and institutions, promoting wealth creation and economic growth. The stability and proper functioning of stock markets are crucial for fostering investor confidence and supporting the overall financial system of a country.

2.3 Volatility of The Stock Market

Volatility is an essential characteristic of the stock market, and is a critical aspect of stock market behavior, as it affects the decisions of investors and traders, influences the cost of capital, and reflects how well the economy is doing. Stock market volatility is the measure of variability in prices of stocks in the stock markets over a period (Rahmani, 2016 and Cohen and Tegnér, 2019). It is crucial for both theoretical and real-world financial applications (Hong and Lee, 2017 and Korkpoe and Junior, 2018 and Atoi, 2014) such that it is implemented by banks and other financial institutions to measure exposure to risk Engle and Patton (2001). The availability of high frequency data adds a new dimension to the modelling of volatility and the forecasting of financial asset returns, and researchers have concentrated on this area(Andersen and Bollerslev, 1998 ; Hansen and Lunde, 2005; Andersen *et al.*, 2010 and (Pypko, 2015)).

The volatility of stocks plays a significant role in the economy, particularly through its impact on the stock markets. Changes in stock prices can have various effects on the overall economic conditions. For example, when stock prices increase, it stimulates investment activities and creates a higher need for credit, which can results in higher interest rates in the economy. Consequently, it is imperative to specify appropriate volatility models that can capture variations in stock market returns, as it directly affects the economy as a whole (Atoi, 2014)

Furthermore, reliable volatility models for stock market returns are essential for investors in making informed decisions regarding risk management and portfolio adjustments. These models assist in understanding and predicting the behavior of stock market volatility, allowing investors to mitigate risks and optimize their investment strategies. According to Engle (1982), a viable volatility model should effectively capture heteroscedasticity in the innovation term and reflect key stylized facts hidden in stock market return series, such as ARCH effect, clustering of volatility and asymmetry.

In literature, the extensively used volatility models are the ARCH model and its extensions, including the GARCH, EGARCH, and GJR-GARCH models. Among these, first-order GARCH models have been comprehensively studied and proven to be feasible for modelling and forecasting financial time series (Bera and Hiigins, 1993; Goudarzi and Ramanarayanan, 2011; Olowe, 2011). These models provide valuable insights into the dynamics of volatility and have been widely employed in empirical studies and practical applications.

2.4 Volatility Models

The first breakthrough in volatility modelling was made by Engle (1982) with the development of the ARCH model. This was a significant advancement as it provided a framework for considering the volatility clustering observed in financial time series.

Following Engle's work, numerous studies recognized the potential of the ARCH model and applied it to model various financial time series. However, it was noted by (Rydberg, 2000) that ARCH models often require large lag values, leading to a higher number of parameters. This can pose challenges in estimation and interpretation, especially when dealing with complex and high-frequency data. Despite the need for many parameters, the ARCH model's ability to capture time-varying volatility patterns made it a valuable tool in financial econometrics. Subsequent research has built upon Engle's original work to develop more sophisticated and efficient models that address the shortcoming of the ARCH model while still capturing the essential characteristics of financial volatility.

In their independent studies, Taylor (1986) and Bollerslev (1986) developed the GARCH model as an extension to the ARCH model. The GARCH model incorporates an Autoregressive Moving Average (ARMA) formulation to achieve greater simplicity in modelling. It models the conditional variance as a function of its lagged values and the squared lagged values of the innovation term. Since its inception, the GARCH family of models have proven to be more effective in considering various stylized facts observed in financial time series. These stylized facts include volatility clustering, risk premium, shock persistence, the leverage effect and mean reversion among other important characteristics of financial time series.

The capacity of the GARCH models and its extensions to capture these stylized facts has led to their widespread application in the field of financial econometrics and provide valuable insights into the dynamics of financial markets. They are instrumental in risk management, derivative pricing, and forecasting volatility, aiding investors, traders, and financial institutions in making informed decisions (Atoi, 2014; Koima *et al.*, 2015; Hong and Lee, 2017 and Korkpoe and Junior, 2018). The GARCH model has shown to be effective at determining the symmetric effect of volatility, but it has several drawbacks, such as a violation of the non-negativity requirements placed on the parameters to be evaluated.

To address these limitations, several extensions to the standard GARCH models have been suggested, such as the EGARCH by Nelson (1991), threshold GARCH (TGARCH) by Zakoian (1994) and GJR-GARCH by Glosten *et al.* (1993) to accommodate the asymmetric nature of volatility. The TGARCH and GJR-GARCH are closely related.

The EGARCH model was introduced to overcome three key deficiencies of the GARCH model. These deficiencies include parameter restrictions that guarantee positive conditional variance, limited sensitivity to the asymmetric response of volatility to shocks, and challenges in evaluating persistence in strongly stationary series Ekong and Onye (2017). In the EGARCH model, the logarithm of the conditional variance is used to represent the leverage effect. This logarithmic transformation signifies that the leverage effect is exponential instead of quadratic, as in the symmetric GARCH model. By expressing volatility in its logarithmic transformation, the EGARCH model avoids restrictions on the parameters to ensure the variance is positive. This is a major benefit of the EGARCH model over the symmetric GARCH model.

The TGARCH (Threshold GARCH) model, introduced by Zakoian (1994), incorporates the conditional standard deviation of the innovation term based on the sign of the lagged innovation. As a result, volatility can react differently to positive and negative shocks. Unlike some other models, the TGARCH specification fails to impose parameter limitations to ensure the conditional variance is positive. Additionally, the error distribution chosen for the TGARCH model also plays a role in maintaining stationarity. The GJR-GARCH model developed by Glosten *et al.* (1993) is closely related to he TGARCH model. Both models address the issue of asymmetric responses in volatility by incorporating the impact of negative and positive shocks differently.

In their study, Ding *et al.* (1993) introduced another version of the standard GARCH model, known as the Power GARCH (PGARCH) model. The PGARCH model extends the GARCH framework by raising the conditional standard deviation to a power, denoted as d, where d is a positive exponent. This power term is then related to a function of the lagged conditional standard deviations and the lagged absolute innovations, both raised to the same power. By allowing for the exponent d to vary, the PGARCH model offers increased flexibility compared to the standard GARCH model. This means that different values of d can be used to determine various patterns and characteristics of volatility in the data. When d is set to two, the PGARCH model simplifies to the standard GARCH model. Therefore, the provision for switching the power exponent allows the PGARCH model to adapt to different volatility dynamics and offers enhanced modelling capabilities.

In addition to the standard GARCH models, a GARCH-in-mean parameterization was developed by Engle *et al.* (1987). This parameterization formalizes the concept that risk is priced by the market and that risk premia changes with volatility. It suggests that the conditional mean of asset returns can be influenced by the conditional volatility, thereby incorporating the relation between return and risk in the model. When analyzing high-frequency data, it is observed that volatility varies slowly and shocks take a long time to decay, exhibiting a long memory property. To capture this behavior, Ballie *et al.* (1996) developed the Fractionally Integrated GARCH (FIGARCH) model. The FIGARCH model introduces fractional integration to the GARCH framework, allowing for the hyperbolic decay of shocks rather than the exponential decay observed in traditional GARCH models. By incorporating long memory into the volatility dynamics, the FIGARCH model provides a more accurate representation of the persistence and decay patterns observed in financial time series.

High frequency series, like stock returns, are well recognized for various stylized features, such as volatility clustering, fat-tail distribution, and asymmetry. Therefore the conventional assumption of normality in volatility modelling of financial time series could reduce the robustness of parameter estimates. Mandelbrot (1963) and Fama (1965) infer that daily stock index returns are not normal and tend to have fat-tailed distribution. Due to this, Bollerslev (1986) relaxed the conventional assumption of normality to include time varying volatility in high frequency data by assuming that such data follows student t-distribution. Furthermore, Bollerslev *et al.* (1994) established that kurtosis and slowly decaying autocorrelations in return series cannot be well explained by the GARCH model with normally distributed errors.

In the field of stochastic volatility modelling, White (2006) proposed a likelihood-based joint estimation and specification testing process that addresses the challenges associated with existing estimators. The key innovation introduced by the authors is the use of a Discrete Nonlinear Filtering (DNF) algorithm. The DNF algorithm enables estimation and specification testing by applying the nonlinear filtering set of equations, which are commonly used in the analysis of nonlinear latent variable problems including stochastic volatility models. The algorithm treats the continuously valued state variable as if it were a discrete Markov variable with a large number of states. This approach allows for a quick and accurate application of the nonlinear filtering equations. By leveraging the DNF algorithm, White (2006) provided maximum likelihood estimates for the general class of nonlinear latent variable problems, specifically focusing on stochastic volatility models. This advancement in estimation and specification testing procedures improves the operational efficiency and accuracy of stochastic volatility modelling, offering researchers and practitioners a valuable tool for analyzing and understanding financial time series data.

Durham (2007) investigated the concept of modelling the shape of the conditional distribution of stochastic volatility using a discrete mixture of normals. This approach offers flexibility in capturing the characteristics of the tails of the returns distribution, providing insights into extreme events and outliers. The proposed model, called SV-mix, was subjected to thorough model diagnostics to assess its performance. The results indicated that SV-mix effectively captured the essential features of the data, suggesting its suitability for modelling stochastic volatility. To compare the performance of SV-mix against other models, several affine-jump models were considered. The evaluation criteria used for comparison were the AIC and SIC. The results of the comparison strongly favored SV-mix, indicating its superior performance in terms of model fit and complexity.

Similarly, Malmsten and Teräsvirta (2010) argued that the first-order EGARCH model with normal errors may not have sufficient flexibility to capture the autocorrelation and kurtosis present in stock returns. To address this limitation, they proposed improving the GARCH model by changing the normal error distribution with a fat-tailed distribution. By raising the kurtosis of the error distribution, the enhanced GARCH model would be better equipped to model the low autocorrelation and kurtosis observed in stock returns. Nelson (1991) previously noted that using a student-t distribution could result in infinite unconditional variance for the errors. Therefore, employing an error distribution with heavier tails than the normal distribution can effectively increase kurtosis and reduce autocorrelation in squared observations. For their proposed model, Malmsten and Teräsvirta (2010) recommended using a Generalized Error Distribution (GED) in the EGARCH model. They found that if the innovation follows a GED, the EGARCH model becomes stationary. By incorporating the GED, the desired features of stock return series, such as kurtosis and autocorrelation can be modelled more effectively.

The idea of implied volatility is widely employed in options pricing and trading. It represents the volatility level implied by the market prices of options. However, in stochastic volatility, traditional methods for calculating implied volatility may not be directly applicable. To address this, Henry-Labordère (2005) introduced a novel approach based on differential geometry and heat kernel techniques. They developed the problem in the framework of a Riemann manifold, which is a mathematical structure used to describe curved spaces. By considering an Abelian connection on this manifold, they were able to derive a general asymptotic expression for implied volatility.

Feunou and Tédongap (2012), developed a discrete time affine stochastic volatility model that has time varying conditional skewness in a unified manner. Their approach allowed current asset returns to be asymmetric conditional on current factors and past information. Analytical formulas were derived for different return moments that are utilized in generalized moments methods (GMM) estimation. Wu (2012), proposed a TGARCH model to define the regime switching in volatility dynamics of assets of financial returns. According to his threshold model, volatility in each regime followed a GARCH process, and the switching between regimes was initiated by an observable threshold variable. Again, theoretical conditions were established to guarantee that the return process in the threshold model was strictly stationary as well as conditions for the finite variance and fourth moment. The finite sample characteristics of the maximum likelihood estimator were further investigated using a simulation study. His proposed model was applied to an empirical data and the findings supported the use of threshold variable to recognize the regime shifts in the volatility processes. Akyildirim *et al.* (2014) developed a general method to reconstruct recombinant tree approximations for stochastic volatility models and implemented on the Heston model for the dynamics of stock price.

Vrontos *et al.* (2000) introduced a Bayesian approach to the analysis of GARCH and EGARCH models. They developed a comprehensive framework that encompassed parameter estimation, model selection, and volatility prediction within the Bayesian paradigm. To perform Bayesian analysis, Vrontos *et al.* (2000) employed Markov-chain

Monte Carlo (MCMC) methods, which are computational techniques for obtaining posterior distributions of model parameters. By utilizing MCMC methodologies, they were able to capture the full uncertainty in parameter estimates and make probabilistic inferences. The authors demonstrated the implementation of their Bayesian framework using data from the General Index of the Athens stock exchange. They provided detailed implementation guidelines and illustrated the results of their analysis. Through this empirical application, they showcased the practicality and effectiveness of their proposed approach in real-world financial data.

As opposed to the popular Bayesian methods, Turatti (2018) developed a classical maximum likelihood estimator for time-varying autoregressive models with stochastic volatility. The estimation technique was as a result of a multivariate extension of the numerically accelerated sampling together with a Rao-Blackwellization step to form a highly efficient estimation method. In addition, a new specification for non-linear time-varying parameter models was established to summarize the time-variation in the coefficients through a common factor structure, and this enhanced estimation and retained the flexibility of the model. The proposed specifications was applied to US consumer price index inflation and it was acknowledged that the proposed specifications was able to consider the observed dynamics in inflation.

Delatola and Griffin (2011) proposed a Bayesian non-parametric approach for analyzing the return distribution in a stochastic volatility (SV) model. Specifically, they focused on modelling the logarithm of squared returns using an infinite mixture of Normal distributions, which provided flexibility in capturing various shapes and characteristics of the distribution. To estimate the model parameters, they developed efficient MCMC methods, which allowed for posterior inference. The proposed method was implemented on simulated and real-world financial data. The outcome showed that the estimated volatilities using the model could differ significantly from those obtained using a normal return distribution, particularly when there was proof of a heavy-tailed return distribution. This highlighted the importance of considering more flexible distributional assumptions when modelling financial returns.

Similarly, Liu and Morimune (2005) introduced modifications to the GARCH model to consider the effect of consecutive negative or positive shocks on volatilities. The new model was tested on the Shanghai SHCOMP and Nikkei225 indices, with a particular focus on the SHCOMP index. Additionally, the EGARCH model was extended along similar lines as the GARCH model. The authors proved the stationarity of the new GARCH (1, 1) model and obtained the asymptotic distribution of the quasi-maximum likelihood estimator. Also in the work of Men (2012), estimation techniques for multivariate and univariate SV models were proposed. For heavy-tailed SV models, the slice sampler algorithm was recommended as the major instrument to sample the proposal distribution in the simulation of latent states, while a simple Metropolis-Hastings method was developed for SV models without heavy tails. The slice sampler was favored because it could adjust to the analytical structure of the underlying density, leading to efficient sampling and fewer discarded samples compared to the original Metropolis-Hastings method.

Yang *et al.* (1999) determined the joint estimation of both multiplicative and additive volatility using the marginally integrated local polynomial estimation. Audrino and Bühlmann (2009) introduced a new GARCH-type model for forecasting volatility in financial time series. Their procedure was based on multivariate B-splines of lagged observations and volatility. The estimation of such B-splines basis expansion was created within the likelihood framework of non-Gaussian observations. Regularized and sparse model fitting method was used, because the dimension for the B-spline was large. The predictive potential of their model was demonstrated through simulated and real data. Their result was compared with other approaches. Yu (2012) generalized the correlation structure in the conventional leverage stochastic volatility model from a linear spline. In his model, the correlation between the volatility innovations and return were time varying and depended non-parametrically on the kind of news that arrives at the market. Theoretical characteristics of the proposed model were investigated and the estimation was done through Bayesian methods. The performance of their estimates was determined through simulations. Their model found evidence of time varying leverage effect in individual stocks. Chockalingam and Muthuraman (2011) considered the issue of pricing American options under stochastic volatility. A transformation technique was established to evaluate the optimal exercise policy and option price which gives assurance for convergence. The accuracy and speed of their technique was compared with other existing methods. Feunou and Tédongap (2012) developed a discrete time affine stochastic volatility model with time varying conditional skewness in a streamlined manner. Their technique enabled current asset returns to be asymmetric conditional on past information and current factors. Analytical formulas were derived for different return moments that are used for generalized method of moments (GMM) estimation.

Fengler (2009) recognized the importance of maintaining an arbitrage-free implied volatility surface in local volatility models, as mispricings caused by arbitrage opportunities can lead to inaccurate pricing and performance. They highlighted that prevalent smoothing algorithms for the implied volatility surface may not ensure the absence of arbitrage, and therefore proposed a method for smoothing the implied volatility smile in an arbitrage-free manner. The key idea behind their approach is to directly incorporate the no-arbitrage condition into the smoothing process. They introduced a penalty term in the smoothing algorithm that penalizes deviations from the no-arbitrage condition, effectively ensuring that the implied volatility surface is free from arbitrage opportunities. By incorporating the no-arbitrage condition into the smoothing algorithm, Fengler (2009) aimed to address the issue of negative transition probabilities and negative local volatilities that can arise when the implied volatility surface is not arbitrage-free. These mispricings can have a significant impact on option pricing and risk management, making the development of an arbitrage-free smoothing approach crucial for accurate pricing and performance evaluation in local volatility models.

Again, For both the drift and the diffusion coefficient of the unobserved diffusion process of a stochastic volatility, Comte *et al.* (2010) proposed non-parametric least square estimators, and provided bounds for their risk. Estimators were selected from a class of functions belonging to a finite dimensional space whose dimension is chosen by a data driven technique. Barndorff-Nielsen and Veraart (2012) developed a new class of stochastic volatility models that incorporates stochastic volatility of volatility (SVV) effects. This class of models is known as Volatility Modulated non-Gaussian Ornstein-Uhlenbeck (VMOU) processes. The VMOU processes allow for the volatility to be modulated by a separate stochastic process, capturing the additional uncertainty and dynamics in the volatility of the underlying asset. This SVV effect is important as it can have a significant impact on the behavior of financial markets. The authors investigated various probabilistic properties of the (integrated) VMOU processes, studying the statistical properties and dynamics of the model. They also examined the effects of SVV on two important aspects of financial markets: the leverage effect and the presence of long memory. The leverage effect refers to the phenomenon where negative returns are associated with higher volatility compared to positive returns. Barndorff-Nielsen and Veraart (2012) analyzed the impact of SVV on the leverage effect and provided insights into how the volatility of volatility affects this relationship. Furthermore, the authors studied the presence of long memory in the VMOU processes. Long memory refers to the persistence of volatility over time, where past volatility values have a significant influence on future volatility. They explored how SVV influences the presence of long memory and investigated the implications for modeling and forecasting financial time series.

Langrock *et al.* (2014) proposed an approach for estimating the conditional distribution in a stochastic volatility model in a non-parametric way, without relying on specific assumptions about its shape. This approach is based on a maximum penalized likelihood estimation framework. To achieve non-parametric estimation, the authors combined the hidden Markov model methodology with penalized B-splines. This combination allowed them to flexibly capture the conditional distribution of the stochastic volatility model without imposing strong assumptions about its functional form. The proposed approach offers an alternative to Bayesian methods that have been previously developed for semi-parametric stochastic volatility modelling. It provides a powerful and flexible tool for estimating the conditional distribution in a non-parametric manner. It is important to note that the model proposed by Langrock *et al.* (2014) does not explicitly consider the leverage effect. The focus of their work was on the non-parametric estimation of the conditional distribution, rather than capturing specific stylized facts such as the leverage effect. The authors demonstrated the feasibility and effectiveness of their approach through a simulation study. By avoiding critical assumptions on the shape of the distribution, their method provides a flexible and robust framework for modelling and estimating the conditional distribution in a non-parametric way.

Kanaya and Kristensen (2016), proposed a general estimation strategy for SV jump-diffusion models that combined a simple, model-free realized volatility estimator with the additional structure imposed by the Markov diffusion model of the volatility process. The asymptotic theory that they developed assumed the volatility process has no jump component. Bandi and Renò (2018) identifies spot volatility as a result of jump robust non-parametric estimates and extracted the parameters and functions that drives the price and volatility dynamics from non-parametric estimates of the bivariate process' infinitesimal moments. Phan *et al.* (2019) proposed a multivariate stochastic volatility (MSV) model with the objective of estimating the time-varying volatility of multivariate neural data and its spatial correlational structure. The MSV model assumes that the volatility series of Intracranial electroencephalography (iEEG) signals follows a latent-variable vector-autoregressive process, and it allowed for the lagged signals of different brain regions to influence each other by specifying a full persistent matrix (typically assumed to be diagonal) in the VAR process for volatility. They used a Bayesian method to evaluate the hidden series in volatility and the parameters of the MSV model using the forward filtering backward sampling and Metropolis Hastings algorithms.

Chung (2014) estimated financial volatility utilizing multivariate adaptive regression splines (MARS) by logarithmic transformation as a initial analysis to determine a non-parametric volatility model. To use the MARS methodology in a time series setting, the predictor variables were chosen to be lagged values which results in a model referred to as adaptive spline threshold autoregression (ASTAR). The estimation was demonstrated through simulations and empirical examples. The performance of the MARS volatility model was compared with the existing parametric and non-parametric models in the literature by using several out-of-sample goodness-of-fit measures. Cassim (2018) combined the aspects of multivariate adaptive regression splines(MARS) model estimation algorithm proposed by Chung (2014) and an algorithm proposed by Bühlmann and McNeil (2002) to develop an algorithm for non-parametric estimation of GARCH (2,2) volatility model.

The characteristics of a linearized stochastic volatility model originally proposed by Harvey *et al.* (1994) was investigated under discrete mixtures of normal by (Xu and Knight, 2013) . They derived the general closed form expressions for the moment conditions and showed that the proposed model determines various tail behavior in a more flexible manner than the Gaussian stochastic volatility model. Coleman *et al.* (2013) proposed an optimization formulation for calibrating a local volatility function for option pricing. The aim was to ensure stability and accuracy in the calibration process. The authors introduced an objective function based on the L1-norm, which provides a trade-off between calibration accuracy and model complexity. To represent the unknown local volatility function, the authors employed a spline kernel function inspired by support vector machine learning. The spline kernel coefficient vector was minimized using the L1-norm regularization term, which effectively controls the complexity of the model. Minimizing the L1-norm corresponds to reducing the number of support vectors in the context of support vector regression, thus enhancing predictability. The proposed approach was illustrated using synthetic market data, showcasing its ability to accurately calibrate the local volatility surface. Additionally, the authors demonstrated the simplicity of the calibrated local volatility surface by applying their method to the S&P 500 market index.

Feil *et al.* (2009) also approximated the local volatility function using a bicubic spline and high dimensional model representation (HDMR) model and thier results were compared. The HDMR model produced more accurate results than the bicubic spline model for option prices. Lee (2014) introduced the B-spline (BS) method as an improvement over the smoothed implied volatility smile (SML) procedure for estimating option implied risk-neutral measures (RNMs). The BS method models the risk-neutral cumulative distribution function (CDF) using quartic B-splines with power tails. This choice of modelling allows for a risk-neutral probability density function (PDF) that exhibits continuity and is free of arbitrage. Through Monte Carlo experiments and applications to S&P 500 index options, the authors demonstrated that the BS method outperforms the SML method. The BS method consistently produced arbitrage-free estimators of the risk-neutral measures and accurately recovered the actual risk-neutral PDFs for various hypothetical distributions. This suggests that the BS method provides more accurate and reliable estimates of option implied RNMs.

Additionally, Zhao (2016) proposed an algorithm to enhance the pricing process, calibration process, and sensitivity analysis of the double Heston model in terms of accuracy and efficiency. The optimization was achieved through the use of an optimized caching technique, which reduced the computation time for pricing by approximately 15%. Ulrich and Walther (2018) addressed the sensitivity of option-implied information, such as forward-looking variance, skewness, and variance risk premium, to the construction of the volatility surface. They observed that different volatility surface construction methods can lead to economically significant differences and systematic biases, particularly for out-of-the-money put options. To mitigate this problem, the authors proposed a volatility surface construction method based on one-dimensional kernel regression. They assessed the statistical accuracy of their proposed approach by comparing it to existing state-of-the-art parametric, semi-parametric, and non-parametric volatility surfaces using leave-one-out cross-validation. Using 14 years of end-of-day and intraday data from S&P 500 and Euro Stoxx 50 options, the study concluded that the one-dimensional kernel regression approach provided a more accurate representation of option market information compared to existing approaches in the literature. This suggests that the proposed method captures the nuances of the option market more effectively, reducing biases and improving the accuracy of option-implied information.

Choi *et al.* (2019) also proposed a continuous-time stochastic volatility model based on an arithmetic Brownian motion: a one-parameter extension of the normal stochastic alpha-beta-rho (SABR) model. Implementing two generalized Bougerol's identities in the literature, the study demonstrated that the model had a closed-form Monte Carlo simulation scheme and that the transition probability for one special case followed Johnson's S_u distribution. It was argued that the S_u distribution served as an analytically better alternative to the normal SABR model due to the fact that the two distributions are empirically same.

2.4.1 Empirical Review

Empirical evidence suggests that global equity market volatility exhibits several stylized facts, including asymmetry, long memory, and spillover effects. One notable observation is that volatility tends to be higher during bearish markets compared to bullish markets, demonstrating asymmetry. This finding implies a negative correlation between current stock returns and future conditional volatility as highlighted by (Black, 1976). In addition to the leverage effect, volatility feedback has been proposed as another explanation for volatility asymmetries, as highlighted by Campbell and Hentschel (1992). From this hypothesis, there is a causal relationship between price volatility and future risk premiums. Positive shocks to volatility lead to an increase in future risk premiums, which, assuming dividends remain constant, results in a decrease in stock prices. This feedback mechanism suggests that volatility changes can impact market dynamics and asset prices beyond their immediate effects. Studies have identified asymmetry in volatility at the aggregate market level in emerging markets, as noted by Chiang and Doong (2001). However, the specific mechanisms underlying this volatility asymmetry at the sector and firm levels remain less explored. It is important to understand how volatility asymmetries manifest across different sectors and individual firms to gain a comprehensive understanding of market dynamics. It is worth noting that the presence of volatility feedback does not negate the existence of leverage effects. In fact, both factors can interact and contribute to volatility asymmetries. Christie (1982) conducted an analysis across a section of firms to test Black's hypothesis and found a strong correlation between asymmetry and leverage. However, they concluded that the leverage effect alone was not sufficient to fully explain the observed asymmetric effects in volatility.

In a study by Pagan and Schwert (1990), different statistical models for monthly stock return volatility were compared, with a focus on U.S. data from 1834 to 1925. The reason for focusing on this time period was that post-1926 data had already been analyzed extensively by other researchers. Additionally, the presence of the Great Depression during that period led to stock volatility levels that were inconsistent with stationary models for conditional heteroskedasticity. The study aimed to address the non-linearities in stock return behavior that were not adequately determined by conventional ARCH or GARCH models, as well as the non-stationarity of stock volatility. Similarly, Ladokhin (2009) conducted a study to determine the accuracy of different models in forecasting stock volatility. The models considered in the study included the Exponential Weighted Moving Average (EWMA), implied volatility, and autoregressive conditional heteroskedastic (ARCH) models. The objective was to assess the performance of these models and determine their ability to accurately capture and forecast stock market volatility. By evaluating these models, researchers aimed to gain insights into the effectiveness of different approaches in capturing the dynamic nature of stock volatility and providing reliable forecasts. The findings of such studies have contributed to the development and refinement of volatility models, enhancing our understanding of the complexities of financial markets and enabling better risk management and investment decision-making.

The GARCH framework utilizes the concept of volatility dependence to quantify the influence of the previous period's forecast error and volatility in determining the current volatility. For example, Niyitegeka and Tewari (2013), used the GARCH-type models to examine the nature of the volatility clustering phenomenon in the Johannesburg Stock Exchange. Their results indicated that there was an existence of volatility clustering in the Johannesburg Stock Exchange but could not establish the fact that an asymmetric effect of negative and positive shocks on conditional volatility existed. Again, Koima *et al.* (2015), also examined the characteristics of volatility of the Kenyan stock markets and its stylized facts using the symmetric GARCH(1,1) model. Their result indicated the evidence of time varying stock return volatility over the sampled period. Abdullah *et al.* (2017) also used several GARCH-type models with Student's t-distribution and normal distribution error assumption to model the volatility of the exchange rate of Bangladesh. Their results showed that the Student t-distribution assumption in error provided more accurate forecast than the normal distribution indicating the existence of fat tails in the exchange rate data used.

Yaya *et al.* (2014) also examined the effect of misspecification of correct sampling probability distribution of GARCH processes using the Monte Carlo Simulation approach. Among the three distributions considered in their work, The AR-GARCH with generalized error distribution (GED) was judged the best. Anton (2012) evaluated the forecasting performance of GARCH-type models in terms of their forecasting accuracy in the case of Romanian stock market. It was found that the TGARCH was the most successful in forecasting the volatility of the Romanian stock market. Ahmed and Suliman (2011) implemented the GARCH models to determine the conditional variance in the daily returns of the principal stock exchange of Sudan (Khartoum Stock Exchange (KSE)). Various extensions to the standard GARCH models have been developed to determine the asymmetric nature of volatility. Examples of such extensions include the exponential GARCH (EGARCH) introduced by Nelson (1991) and the threshold GARCH (TGARCH) proposed by Glosten *et al.* (1993) and Zakoian (1994). In a study by Pagan and Schwert (1990), it was observed that the EGARCH model provided slightly better predictions for monthly US stock index volatility compared to the standard GARCH model. However, Franses and van Dijk (1996) argued that asymmetric models, including EGARCH, did not outperform simple GARCH models when forecasting the weekly volatility of European stock market indices. For the Australian stock market, Brailsford and Faff (1996) found that the TGARCH model performed slightly better than other simple models such as random walk, historical average (HA), moving average (MA), and exponentially weighted moving average (EWMA).

Bates (1996) among others investigated the empirical performance of an affine stochastic volatility with jump model using index returns and option data. Hautsch and Ou (2008) reviewed the most common specifications of the discrete time stochastic volatility models and illustrated the major principles of corresponding MCMC based statistical inference. Karali *et al.* (2011) used the Bayesian state-space techniques to evaluate the stochastic volatility of future prices for three storable commodities; soybeans, corn and wheat with available futures contract maturities. Fatone *et al.* (2014), analyzed some stochastic volatility models summarising advantages and shortcomings of each of them. Saltik *et al.* (2016) analyse the return of volatility of spot market prices of crude oil and natural gas for two different terms with different versions of the GARCH class models.

In a study conducted by Krichene (2003), a stochastic volatility model was employed to determine the volatility of three stock indices. The model assumed that volatility was driven solely by a latent variable referred to as "news." To estimate the model parameters and filter volatilities, a Markov Chain Monte Carlo algorithm was utilized. The results indicated a high degree of volatility persistence, which was consistent with both volatility clustering and mean reversion phenomena. The filtering process revealed highly volatile markets, reflecting the frequent occurrence of relevant news events. The diagnostic tests did not indicate any model failure, although there was a possibility for specification improvements. The findings of the study aligned with well-known patterns observed in volatility modelling and suggested that the model could offer value to market participants in asset pricing and risk management. Additionally, the model could assist policymakers in designing macroeconomic policies that promote less volatile financial markets. Joubert and Vencatasawmy (2005) presented some empirical observations concerning volatility, and considered the impact of volatility on actuarial work.

Li (2007) studied the connection in the developed markets in Hong Kong and the US and emerging stock exchanges in mainland China using a multivariate GARCH approach. The researchers employed a four-variable asymmetric GARCH model based on the BEKK framework proposed by Engle and Kroner (1995) to capture the regularities observed in the share price indices. The aim was to test for the transmission of volatility and returns across these markets. The findings of the study revealed no direct connection between the stock exchanges in mainland China and the US market. However, proof for uni-directional volatility spillovers was found from the stock exchange in Hong Kong to those in Shanghai and Shenzhen.

Dao and Wolters (2008), employed a multivariate stochastic volatility (SV) model to assess the presence of usual stochastic trends in the weekly volatilities of the Hang Seng, Dow Jones, Nikkei, and Strait Times index. The study aimed to evaluate the correlation and potential shared patterns among the volatility innovations in these stock indexes. The results revealed a remarkably high correlation among the volatility innovations, indicating a strong interdependence between the volatilities of the different indexes.

In their research, Agnolucci (2009) conducted a comparative analysis of different volatility models for predicting West Texas Intermediate (WTI) future contract volatility. The study evaluated GARCH-type models estimated from time series data and an implied volatility model derived from options pricing. The researchers assessed the models' performance using statistical and regression-based criteria and investigated the presence of asymmetric effects, the influence of error distribution on GARCH parameters, and the potential improvement of incorporating a time-varying long-run mean in volatility estimation. The findings provided insights into model effectiveness and suitability for forecasting WTI future contract volatility, benefiting traders, investors, and risk managers in the oil market. Takaishi (2009) employed Bayesian estimation techniques to estimate a GARCH model for the exchange rate between US Dollar and Japanese Yen. They utilized the Metropolis-Hastings algorithm with an adaptive construction scheme for the proposal density. The proposal density, assumed to follow a multivariate Student's t-distribution, was updated adaptively during the Markov Chain Monte Carlo simulations using the sampled data. In a similar vein, the study conducted by Valadkhani *et al.* (2010) aimed to investigate the link between stock market returns and volatility, specifically examining the impact of the Asian and global financial crises that occurred in 1997-1998 and 2008-2009. The research focused on four countries, namely Singapore, Australia, the United States, and the United Kingdom, and employed a multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) model. By utilizing this model, the study sought to analyze the relationship between stock market returns and volatility during periods of financial turmoil, providing insights into the dynamics and interactions between stock returns and volatility in different market contexts. Based on their analysis of the mean return equations, the researchers found no significant effect on returns coming from the Asian crisis or the more recent global financial crises in the four markets under investigation.

Arouri *et al.* (2011) used a generalized VAR-GARCH approach to investigate the magnitude of volatility transmission between stock and oil markets in Europe and the United States at the sector-level. The study revealed the evidence of significant volatility spillovers between the stock and oil returns. The estimated cross-market volatility spillovers obtained from the VAR-GARCH models often resulted in diversification benefits and improved hedging effectiveness compared to commonly used multivariate volatility models such as the CCC-GARCH proposed by Bollerslev (1990), the diagonal BEKK-GARCH introduced by Engle and Kroner (1995), and the DCC-GARCH developed by Engle (2002). These findings suggest that considering the interdependence between stock and oil markets at the sector-level can provide valuable insights for portfolio management and risk hedging strategies. By accounting for the volatility spillovers, investors can potentially enhance their portfolio diversification and achieve more effective hedging strategies in the presence of oil-market-related risks.

Busch *et al.* (2011) focused on predicting future realized volatility in the bond markets, foreign exchange and stock using a range of information variables that includes implied volatility derived from option prices. The findings of the study indicated that implied volatility provides additional information about future volatility in all three markets when compared to past continuous and jump components alone. Furthermore, the study demonstrated that implied volatility serves as an unbiased forecast in the foreign exchange and stock markets.

Du and Kapadia (2012) proposed a novel approach to analyze the influence of jumps on the VIX index of the Chicago Board Options Exchange. The researchers developed a model-free jump and tail index, which allowed them to assess the impact of jumps without relying on specific models or assumptions. By employing this approach, they were able to examine the presence and significance of jumps in the VIX index, providing valuable insights into the behavior and dynamics of this important volatility measure. This index was constructed by creating a portfolio of risk-reversals using 30-day index options, allowing for the measurement of changes in the intensity of return jumps over time. By utilizing this jump and tail index, the researchers were able to document a significant increase in jump fears during the financial crisis. Specifically, they observed a 50-fold surge in the intensity of return jumps, indicating heightened market volatility and uncertainty during that period. The construction of the model-free jump and tail index provided valuable insights into the dynamics of market jumps and their impact on the VIX index. By quantifying the intensity of return jumps, market participants and analysts can better understand and monitor market conditions, precisely during times of heightened market stress and uncertainty such as financial crises.

Singania and Anchalia (2013) investigated the effects of the global financial turmoil on stock market return volatility in Japan, Hong Kong, India and China during the sub-prime crisis and Eurozone debt crisis. The researchers employed the EGARCH model and analyzed the daily return data from 2005 to 2011 for major stock market indices in these countries. The findings of the study revealed that the sub-prime crisis had a positive effect on the volatility of stock returns in China, Japan, and India. However, it did not have a significant impact on the volatility of returns in Hong Kong. On the other hand, during the Eurozone debt crisis period, it was observed that the already highly volatile stock returns in China and India experienced a negative impact on their volatility. These results highlight the differential effects of global crises on stock market volatility across the studied countries. The positive impact of the sub-prime crisis on volatility in Japan, China, and India suggests increased market uncertainty and turbulence during that period. Similarly, the negative impact of the Eurozone debt crisis on the volatility of extremely volatile stock returns in China and India indicates further destabilization of their markets during that particular crisis period.

Crisóstomo (2014) focused on the implementation of the Heston Stochastic Volatility Model. This model is widely used in financial mathematics to determine the changing trends of prices of asset and their associated volatilities. Additionally, Rasmussen (2016) conducted a comprehensive evaluation of five different methods for calibrating the local volatility function. The study employed a uniform testing framework to compare these methods in terms of accuracy, smoothness, speed, and robustness. By assessing these key metrics, the researchers aimed to determine the performance and suitability of each calibration method. The evaluation criteria allowed for a thorough analysis of the methods, considering their accuracy in reproducing observed market prices, the smoothness of the resulting volatility surface, computational speed, and the robustness of the calibration process. By comparing the different calibration methods within a consistent framework, the study shed light on their relative strengths and weaknesses. Vasquez (2017) conducted an analysis of firms as a result of the gradient of the volatility structure and examined the returns for straddle portfolios. Remarkably, the findings revealed that straddle portfolios with high gradient of the volatility structure consistently outperformed straddle portfolios with low gradient. This outperformance was both economically and statistically significant. The observed results remained robust across different empirical setups, indicating the reliability and consistency of the findings. Furthermore, the superior performance of high-slope straddle portfolios could not be explained by traditional factors commonly considered in finance.

Hofmann and Uhrig-Homburg (2018) examined the relationship between fitting errors of equity option-implied volatility surfaces and intermediary frictions in financial markets. They focused on quantifying the goodness of fit between observed implied volatilities of options and estimates obtained from Option Metrics' smoothed volatility surface. For each stock and day, the researchers calculated the root-mean-square errors of fitting, which captured the discrepancy between observed implied volatilities and estimated values. They found that this error metric increased with idiosyncratic stock volatility, as well as various measures of option and stock illiquidity. Building on these findings, Hofmann and Uhrig-Homburg (2018) proposed an overarching measure for intermediary frictions. This measure was derived from the value-weighted average of the stock-specific fitting errors, indicating the entire level of frictions present in the market. By considering the fitting errors across a broad range of stocks, the researchers were able to capture the impact of intermediary constraints on equity and debt markets. Furthermore, the study uncovered a close link between volatility noise and the constraints faced by intermediaries in the equity and debt markets. The findings suggested that higher levels of volatility noise were associated with tighter constraints on intermediaries, which could have implications for market liquidity and stability.

Rahahleh and Kao (2018) assessed the forecasting performance of linear and non-linear GARCH models for the Tadawul All Share Index (TASI) and the Tadawul Industrial Petrochemical Industries Share Index (TIPISI) in the context of petrochemical industries. The researchers compared the in-sample and out-of-sample forecasting accuracy of various GARCH-class models. By evaluating the performance of both linear and non-linear GARCH models, Rahahleh and Kao (2018) aimed to determine which type of model provided more accurate forecasts for the TASI and TIPISI indices. The in-sample analysis assessed the models' ability to fit historical data, while the out-of-sample analysis examined how well the models predicted future movements in the indices. Neha and Singhania (2018) examined the presence of volatility spillover effects in frontier markets and investigated whether any interlinkages existed among these markets. The analysis utilized monthly data from 2009 to 2016 for regional frontier markets. To investigate the volatility spillover effects and market linkages, the researchers employed multivariate GARCH models, specifically the BEKK . Using the BEKK test, it was found that the effect of a shock originating from own market did not have a lasting impact. However, shocks from other markets exhibited greater persistence. Additionally, from the DCC test, volatility spillover effects were observed across all the markets analyzed.

Kuhe (2018) focused on examining volatility asymmetry and persistence with exogenous breaks in the Nigerian stock market. The analysis utilized daily closing quotations of stock prices from the Nigerian stock exchange, covering the period from 3rd July 1999 to 12th June 2017. To measure the persistence of shocks and leverage effects in varying distributional assumptions, the researchers employed standard symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models. These models were estimated both with and without considering structural breaks. The empirical findings of the study revealed a high persistence of shocks in the return series.Nevertheless, when incorporating structural breaks into the models, a significant reduction in shocks persistence was observed. This suggests that the occurrence of structural breaks plays a crucial role in understanding the dynamics of volatility in the Nigerian stock market. The study also revealed evidence of asymmetry without leverage effects in the Nigerian stock market. Specifically, the presence of asymmetry in volatility was detected, but the leverage effect, was not significant. Similarly, Botshekan *et al.* (2018) aimed to identify volatility spillovers in the capital market. The study confirmed the existence of asymmetric volatility spillovers from the dollar exchange return, as well as conditional shocks from gold coin and crude oil returns, to the stock index.

Salameh and Alzubi (2018) aimed to examine the sources of volatility shocks in the Dubai Financial Market Index. Specifically, they investigated whether these shocks originated from the index's own past shocks or from external shocks such as the FTSE and S & P 500. Empirical analysis found that the volatility of the Dubai Financial Market Index was primarily driven by its own shocks, indicating a significant impact of endogenous factors on the index's volatility. Additionally, a portion of the volatility was found to be influenced by external shocks, particularly those originating from the S & P 500. However, the study did not find any significant contribution to the Dubai Financial Market Index volatility from external shocks related to the FTSE. Moreover, the findings suggested that the Abu Dhabi stock Exchange (APX) affected the volatility of the Dubai Financial Market Index. This indicates a significant relationship between the two stock exchanges, with the Abu Dhabi stock Exchange influencing the volatility dynamics of the Dubai Financial Market Index.

Oikonomikou (2018) examined the equity market linkages among Ukraine, Russia, Poland and the Czech Republic during the period from January 2005 to December 2014. They employed a multivariate asymmetric EGARCH model to analyse the relationship between these markets. The empirical findings provided by the study demonstrated significant return and volatility spillover effects across different periods, the "Great Recession," and the Ukrainian political crisis. Throughout the entire sample period, there was proof of return co-movements, indicating a degree of interdependence among the equity markets. Additionally, the results revealed a strong persistence in volatility, suggesting that shocks and fluctuations in volatility tend to persist over time. The own return effects of each market were found to be more influential compared to the cross-market effects, and the correlations between the markets increased. This indicates that market-specific factors played a dominant role during this period, leading to a stronger impact on individual market returns.

Cao *et al.* (2019) examined the connection between the uncertainty of volatility measured as the volatility of volatility and future delta-hedged equity option returns. The findings consistently showed that delta-hedged option returns decreased in the face of higher uncertainty of volatility. This result held true for different measures of volatility, including the implied volatility, EGARCH from daily returns, and realized volatility from high-frequency data.

McAlinn *et al.* (2020) investigated the impact of leverage effect on individual stocks by relaxing the assumption of linearity. They proposed nonlinear generalizations of the leverage effect within the Bayesian stochastic volatility framework to determine more flexible leverage structures. To evaluate this effect in a practical way, they developed an efficient Bayesian sequential computation method. The study utilized a dataset of 615 stocks comprising the S&P500 and Nikkei 225 indices. The empirical results showed that the proposed nonlinear leverage effect model significantly improved the predictive performance for 89% of all stocks compared to the classical stochastic volatility model.

CHAPTER 3

MATERIALS USED

3.1 Overview

This chapter serves as a comprehensive overview of essential mathematical concepts and models used in the study. The discussions on stochastic processes, polynomial interpolation, splines, GARCH-type models, and model performance evaluation lay the foundation for the subsequent chapters, providing the necessary theoretical framework for the proposed model.

3.2 Stochastic Processes

Any process of random variables that are indexed by a parameter such as time is termed a stochastic process (Ming, 2015). This index may be either discrete or continuous and the possible values of these random variables are called the states of the process. Commodity prices, interest rates, foreign exchange rates and stock prices all follow the stochastic process (Murara, 2019). That is they cannot easily be predicted.

If a probability space $(\Omega,\mathcal{F},\mathbb{P})$, is considered, then a random variable Z_t is a measurable function $Z_t : \Omega \longrightarrow \mathbb{R}$, where $\mathcal{F}$ is the $\sigma-$field, Ω is the sample space, and $\mathbb{P}$ is a function that assigns probability to the events in $\mathcal{F}$.

3.2.1 Filtration

Given each time t, a subset of events $\mathcal{F}_t \subset \mathcal{F}$ is the set of those events whose truth or otherwise are known at time t. As t increases, so does $\mathcal{F}_t : \mathcal{F}_t \subset \mathcal{F}_u, t \leq u$. The family $(\mathcal{F}_t)_{t\geq 0}$ taken collectively is called the filtration in connection with the stochastic process $Z_t, t \geq 0$. Thus, the filtration $\mathcal{F}_t$ represent the history of the process up to time t.

A stochastic process Z_t on the same time set u is adapted to the filtraton if, $\forall t \in u$, Z_t is $\mathcal{F}_t$-measurable. Thus, an adapted process is a process whose random value is known at all times.

3.2.2 Standard Brownian Motion

A Standard Brownian Motion (SBM) sometimes referred to as the Wiener Process is a stochastic process $W(t)_{t\geq 0}$ with the following properties

i. $W(0) = 0$

ii. $W(t)_{t\geq 0}$ has stationary and independent increment.

iii. The increment $W(t) - W(s)$ is normally distributed with mean 0 and variance $t - s$ for $0 \leq s < t$.

3.2.3 Martingale

A process Z_t adapted to $\mathcal{F}$ and satisfying $E[Z_t] < \infty, \forall t \in [0, u]$ is called a submartingale if

$$E[Z_u|\mathcal{F}] \geq Z_t, \forall t \in [0, u]$$

and a supermartingale if

$$E[Z_u|\mathcal{F}] \leq Z_t, \forall t \in [0, u].$$

Z_t is a martingale if it is both a submartingale and a supermartingale, that is if $E[Z_u|\mathcal{F}] = Z_t$.

3.3 Interpolation

Interpolation refers to the process of constructing a smooth curve that passes through a certain set of points, typically representing the graph of a function. It finds applications in various fields such as data analysis industrial design, signal processing and numerical analysis. The goal of interpolation is to estimate values between the given points, enabling the prediction or representation of intermediate data points with a continuous curve (Figuero *et al.*, 2016). Thus,interpolation is a basic tool for the approximation of given functions. Consider a family of functions of a single varible x,

$$\theta(x; a_0, ..., a_n)$$

having $n+1$ parameters $a_0, ..., a_n$ whose values demonstrate the individual functions in this family. The problem of interpolation for θ consist of evaluating these parameters a_i so that for $n+1$ given real or complex pairs of numbers (x_i, f_i), $i = 0, \cdots, n$, $\forall x_i \neq x_k$ for $i \neq k$,

$$\theta(x; a_0, ..., a_n) = f_i,\ i = 0, \cdots, n$$

exists. The pairs (x_i, f_i) would be referred to as the support points.

3.3.1 Polynomial Interpolation

Polynomials are commonly used for approximations because they possess several advantageous properties. They can be evaluated, differentiated, and integrated efficiently using basic arithmetic operations such as addition, subtraction, and multiplication. This makes computations involving polynomials relatively straightforward and computationally efficient. Additionally, polynomials offer flexibility in terms of degree, allowing for a trade-off between accuracy and computational complexity. The simplicity and versatility of polynomials make them a popular choice for approximation techniques in various mathematical and computational applications(de Boor, 2001). A polynomial of order n is a function of the form

$$p(x) = a_0 + a_1 x + \cdots + a_n x^n = \sum_{j=0}^{n} a_j x^j \tag{3.1}$$

The collection of all polynomials of order n forms a linear space (de Boor, 2001). In the context of interpolation, the polynomial used for fitting the given set of points has a degree that matches the number of degrees of freedom required to accurately represent the function at those points. The degrees of freedom in the polynomial correspond to the number of parameters that can be adjusted to ensure the polynomial passes through each of the specified points.

3.4 Splines

Spline is commonly used to describe a broad class of functions utilized in various applications that involve data interpolation and smoothing (Orosi, 2012). Splines are particularly useful when there is a need to represent a smooth curve or surface that passes through a given set of points. These functions are often employed in fields such as computer graphics, computer-aided design, data analysis, and numerical modelling. Splines offer flexibility and versatility, allowing for efficient interpolation and smoothing of data by providing a continuous and differentiable representation of the underlying phenomenon. The foundations of modern mathematical theory of spline approximation were laid by I. J. Schoenberg in 1946 (Schoenberg, 1959). In that paper, splines was established for use in new approach to smoothing in statistical data. Let $X = (x_0, x_1 \cdots x_k)$ be a vector of reals such that $x_i \leq x_{i+1}$. A function S is called a (polynomial) spline function of degree $m+1$ (order m) if it satisfies the following two conditions:

1. S is a polynomial of degree $m-1$ on each subinterval (x_i, x_{i+1})
2. S and its derivatives of order $1, 2, \cdots m-2$ are everywhere continuous, that is $S \in C^{[m-2]}$

The points x_i are called the knots and X is the knot vector.

3.4.1 Knot Vector

In the context of spline interpolation and approximation, a knot vector refers to a collection of increasing parameter values that define the boundaries of the knot spans within the parameter space. The parameter space can be thought of as an abstract space where the basis functions used in the spline representation are defined. The knot vector divides the parameter space into contiguous intervals called knot spans, with each span corresponding to a specific set of basis functions. The knot vector is written as $E = [t_1, t_2 \cdots t_{n+p+1}]$ where t_i is the i^{th} knot, i is the knot index, p is the polynomial order and n is the number of basis functions used to create the B-spline curve. The knot vector serves as a fundamental component in the generation of B-splines. It provides the necessary information to construct the basis functions used in B-spline interpolation or approximation. If the knot sequence is uniform and also a subset of Z, then the spline is referred to as a cardinal spline.

3.4.2 B-Spline

A B-spline curve is a mathematical function that can be represented as a combination of B-spline basis functions. These basis functions are piecewise polynomial functions with local support, meaning they are only nonzero over a specific interval (Sherar, 2004). For a partition of knot sequence (l_i), the B-splines of order 1 for this knot sequence are the characteristic functions of this partition and it is defined as;

$$B_{i1}(l) = \begin{cases} 1, & \text{if } l_i \leq l < l_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{3.2}$$

The key requirement is that the B-splines must collectively form a complete and non-overlapping set, ensuring that their sum at any given point is always equal to 1. That is

$$\sum_i B_{i1}(l) = 1, \forall l \tag{3.3}$$

In particular, $l_i = l_{i+1}$ implies $B_{i1} = X_i = 0$.

From the first order B-splines, higher-order B-splines can be obtained by recurrence:

$$B_{ik} := \psi_{ik} B_{i,k-1} + (1 - \psi_{i+1,k}) B_{i+1,k-1} \tag{3.4}$$

where

$$\psi_{ik}(l) := \begin{cases} \frac{l-l_i}{l_{i+k-1}-l_i}, & \text{if } l_i \neq l_{i+k-1} \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

Thus, the second-order B-spline is expressed as

$$B_{i2} = \psi_{i2}X_i + (1 - \psi_{i+1,2})X_{i+1} \tag{3.6}$$

and in general, consists of two nontrivial linear pieces which join continuously to form a piecewise linear function which becomes zero outside the interval $[l_i, l_{i+2})$.

The third-order B-spline is given by

$$\begin{aligned} B_{i3} &= \psi_{i3}B_{i2} + (1 - \psi_{i+1,3})B_{i+1,2} \qquad (3.7) \\ &= \psi_{i3}\psi_{i2}X_i + (\psi_{i3}(1 - \psi_{i+1,2}) + (1 - \psi_{i+1,3})\psi_{i+1,2})X_{i+1} \\ &\quad +(1 - \psi_{i+1,3})(1 - \psi_{i+2,2})X_{i+2} \end{aligned}$$

This shows that, in general B_{i3} consists of 3 (nontrivial) quadratic pieces.

After $k-1$ steps of the recurrence, B_{ik} is obtained in the form

$$B_{ik} = \sum_{j=i}^{i+k-1} b_{jk}X_j \tag{3.8}$$

with each b_{jk} a polynomial of degree less than k since it is the sum of products of $k-1$ linear polynomials. Thus a B-spline of order k consists of polynomial pieces of degree less than k.

3.4.3 Evaluation of B-spline

The recurrence relations imply that

$$s = \sum_i B_{ik}a_i = \sum_i B_{ik-1}a_i^{[1]} \tag{3.9}$$

with

$$a_i^{[1]} = (1 - \psi_{ik})a_{i-1} + \psi_{ik}a_i \tag{3.10}$$

Note that $a_i^{[1]}$ is not a constant, but is the straight line through the points (l_i, a_{i-1}) and (l_{i+k}, a_i). In particular, $a_i^{'}(l)$ is a convex combination of a_{i-1} and a_i if $l_i \leq l \leq li + k - 1$. After $k - 1$ fold iteration of this procedure,

$$s = \sum_i B_{i1}a_i^{[k-1]} \tag{3.11}$$

which shows that

$$s = a_i^{[k-1]} \tag{3.12}$$

on $[l_i, l_{i+1})$. See de Boor (2001) for details.

3.5 ARCH/GARCH-TYPE MODELS

With the swift increase of the financial market and the increasing complexity of financial instruments, there is a growing need for sophisticated statistical methods to gain a deeper understanding of financial time series. It is widely recognized that predicting daily returns of financial assets, such as stock returns, is a challenging task (Yanan, 2014). In this regard, time-varying volatility models have been widely employed in the analysis of time series data, with the ARCH model being the simplest and most commonly used approach. These models allow for the modelling of changing volatility patterns over time, enabling researchers to better capture and analyze the dynamics of financial markets.

3.5.1 ARCH MODEL

The ARCH models have been used extensively to model volatility. The ARCH(p) model for the series x_t is defined by specifying the conditional distribution of x_t given the filtration. Let $\mathcal{F}_{t-1}$ be the filtration up to time $t - 1$. The ARCH(p) model for the series x_t is given by

$$x_t|\mathcal{F}_{t-1} \sim N(0, h_t) \tag{3.13}$$

$$h_t = \alpha_o + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 \tag{3.14}$$

where $\alpha_o > 0, \alpha_i \geq 0\ \forall i$ and $\sum_{i=1}^{p} \alpha_i < 1$ are required to be satisfied to ensure non-negative and finite unconditional variance of stationary x_t series. Despite its usefulness, the ARCH model has some limitations. One of the challenges is that when using a large order of the ARCH model, a significant number of parameters need to be computed. This can be computationally intensive and may lead to issues of overfitting (Lama *et al.*, 2015). Another shortcoming of the ARCH model is related to the conditional variance. In an ARCH(p) model, the unconditional autocorrelation function (ACF) of squared residuals, if it exists, tends to decay rapidly. However, in practice, this decay is often slower than what is typically observed. This discrepancy is especially noticeable unless the maximum lag p is set to a large value (Lama *et al.*, 2015).

3.5.2 Evaluating the ARCH(p) Model

The ARCH(p) model model parameters are evaluated by maximizing the likelihood function. Given the assumption of normality, the likelihood function for an ARCH(p) model can be formulated as follows:

$$\begin{aligned} f(x_1 \cdots, x_T|\alpha) &= f(x_T|\mathcal{F}_{T-1}) f(x_{T-1}|\mathcal{F}_{T-2}) \cdots f(x_{p+1}|\mathcal{F}_p) f(x_1, \cdots, x_p|\alpha) \\ &= \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi h_t}} exp\left(-\frac{x_t^2}{2h_t}\right) \times f(x_1 \cdots x_p|\alpha) \end{aligned} \tag{3.15}$$

Here, $\alpha = (\alpha_0, \alpha_1, \cdots \alpha_p)'$ and $f(x_1 \cdots x_p|\alpha)$ is the joint probability density function of $x_1 \cdots x_p$.

Often the conditional likelihood function

$$f(x_{p+1} \cdots, x_T|\alpha, x_1 \cdots, x_p) = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} exp\left(-\frac{x_t^2}{2\sigma_t^2}\right) \tag{3.16}$$

is used because the specific form of $f(x_1 \cdots x_p|\alpha)$ is complex. σ_t^2 can be computed recursively when using the conditional likelihood (Tsay, 2005). Taking the logarithm of the conditional likelihood simplifies its usage. The logarithm of the conditional likelihood is

$$\mathcal{L}(x_{p+1}\cdots, x_T|\alpha, x_1 \cdots, x_p) = \sum_{t=p+1}^{T} \left(-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln h_t - \frac{1}{2}\frac{x_t^2}{h_t}\right) \tag{3.17}$$

3.5.3 GARCH MODELS

The GARCH model, established by Bollerslev (1986), was proposed as an improvement over the shortcomings of the ARCH model. In the GARCH model, the conditional variance is not only dependent on the square of previous shocks but also on its own lagged values Dralle (2011). It is formulated as follows:

$$\begin{aligned} x_t &= \epsilon_t\sqrt{h_t} \\ h_t &= \alpha_o + \sum_{i=1}^{p}\alpha_i x_{t-i}^2 + \sum_{j=1}^{q}\beta_j h_{t-j} \end{aligned} \tag{3.18}$$

where $\epsilon_t \overset{iid}{\sim} N(0,1)$. A sufficient requirement for the conditional variance to be positive is: $\alpha_o > 0, \alpha_i \geq 0, i = 1, 2, \cdots, p, \beta_j \geq 0, j = 1, 2, \cdots, q$. The GARCH(p,q) process is weakly stationary iff $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1$ (Lama *et al.*, 2015). The most popular GARCH model in applications is the GARCH(1,1) model.

The GARCH(1,1) Model

The GARCH(1,1) model is given by

$$\begin{aligned} x_t &= \epsilon_t\sqrt{h_t} \\ h_t &= \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 h_{t-1} \end{aligned} \tag{3.19}$$

According to Equation 3.19, it is evident that when x_{t-1}^2 or h_{t-1} takes on a large value, it leads to a huge value for h_t. Consequently, a large value of x_{t-1}^2 tends to be followed by another large value of x_t^2. This phenomenon gives rise to clustering of volatility, which is commonly observed in financial time series Tsay (2005).

The GARCH(1,1) model can be defined as

$$x_t^2 = \alpha_0 + (\alpha_1 + \beta_1)x_{t-1}^2 + v_t - \beta_1 v_{t-1} \tag{3.20}$$

where $v_t = x_t^2 - h_t$. The expression indicates that the squared errors process follows an ARMA(1,1) model with uncorrelated v_t terms Box *et al.* (2016). This representation of the model is valuable for analyzing the characteristics of the GARCH(1,1) model.

Parameter Estimation For the GARCH(1,1) Model

To estimate the parameters of a GARCH(1,1) model, it is necessary to provide a starting value for the past conditional variance. Bollerslev (1986) proposes using the unconditional variance of x_t^2 as a suitable starting point for this variance. Hence

$$h_t = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \tag{3.21}$$

can be used to determine the starting point for the past conditional variance (Dralle, 2011).

In the case of normality assumption, the maximum likelihood estimation method can be employed to evaluate the parameters ($\theta = (\alpha_0, \alpha_1, \beta_1)'$) of the GARCH(1,1) model. The likelihood can be expressed as

$$\begin{aligned} f(x_1 \cdots, x_T, h_1, h_2, \cdots, h_T|\boldsymbol{\theta}) &= f(x_T, h_T|\mathcal{F}_{T-1})f(x_{T-1}, h_{T-1}|\mathcal{F}_{T-2}) \times \\ &\quad \cdots \times f(x_2, h_2|\mathcal{F}_{T-2})f(x_1, h_1|\boldsymbol{\theta}) \\ &= \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi h_t}} exp\left(-\frac{x_t^2}{2h_t}\right) \times f(x_1, h_1|\boldsymbol{\theta}) \end{aligned} \tag{3.22}$$

where $\theta = (\alpha_0, \alpha_1, \beta_1)'$. The precise form of $f(x_1, h_1|\boldsymbol{\theta})$ is complex and it is therefore often simpler to condition on x_1 and h_1 and then to use the conditional likelihood in equation (3.23)

$$
\begin{aligned}
f(x_2 \cdots, x_T, h_2, \cdots, h_T|\boldsymbol{\theta}; x_1, h_1) &= f(x_T, h_T|\mathcal{F}_{T-1}) \cdots f(x_2, h_2|\boldsymbol{\theta}; x_1, h_1) \\
&= \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi h_t}} exp\left(-\frac{x_t^2}{2h_t}\right) \qquad (3.23)
\end{aligned}
$$

to evaluate θ. The conditional log-likelihood is expressed as

$$
\begin{aligned}
\mathcal{L}(\theta|x_1, h_1) &= \ln f(x_2, x_3, \cdots, x_T, h_2, h_3, \cdots, h_T|x_1, h_1; \boldsymbol{\theta}) \\
&= \sum_{t=2}^{T} \left(-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln h_t - \frac{1}{2}\frac{x_t^2}{h_t}\right) \qquad (3.24)
\end{aligned}
$$

(Francq and Zakoian, 2010).

Evaluating Parameter with Non-Normal Distributions

When fitting GARCH models to real data, the normality assumption is often ignored. This violation can lead to several issues. First and foremost, the parameter value may become inconsistent. Again, it becomes impossible to give legitimate conditional forecasting intervals for x_{T+1} given $\mathcal{F}_T$. Therefore, it is beneficial to use a distribution that exhibits leptokurtosis, indicating heavy tails and excess kurtosis (Herwatz, 2004).

GARCH with Generalized Error Distribution (GED)

The probability density function (PDF) of a random variable x_t with shape parameter v, mean zero, and variance h_t can be expressed as follows:

$$
f(x_t|\theta, v) = v exp\left(-\frac{1}{2}\left|\frac{x_t}{\lambda \cdot \sqrt{h_t}}\right|^v\right)\left(2^{\frac{v+1}{v}}\Gamma\left(\frac{1}{v}\right)\lambda \cdot \sqrt{h_t}\right)^{-1} \qquad (3.25)
$$

where λ is given by

$$
\lambda = \left(\frac{\Gamma\left(\frac{1}{v}\right)}{2^{\frac{2}{v}}\Gamma\left(\frac{3}{v}\right)}\right)^{\frac{1}{2}} \qquad (3.26)
$$

3.5.4 EGARCH MODEL

The EGARCH model offers another asymmetric model by incorporating the leverage effects, which account for the effect of changes in price on the conditional variance. (Mohammed *et al.*, 2020). It was first introduced by (Nelson, 1991). It can be expressed as

$$\begin{aligned} x_t &= \epsilon_t \sqrt{h_t} \\ \ln h_t &= \omega + \sum_{j=1}^{q} \beta_j \ln h_{t-j} + \sum_{i=1}^{p} \gamma_i \left(\frac{|x_{t-i}|}{\sqrt{h_{t-i}}} - E\left(\frac{|x_{t-i}|}{\sqrt{h_{t-i}}} \right) \right) + \sum_{i=1}^{p} \alpha_i \frac{x_{t-i}}{\sqrt{h_{t-i}}} \end{aligned} \tag{3.27}$$

By using the logarithmic form, the parameters in the model can take negative values while ensuring that the conditional variance remains non-negative (Dash and Dash, 2016).

3.5.5 GJR-GARCH MODEL

This model, developed by Glosten *et al.* (1993), is another asymmetric model that incorporates the leverage effect. The model is written as

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{i=1}^{q} \beta_j h_{t-j} + \sum_{k=1}^{p} \gamma_k x_{t-k}^2 I_{t-k} \tag{3.28}$$

where;

$$I_{t-k} = \begin{cases} 1, & \text{if } x_{t-k} < 0 \\ 0, & \text{if } x_{t-k} \geq 0 \end{cases}$$

I_{t-k} is an indicator function.

3.5.6 APARCH

The general structure of the Asymmetric Power ARCH (APARCH) as introduced by Ding *et al.* (1993) is as follows;

$$x_t = \epsilon_t \sqrt{h_t} \tag{3.29}$$

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i(|x_{t-i}| - \gamma_i x_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \tag{3.30}$$

where $\alpha_0 > 0$, $\delta \geq 0$, $\alpha_i \geq 0$, $i = 1, \cdots, p$, $-1 < \gamma_i < 1$, $\beta_j \geq 0$, $j = 1, \cdots, q$ and $\sigma_t = \sqrt{h_t}$. The model enforces a BoxCox power transformation of the conditional standard deviation process.

3.6 Testing for ARCH Effect

To examine the presence of ARCH effect, the residuals x_t from the mean equation of the return series are used. Two commonly employed tests can be used to assess the ARCH effect. The first test utilizes the Ljung-Box statistics $Q(m)$, which are applied to the series x_t^2. The null hypothesis of this test assumes that the first m lags of the autocorrelation function of the x_t^2 series are zero (Tsay, 2005). The Ljung-Box statistic is given by

$$Q(m) = T(T+2)\sum_{k=1}^{m} \frac{\hat{\rho}_k}{T-k} \tag{3.31}$$

where T is the sample size, m is the number of lags, and $\hat{\rho}_k$ is the estimate of the k^{th} autocorrelation of the squared residuals. $\hat{\rho}_k$ is given by

$$\hat{\rho}_k = \frac{\sum_{t=k+1}^{T} \left(x_t^2 - \hat{\mu}\right)\left(x_{t-k}^2 - \hat{\mu}\right)}{\sum_{t=1}^{T} \left(x_{t-k}^2 - \hat{\mu}\right)^2} \tag{3.32}$$

where $\hat{\mu}$ is the sample mean which is given by

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} x_t^2 \tag{3.33}$$

When the null hypothesis is true, the Ljung-Box statistics $Q(m)$ follows an asymptotic chi-squared distribution with m degrees of freedom (Box *et al.*, 2016). The null hypothesis is rejected if $Q(m) > \chi_m^2(\alpha)$.Here, $\chi_m^2(\alpha)$ is the $100(1-\alpha)$ percentile of a chi-squared distribution with m degrees of freedom (Tsay, 2005)

The second test is the Lagrange multiplier test. The Lagrange multiplier test is equivalent to the F statistic for testing $\alpha_i = 0$ for $i = 1, 2, \cdots, m$ in the regression

$$x_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \cdots + \alpha_m x_{t-m}^2 + \epsilon_t \tag{3.34}$$

for $t = m+1, \cdots, T$, where ϵ_t is the error term, m is a specified integer, and T is the sample size (Lee, 1991). The null hypothesis is then

$$H_0 : \alpha_1 = \cdots = \alpha_m = 0 \tag{3.35}$$

Under the null hypothesis, the test statistic given by

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T - 2m - 1)} \tag{3.36}$$

is asymptotically distributed as a chi-squared distribution with m degrees of freedom. The null hypothesis is rejected if $F > \chi_m^2(\alpha)$, where $\chi_m^2(\alpha)$ is the upper $100(1-\alpha)$ percentile of a chi-squared distribution with m degrees of freedom (Tsay, 2005). Here,

$$SSR_0 = \sum_{t=m+1}^{T} (x_t^2 - \bar{u})^2 \tag{3.37}$$

and

$$SSR_1 = \sum_{t=m+1}^{T} (\hat{\epsilon}_t)^2 \tag{3.38}$$

where $\bar{u}$ is the mean of x_t^2 and $\hat{\epsilon}_t$ is the least squares residual from the regression in equation 3.34.

3.7 Realized Volatility

The realized volatility is a model-free estimate of volatility which is obtained by summing the squared intraday returns (Floros *et al.*, 2020 ; Zhang *et al.*, 2021) It was proposed by Andersen and Bollerslev (1998) and is defined as

$$RV_{t,n} = \sum_{j=2}^{n+1} r_{t,j}^2 \tag{3.39}$$

where $r_{t,j} = 100(\ln P_{t,j} - \ln P_{t,j-1})$ is an intraday return $(j = 2, ..., n+1)$ on day t . $P_{t,j}$ is the last price at time j on day t . Therefore, there are n intervals and $n+1$ intraday closing prices in one trading day. The realized volatility is a valuable tool as it offers a relatively precise measure of volatility, serving various purposes such as volatility prediction and evaluation of predictions.

3.8 Model Performance Evaluation

The performance of the models in accurately predicting realized volatility is evaluated using four different loss functions, as it is not clear which one is most suitable for assessing volatility models (Hansen and Lunde, 2005). The employed loss functions include RMSE, MAPE, TIC, and QLIKE.

3.8.1 RMSE

The RMSE is used to evaluate the predicting performance of the proposed model. It is the most favoured measure among practitioners and academics Lim and Kun (2013). It is given by

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(RV_t - h_t)^2} \tag{3.40}$$

The RMSE gives equal weights to all the errors, irrespective of any time period.

3.8.2 MAPE

The Mean Absolute Percentage Error (MAPE) is a measure of the accuracy of a forecast or prediction. It is calculated as the average of the absolute percentage errors between the forecasted value and the real value.

The formula for MAPE is:

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{RV_t - h_t}{RV_t}\right| \times 100 \tag{3.41}$$

MAPE is expressed as a percentage, and a lower value indicates a better fit between the actual and forecasted values.

3.8.3 Theil's Inequality Coefficient (TIC)

TIC is a statistical measure used to assess the degree of inequality in a distribution. It was first introduced by *Theil, H.* (1969) as a way to quantify the gap between an ideal situation where everyone has the same income (or any other variable of interest) and the actual situation. TIC measures the relative difference between the observed values and the expected values under perfect equality. The formula for TIC is:

$$TIC = \frac{\sqrt{\frac{1}{T}\sum_t (RV_t - h_t)^2}}{\sqrt{\frac{1}{T}\sum_t RV_t^2} + \sqrt{\frac{1}{T}\sum_t h_t^2}} \tag{3.42}$$

In general, a model with a smaller TIC value is considered to be better than a model with a larger TIC value. This is because a smaller TIC value indicates that the model is better at predicting the actual outcomes.

3.8.4 QLIKE

QLIKE is a loss function used in statistical modelling to assess the accuracy of a prediction model. According to Hansen and Lunde (2005), the QLIKE loss function is defined as:

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T}\left(\ln(h_t^2) + \frac{RV_t^2}{h_t^2}\right) \tag{3.43}$$

QLIKE penalizes more heavily the predictions that deviate from the target quantile for the given level, while still taking into account the overall accuracy of the model. The lower the value of QLIKE, the better the performance of the model.

3.8.5 Average Superior Predictive Ability (aSPA)

The aSPA is used to test for average multi-horizon superior predictive ability. It enables the possibility of compensating for inadequate performance at certain horizons with exceptional performance at other horizons. It was proposed by Quaedvlieg (2021). The test for aSPA is based on the loss differential given by

$$d_{ijt} = L_{i,t} - L_{j,t} \tag{3.44}$$

The associated hypothesis is given as

$$\begin{aligned} H_{0,aSPA} &: \mu_{i,j}^{Avg} \leq 0 \\ H_{1,aSPA} &: \mu_{i,j}^{Avg} > 0 \end{aligned} \tag{3.45}$$

A simple studentized statistics takes the form

$$t_{aSPA,ij} = \frac{\sqrt{T}\bar{d}_{ij}}{\zeta_{ij}} \tag{3.46}$$

Here, $\mu_{ij} = E(d_{ij,t})$ and $\zeta_{ij} = \sqrt{\boldsymbol{w}'\boldsymbol{\Omega}_{\boldsymbol{ij}}\boldsymbol{w}}$. $\boldsymbol{w}$ are weight given to the loss differential and Ω_{ij} is the covariance matrix Quaedvlieg (2021)

CHAPTER 4

PROPOSED MODEL

4.1 Overview

This chapter presents the model formulation. This chapter describes the development of the BSGARCH (1,1) model, including its mathematical expressions and the assumptions made during its derivation.

4.2 Model Formulation

Let (Φ,$\mathcal{F}$,$\mathbb{P}$) be a probability space and let $\mathcal{F}_{t-1}$ be the information through $t-1$ which is produced by a stochastic process X_t and assume that under the probability measure $\mathbb{P}$, X_t is given by

$$X_t = \mu_t + \sigma_t v_t$$
$$\sigma_t^2 = f(X_{t-1}, \sigma_{t-1}^2) \tag{4.1}$$

Here, v_t is i.i.d. random variable such that $v_t \sim N(0,1)$, $\mu_t = \mathbb{E}(X_t|\mathcal{F}_{t-1})$ and $\sigma_t^2 = var(X_t|\mathcal{F}_{t-1})$. $\mathcal{F}_{t-1}$ is the filtration generated by the stochastic process X_t. Generally, in financial application,it is not necessary to enable a large degree of flexibility in the dynamics of the conditional mean (Audrino and Bühlmann, 2009). Thus, the conditional mean is assumed to be zero. That is,

$$\mu_t = 0 \tag{4.2}$$

The focus will be on modelling the changing dynamics of volatility over time. $\sigma_t^2 = var(X_t|\mathcal{F}_{t-1})$. Estimating and predicting volatility play a crucial role in the financial industry due to its fundamental significance in various practical applications.

Achieving accurate volatility predictions is a primary objective in both academic research and practical applications, driving the search for effective methodologies.

The squared volatility dynamics are captured using a method that involves decomposing the lagged squared volatility residuals into a basic univariate B-spline basis function, resulting in an additive expansion. In details, the volatility is modelled as

$$\sigma_t^2 = g(x_{t-1}, \sigma_{t-1}^2) + \sum_{i=1}^{T} \delta_i \varrho_i (v_{t-1})_+ \tag{4.3}$$

Here, the univariate B-spline basis function $\sum_{i=1}^{n} \delta_i \varrho_i (v_{t-1})_+$ is used to improve the simple parametric initial function $g(x_{t-1}, \sigma_{t-1}^2)$. Where,

$$\varrho_i(v_{t-1})_+ = \begin{cases} \varrho_i(v_{t-1}), & \text{if } \varrho_i(v_{t-1}) \geq 0 \\ 0, & \text{if } \varrho_i(v_{t-1}) < 0 \end{cases} \tag{4.4}$$

The inclusion of Equation 4.4 is essential to guarantee that Equation 4.3 remains positive at all times. $\varrho_i(v_{t-1})$ is the basis function. v_{t-1} is the residual at time $t-1$ from the classical GARCH (1,1) model. Assuming that all $\delta_i \equiv 0$ which is feasible, then the conventional parametric GARCH (1,1) model is achieved.

B-splines provide a high degree of flexibility in shaping the conditional variance function, relying on the choice of number of knots and degree for each basis function. In this thesis, a B-spline of degree 3 (order 2) is selected to allow the residuals from squared volatility lags to follow a quadratic function. The number of knots plays a role in determining the approximation accuracy. Generally, a huge number of knots leads to a good approximation.

4.3 Parameter Estimation for The Proposed Model

The parameters in the first term of equation (4.3) are evaluated using same method as the conventional parametric GARCH (1,1) model, which is the maximum likelihood estimation. The main emphasis here is placed on evaluating the parameters of the second term of equation (4.3). Considering equation 4.5

$$\hat{f} = \sum_{t=1}^{n} \delta_i \varrho_i(v_{t-1}) \tag{4.5}$$

where δ_i represent control points and $\varrho_i(.)$ is the basis function such that $\sum_{i=1}^{n} \varrho_i(v_{t-1}) = 1$. The estimate is found by minimizing the residual sum of squares. That is

$$\arg\min_f \left(\sum_{i=1}^{n} (v_t - f(v_{t-1}))^2 + \lambda \left\| f''(v_{t-1}) \right\|_2^2 \right) \tag{4.6}$$

This is an infinite-dimensional optimization problem through all functions f for which the criterion is finite. The criterion balances the trade-off between minimizing the least squares error and other considerations. In this context, the term $\lambda \left| f''(v_{t-1}) \right|_2^2$ serves as a roughness penalty that regulates the variability and smoothness of the function. The tuning parameter λ is a positive value that determines the robustness of the penalty. In vector representation, Equation (4.5) is expressed as

$$\hat{f} = \delta \Psi \tag{4.7}$$

with

$$\Psi_{ij} = \varrho_j(\mathbf{v}_{t-1}) \tag{4.8}$$

Equation (4.6) can now be written as

$$\begin{aligned} S &= \arg\min_f \left((\mathbf{y} - \mathbf{f})^T (\mathbf{y} - \mathbf{f}) + \lambda \left\| \mathbf{f}'' \right\|_2^2 \right) \\ &= \arg\min_f \left((\mathbf{y} - \delta \mathbf{\Psi})^T (\mathbf{y} - \delta \mathbf{\Psi}) + \lambda \delta^T \Omega \delta \right) \end{aligned} \tag{4.9}$$

The matrix Ω is intentionally designed to be positive semidefinite, serving as the penalty matrix. It defines a seminorm on $\mathbb{R}^n$, allowing us to express the seminorm $\left| \mathbf{f}'' \right|_2$ of f in terms of the basis expansion parameters represented by ϱ_i. Therefore, Ω

can be formulated as follows:

$$\Omega_{ij} = \int_a^b \varrho_i''(v_{t-1})\varrho_j''(v_{t-1})dv_{t-1} \tag{4.10}$$

where a and b are consecutive knots.

The partial derivative of S with respect to δ is

$$\frac{\partial S}{\partial \delta} = \delta\left(\Psi\Psi^T + \lambda\Omega\right) - \Psi^T\mathbf{y} \tag{4.11}$$

Equating $\frac{\partial S}{\partial \delta}$ to 0 and finding δ, we get

$$\hat{\delta} = \left(\Psi\Psi^T + \lambda\Omega\right)^{-1}\Psi^T\mathbf{y} \tag{4.12}$$

Therefore, equation (4.7) can now be rewritten as

$$\hat{f} = \Psi\left(\Psi\Psi^T + \lambda\Omega\right)^{-1}\Psi^T\mathbf{y} \tag{4.13}$$

where Ω is determined as follows;

Let $\varrho_i''(v_{t-1})\varrho_j''(v_{t-1}) = q_{ij}(v_{t-1})$ then, Equation (4.10) can be expressed as

$$\int_a^b q_{ij}(v_{t-1})dv_{t-1} \tag{4.14}$$

Newton's divided difference polynomial is used to express $q_{ij}(v_{t-1})$ as

$$q_{ij}(v_{t-1}) = b_0 + b_1(v_{t-1} - a) + b_2(v_{t-1} - a)\left(v_{t-1} - \frac{a+b}{2}\right) \tag{4.15}$$

where

$$\begin{aligned} b_0 &= q_{ij}(a) \\ b_1 &= \frac{q_{ij}\left(\frac{a+b}{2}\right) - q_{ij}(a)}{\frac{a+b}{2} - a} \\ b_2 &= \frac{\frac{q_{ij}(b)-q_{ij}\left(\frac{a+b}{2}\right)}{b-\frac{a+b}{2}} - \frac{q_{ij}\left(\frac{a+b}{2}\right)-q_{ij}(a)}{\frac{a+b}{2}-a}}{b-a} \end{aligned} \tag{4.16}$$

Integrating Newton's divided difference polynomial gives

$$\frac{b-a}{6}\left[q_{ij}(a) + 4q_{ij}\left(\frac{a+b}{2}\right) + q_{ij}(b)\right] = \Omega \tag{4.17}$$

The optimal tuning parameter λ must be determined now that the penalty matrix Ω has been formed. In order to produce accurate curve estimator, it is crucial to identify the optimal tuning parameter, λ.

4.3.1 Generalized Cross Validation

Various methods have been proposed for evaluating the optimal tuning parameter λ, which include the ordinary cross validation (OCV), generalized cross validation (GCV), and modifications of OCV such as the leave-(2l+1)-out model proposed in (Chu and Marron, 1991). For this study, the GCV method described in Maharani and Saputro (2021) was adopted due to its asymptotic optimality and other advantages over other methods. The GCV method is expressed as:

$$GCV_\lambda = \frac{\frac{1}{n}\sum_{i=1}^{n}\left[v_t - \hat{f}(v_{t-1})\right]^2}{\left[1 - \frac{df}{n}\right]^2} \tag{4.18}$$

where,

$$df = trace(\Psi\left(\Psi\Psi^T + \lambda\Omega\right)^{-1}\Psi^T) \tag{4.19}$$

CHAPTER 5

RESULTS, ANALYSIS AND DISCUSSION

5.1 Overview

In this chapter, the performance of the new BSGARCH (1,1) model in forecasting volatility of both simulated and real financial time series is compared with the performance of other popular models such as GARCH, EGARCH, GJRGARCH, and APARCH using performance metrics such as RMSE, MAPE, TIC and QLIKE.

5.2 Simulated Data

Following a similar approach to Li and Mark (1994), the data producing process is represented by a time series W_t that satisfies equation (5.1).

$$W_t = \phi W_{t-1} + \varepsilon_t \tag{5.1}$$

First, $\varepsilon_t \sim N(0, \sigma)$ where the conditional variance σ^2 is expressed in equation (5.2)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \tag{5.2}$$

The simulated data was divided into two periods: an in-sample period consisting of the first 70% of the data, and an out-of-sample testing period consisting of the 30% left. The estimation of the models focused solely on the conditional variance, with the conditional mean assumed to be zero. During the in-sample period, all model parameters were estimated using the available data. These estimated parameters were used for forecasting purposes. The forecasting performance of five models - GARCH(1,1), EGARCH(1,1), GJRGARCH(1,1), APARCH(1,1) and the proposed BSGARCH(1,1) was evaluated under six different scenarios. The process was repeated 1000 times, and the average forecasting performance across these repetitions was analysed.

Scenario 1

This scenario involves the generation of 255 observations using the parameter values: $\phi = 0.1, \alpha_0 = 0.1, \alpha_1 = 0.3$. The forecasting performance of the five different models, including BSGARCH (1,1), APARCH (1,1), GARCH (1,1), GJRGARCH (1,1) and EGARCH (1,1), was evaluated and compared. The results of this evaluation are presented in Figure 5.1. It was indicated that the BSGARCH (1,1) model was superior to the conventional GARCH-type models in terms of QLIKE and RMSE, which are indicators of volatility prediction accuracy. Although the disparities in TIC and MAPE between the models were relatively minor, the results suggest that the new BSGARCH (1,1) model is a superior option compared to the classical GARCH-type models for this specific scenario, as it delivers more accurate volatility predictions.

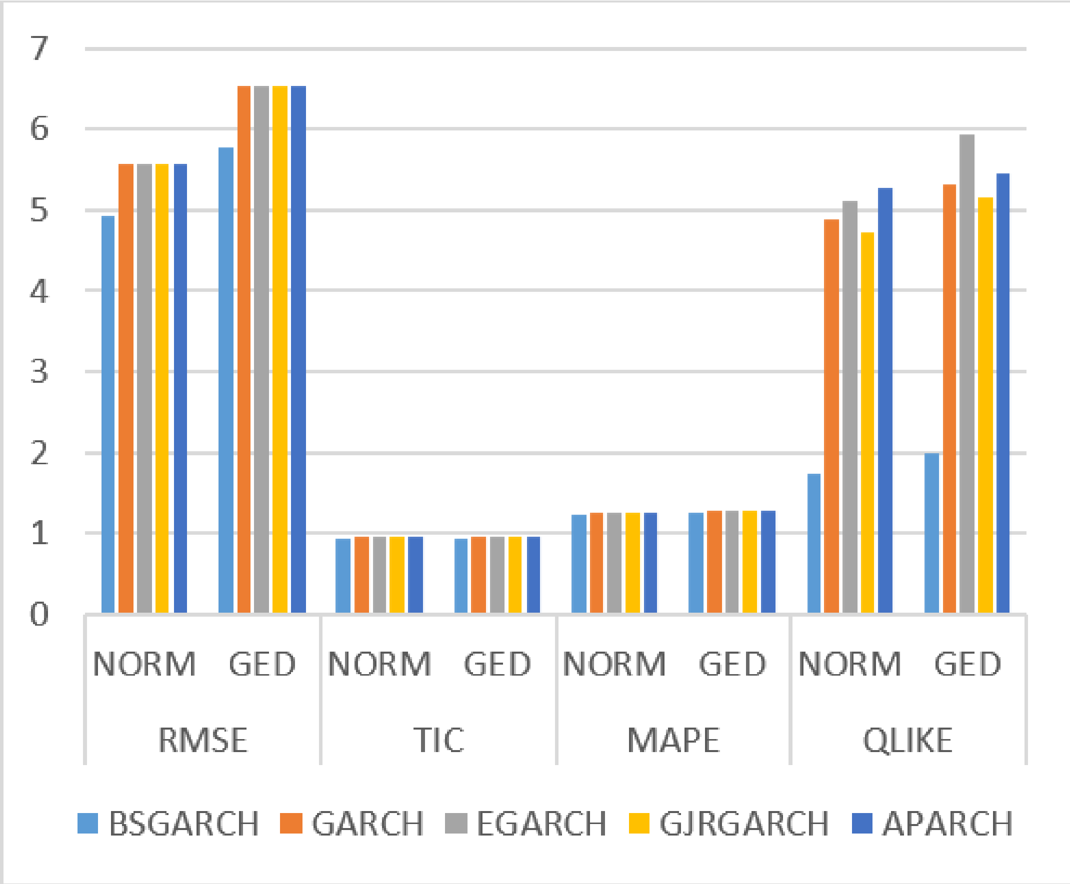

Figure 5.1 Performance Evaluation: Averaged Results for $n = 255$ Data Points with Parameters $\alpha_0 = 0.1$, $\phi = 0.1$, $\alpha_1 = 0.3$

Scenario 2

In this scenario, a dataset of 2000 observations was generated using specific parameter values $\alpha_0 = 0.1, \phi = 0.1, \alpha_1 = 0.3$. As in scenario 1, the forecasting performance of the models were evaluated. The results are presented in Figure 5.2. It was revealed that the new BSGARCH (1,1) model consistently outperformed the conventional GARCH-type models across all performance metrics implemented in this thesis. This suggests that the BSGARCH (1,1) model is better suited for predicting future volatility of financial time series with low ARCH parameters, such as the one generated in this scenario. These results highlight the potential of the BSGARCH (1,1) model for improving the accuracy of volatility forecasts in financial time series analysis, especially in scenarios where the ARCH parameter is relatively low.

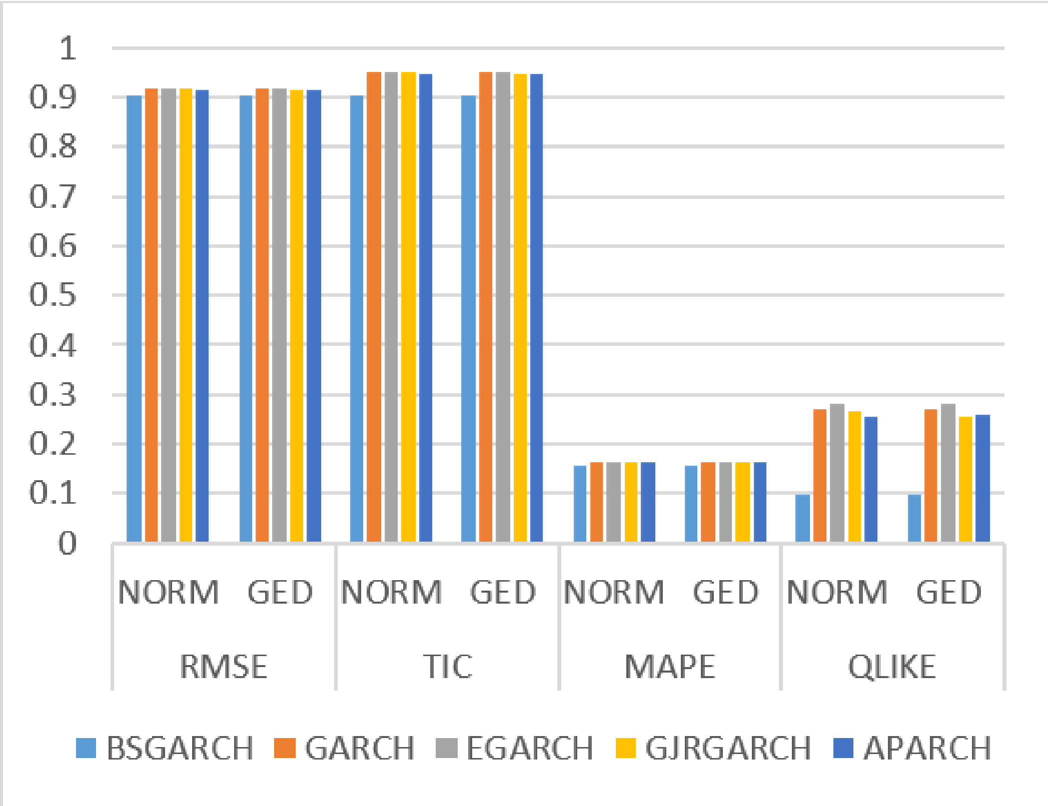

Figure 5.2 Performance Evaluation: Averaged Results for $n = 2000$ **Data Points with Parameters** $\alpha_0 = 0.1, \phi = 0.1, \alpha_1 = 0.3$

Scenario 3

In Scenario 3, 255 data points were produced using the parameter values; $\phi = 0.6, \alpha_0 = 0.4, \alpha_1 = 0.6$ and evaluated the forecasting performance of different models using various performance metrics, which are presented in Figure 5.3. The results revealed that in this particular scenario, there was little variation in the forecasting performance among all the models. This means that none of the models performed significantly better or worse than the others in forecasting the volatility of financial time series with the given parameters.

While this may seem like a negative result, it is an important finding as well.

It suggests that for financial time series with similar characteristics to those generated in this scenario, different GARCH-type models and the BSGARCH model may be equally effective for forecasting their volatility. This could give practitioners more flexibility in choosing which model to use, depending on factors such as their familiarity with the model or computational efficiency.

However, it is worth noting that this finding may not necessarily hold true for financial time series with different characteristics, and practitioners should still carefully consider which model to use based on their specific needs and the properties of the data they are working with.

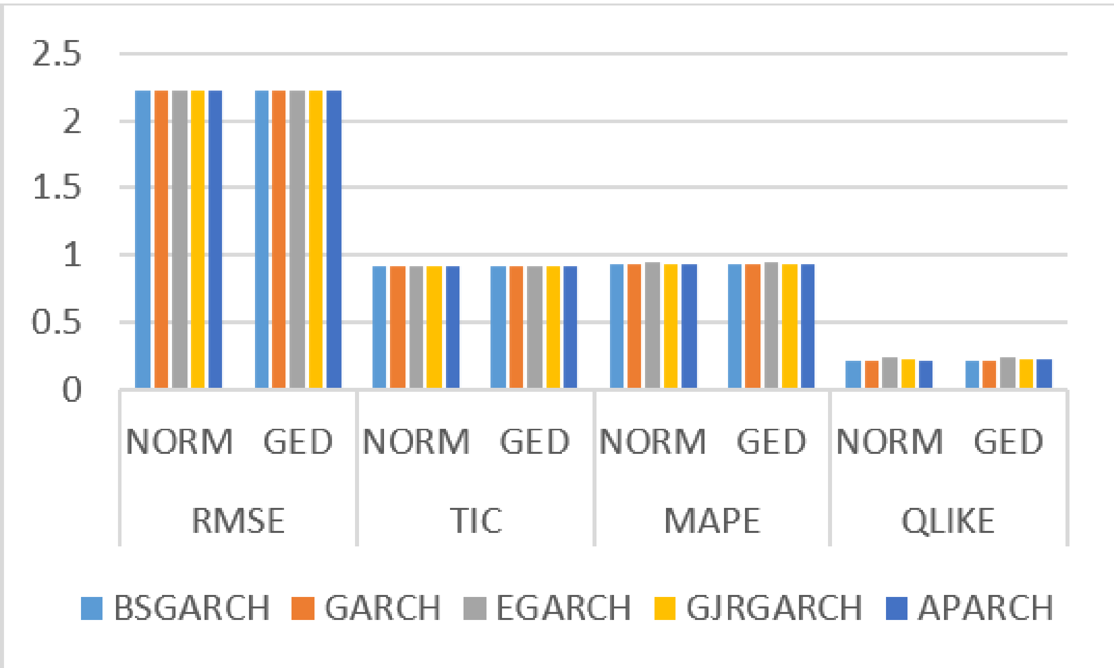

Figure 5.3 Performance Evaluation: Averaged Results for $n = 255$ **Data Points with Parameter Values** $\alpha_0 = 0.4, \phi = 0.6, \alpha_1 = 0.6$

Scenario 4

In Scenario 4, 2000 data points were produced via the parameter values; $\phi = 0.6, \alpha_0 = 0.4, \alpha_1 = 0.6$ and evaluated the forecasting performance using various performance metrics, which are presented in Figure 5.4. The findings indicated that the new BSGARCH (1,1) model consistently outperformed the conventional GARCH-type models across all performance metrics in this particular scenario. This is an important finding, as it suggests that the BSGARCH model can be a useful tool for practitioners who want to forecast the volatility of financial time series with moderate levels of ARCH effects.

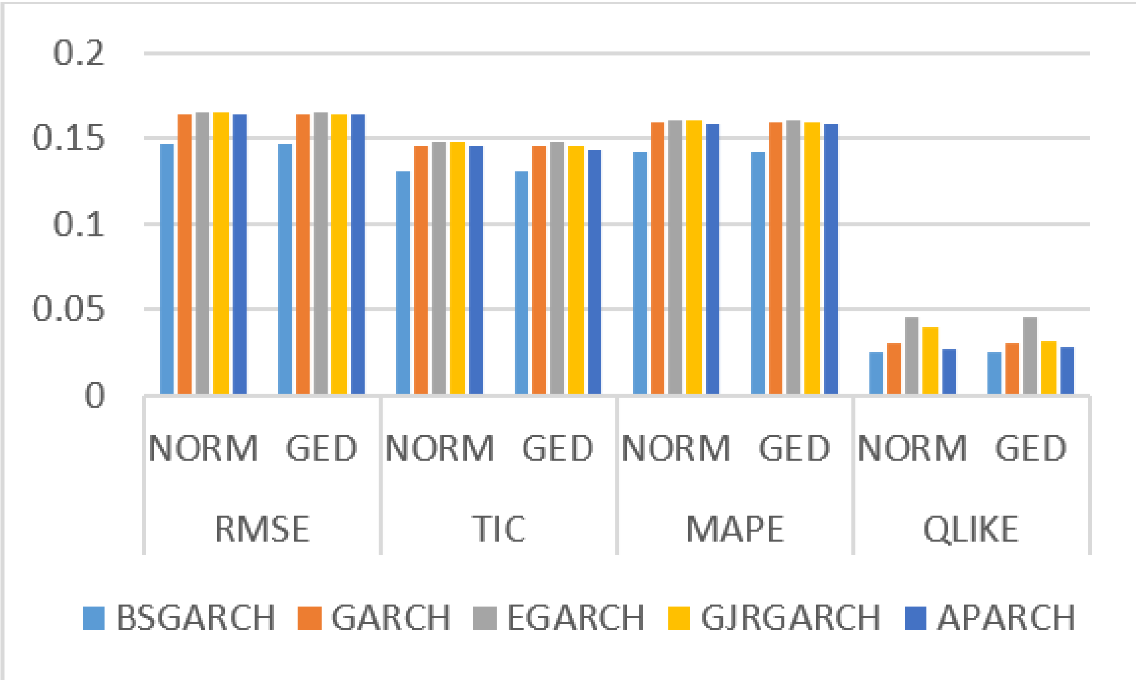

Figure 5.4 Performance Evaluation: Averaged Results for $n = 2000$ Data Points with Parameter Values $\alpha_0 = 0.4, \phi = 0.6, \alpha_1 = 0.6$

Scenario 5

In scenario 5, a financial time series was generated with 255 observations using the parameter values $\alpha_0 = 0.6, \phi = 0.8, \alpha_1 = 0.9$. The predictive performance of different models was determined and compared using the various performance metrics. The results as given in Figure 5.5 showed that the new BSGARCH (1,1) model exhibited better performance as compared to the conventional GARCH-type models in predicting the volatility of the financial time series in this particular scenario. The predictive performance of BSGARCH (1,1) was measured using the various performance metrics, and it had smaller values for all of them compared to the other models considered. This shows that the BSGARCH (1,1) model is effective in predicting the volatility of financial time series with the given parameter values.

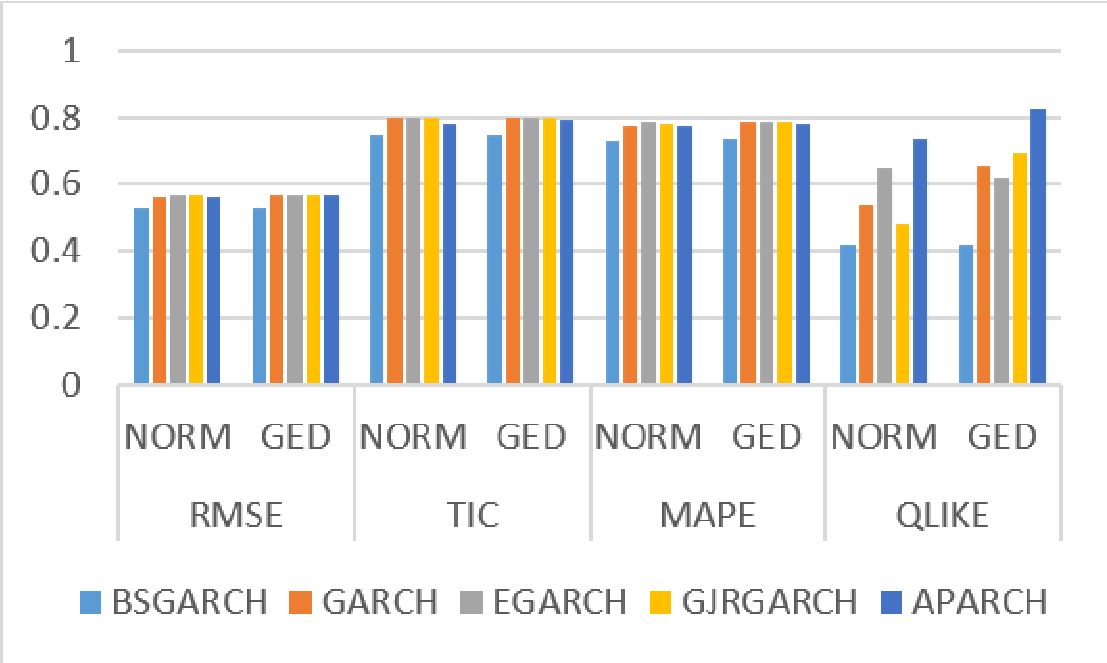

Figure 5.5 Performance Evaluation: Averaged Results for $n = 255$ Data Points with Parameters $\alpha_0 = 0.6, \phi = 0.8, \alpha_1 = 0.9$

Scenario 6

In Scenario 6, the performance of various models was evaluated for 2000 observations produced with the parameter values $\alpha_0 = 0.06, \phi = 0.8, \alpha_1 = 0.9$. The forecasting performance was assessed using different metrics, and the results were given in Figure 5.6. The new BSGARCH (1,1) model was found to have the lowest values in all the performance metrics, indicating its superior forecasting accuracy. This implies that the BSGARCH (1,1) model is more effective than conventional GARCH-type models in predicting the volatility of financial time series with high ARCH parameters.

The results of this scenario are consistent with the findings of the previous scenarios, where the new BSGARCH (1,1) model consistently outperformed other models, indicating the robustness and effectiveness of the proposed model.

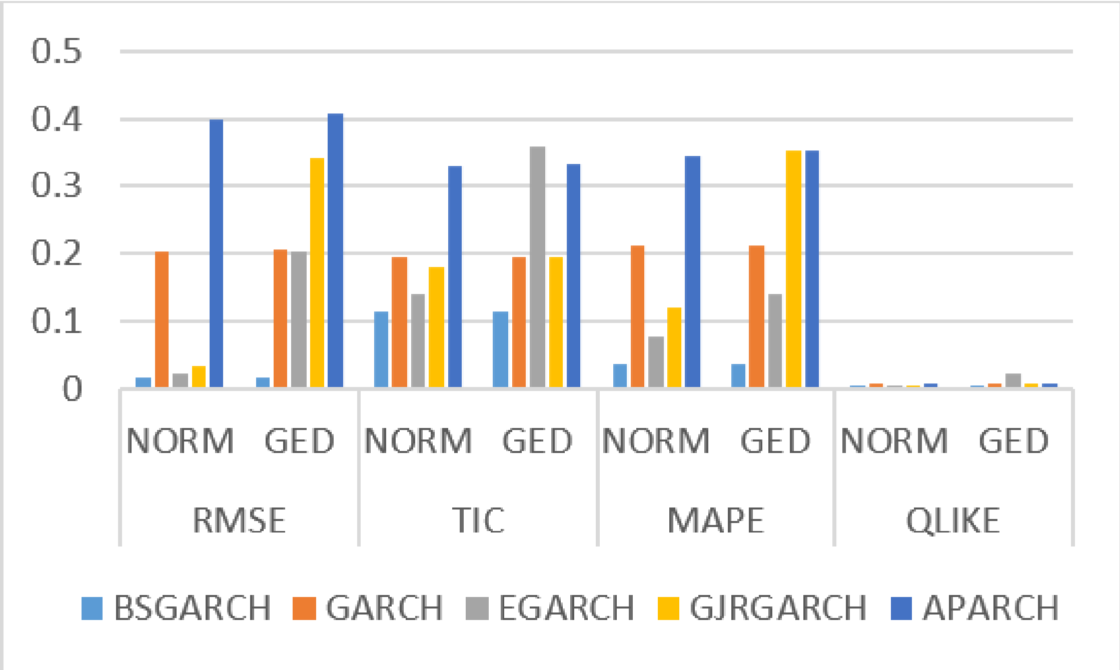

Figure 5.6 Performance Evaluation: Averaged Results for $n = 2000$ Data Points with Parameter Values $\alpha_0 = 0.6, \phi = 0.8, \alpha_1 = 0.9$

The new BSGARCH (1,1) model outperformed the GARCH-type models in all the performance metrics used in all the 6 scenarios, except for Scenario 4, where the difference in forecasting performance was insignificant. These results indicate that the BSGARCH (1,1) model could be a useful tool for forecasting the volatility of financial time series in various market conditions.

In addition to the 6 scenarios discussed earlier, another simulation was conducted to test the performance of the models when ε_t follows a normal distribution with a time-varying deterministic volatility component. The volatility component was set to follow equation (5.3) with parameter values $\alpha_0 = 0.6, \phi = 0.8, \alpha_1 = 0.9$ and the results were averaged over 1000 replications with 255 and 2000 observations.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + g_t \tag{5.3}$$

where g_t represents a time-varying persistent deterministic volatility component, which was defined according to equation (5.4).

$$g_t = \sin(\frac{2\pi t}{n}) \tag{5.4}$$

The performance of the models was then determined using the various metrics in the previous section and the results for $n = 255$ and $n = 2000$ observations are presented in Figure 5.7 and Figure 5.8 respectively. The BSGARCH (1,1) model was found to outperform the conventional GARCH-type models in all metrics in Figure 5.8. However, in Figure5.7, the performance of all models was comparable, except for QLIKE metric where the new BSGARCH (1,1) model outperformed the other models. These findings suggest that the new BSGARCH (1,1) model can effectively forecast the volatility of financial time series under different scenarios, including those with normal error distribution and time-varying deterministic volatility component.

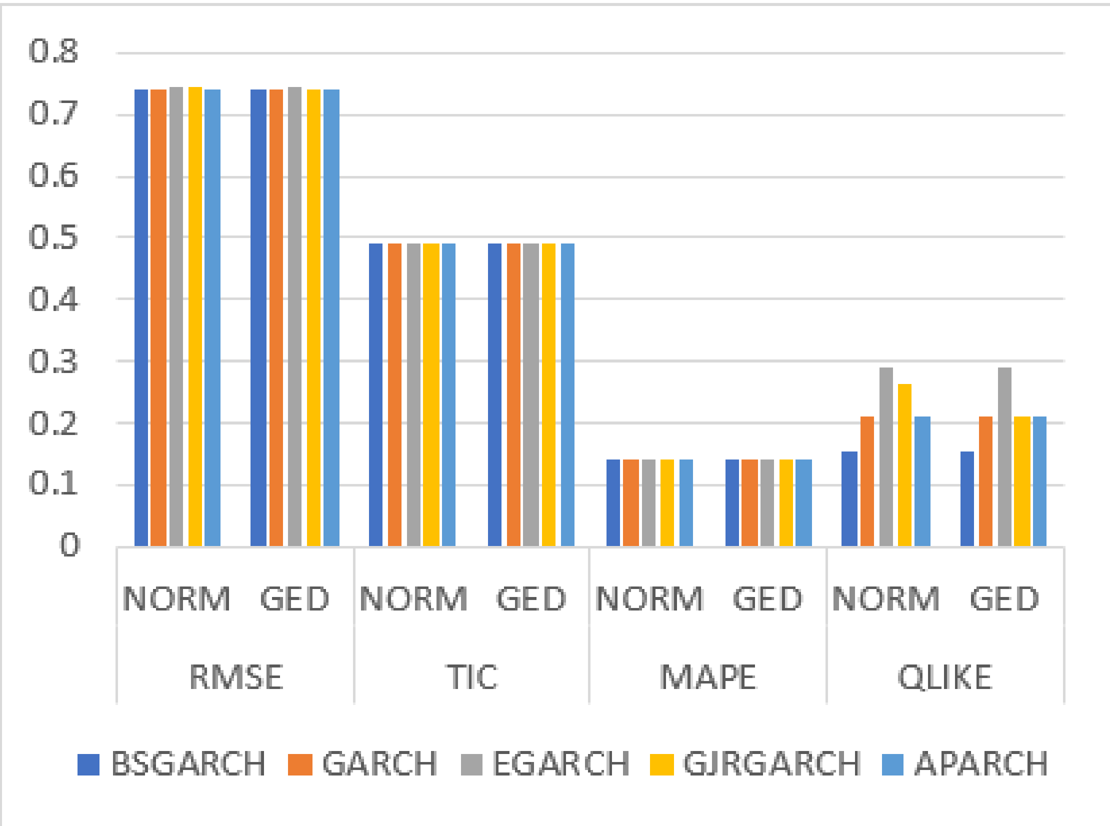

Figure 5.7 Performance Evaluation: Averaged Results for $n = 255$ Data Points with Parameter Values $\alpha_0 = 0.6, \phi = 0.8, \alpha_1 = 0.9$ and $h_t = \alpha_0 + \alpha_1\varepsilon^2_{t-1} + g_t$

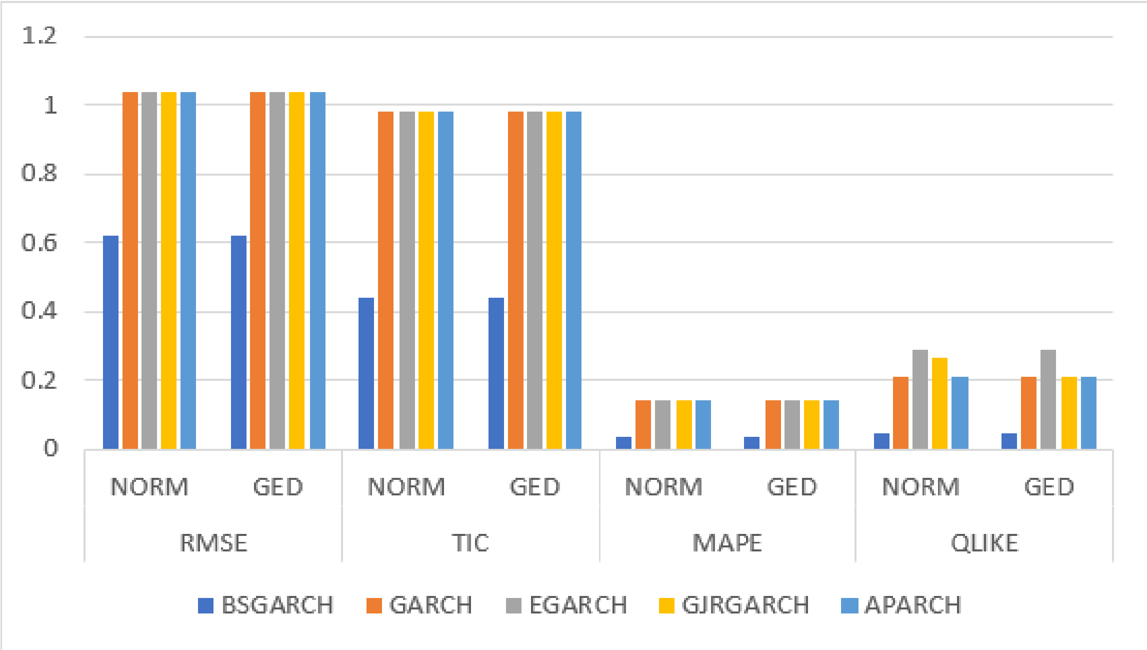

Figure 5.8 Performance Evaluation: Averaged Results for $n = 2000$ **Data Points with Parameter Values** $\alpha_0 = 0.6, \phi = 0.8, \alpha_1 = 0.9$ **and** $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + g_t$

In summary, simulation study was undertaken in this session to assess and compare the forecasting performance of the new BSGARCH (1,1) model with that of conventional GARCH-type models in predicting the volatility of financial time series. The study considered six different scenarios using data generated from equation (5.1) and equation (5.2). Each scenario represented a varying degree of ARCH effect in the data. The first two scenarios were characterized by a low ARCH parameter, indicating a relatively weak presence of autoregressive conditional heteroscedasticity. The third and fourth scenarios had a moderate ARCH parameter, representing a moderate level of volatility clustering. Finally, the fifth and sixth scenarios had a high ARCH parameter, signifying a strong presence of volatility persistence and pronounced clustering. By examining these different scenarios, the study aimed to assess how well the new BSGARCH (1,1) model and conventional GARCH-type models performed in capturing and forecasting volatility under varying degrees of ARCH effect. The analysis of the simulated data allowed for a comprehensive evaluation of the models' forecasting accuracy and effectiveness in capturing the underlying volatility patterns. In all these scenarios, the new BSGARCH (1,1) model outperformed the conventional

GARCH-type models in terms of TIC, MAPE, QLIKE and RMSE performance metrics. However, in some scenarios, the difference in performance between the models was not significant, such as in the third scenario.

Additionally, another scenario was considered where a deterministic time varying volatility component was added to the conditional variance. The new BSGARCH (1,1) model again, outperformed the traditional GARCH-type models. The simulation study suggests that the new BSGARCH (1,1) model can be used as an alternative to classical GARCH-type models for forecasting the volatility of financial time series, especially for series with significant ARCH effects.

5.3 Real Data

Real financial data were utilized to evaluate the performance of the BSGARCH(1,1) model. The data sets consisted of the Standard and Poor 500 (S&P 500), Nasdaq 100 (NASDAQ100), Dow Jones Industrial Average (DJIA), Nikkei 225 Stock Average (NIKKEI225) and the Ghana Stock Exchange Composite Index (GSE-CI). The objective of using data from different stock markets across different continents was to investigate the performance of the new BSGARCH (1,1) model in diverse financial settings.

In finance, return series are often preferred over price series as they are easier to analyse and provide a scale-free summary of the data (Tsay, 2005). Thus, in this thesis, the daily return series is calculated using the daily closing price index, given by

$$r_t' = \frac{y_t - y_{t-1}}{y_{t-1}} \tag{5.5}$$

where y_t and y_{t-1} are the daily closing price index at times t and $t-1$ respectively. It is also common to use the log returns because it reduces the variation of time series which makes it easier to fit models to it (Ruppert and Matteson, 2010). The daily log return is given by

$$r_t = \ln\left(\frac{y_t}{y_{t-1}}\right) \tag{5.6}$$

5.4 Data Exploration

Detailed description of the data used are as follows.

5.4.1 Standard and Poor 500 (S & P 500)

The dataset used in this study consists of 2403 data points spanning from 5th July, 2011 to 30th June 2021. The data was retrieved from the Federal Reserve Economic Data (FRED) and represents the S&P 500 stock market index. The S&P 500 index monitors the performance of 500 large companies listed on stock exchanges in the United States. Figure 5.9 provides a plot of the daily stock index values, revealing periods of both large and small price movements. This indicates clustering of volatility in the series. The log return series, which consists of 2402 data point (one observation lost during return computation), is depicted in Figure 5.10. The plot of log returns provides further proof of clustering of volatility in the series.

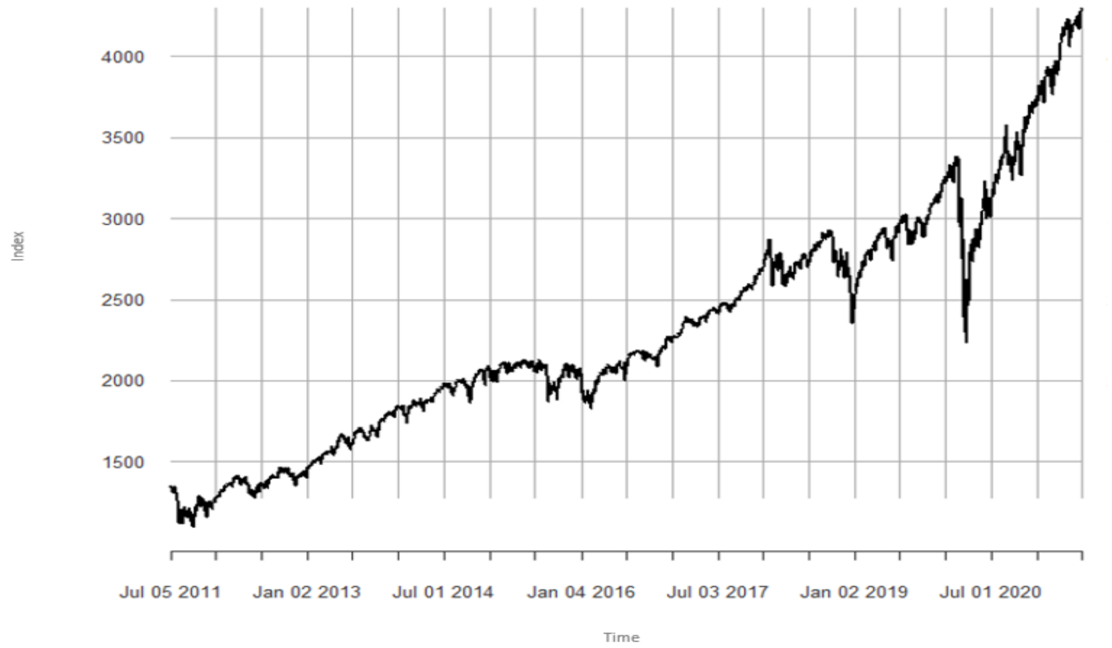

Figure 5.9 S& P 500 Daily Stock Index

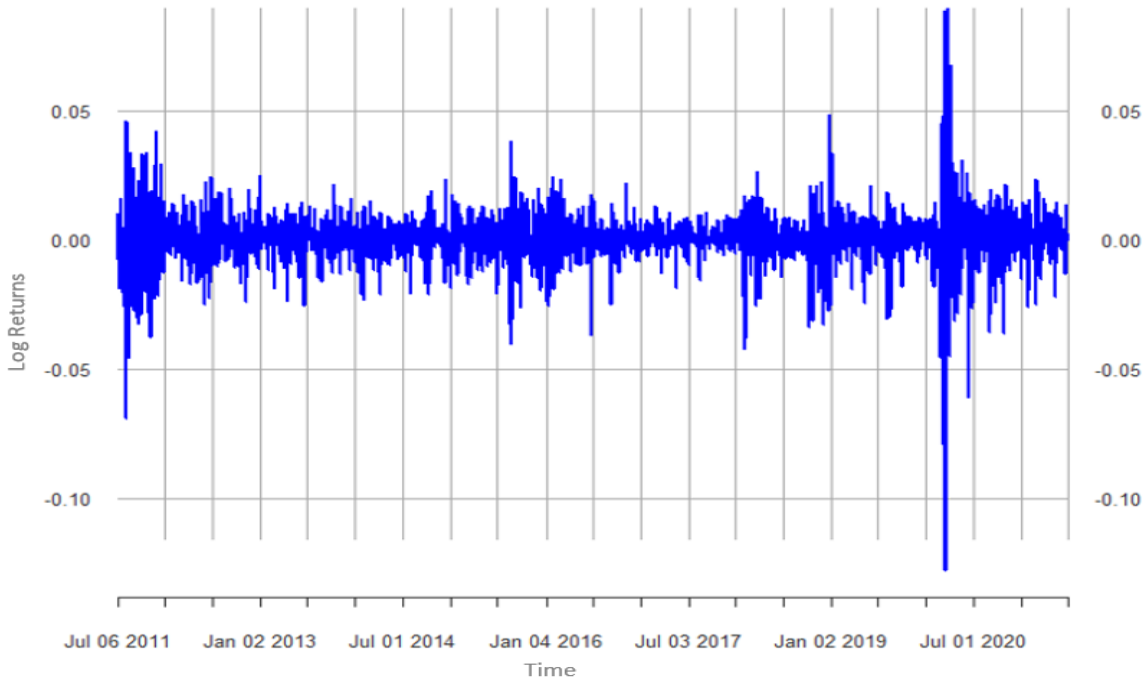

Figure 5.10 S& P 500 Log Returns

Table 5.1 provides some basic statistics for the return series. The statistics indicate that the return series exhibits high kurtosis, suggesting a departure from a normal distribution. This observation is further supported by the tests for normality presented in Table 5.2, as well as the histogram and density plot of the log returns shown in Figure 5.11. In all the tests presented in Table 5.2, the p-values are less than the significance level of 0.05, leading to the rejection of the null hypothesis of normality.

Table 5.1 Summary Statistics of Daily Log Returns of S & P 500

Statistic	S&P 500
Mean	0.000473
Skewness	-0.929785
Maximum	0.089683
Kurtosis	17.7375
Minimum	-0.127652
Standard Deviation	0.011064

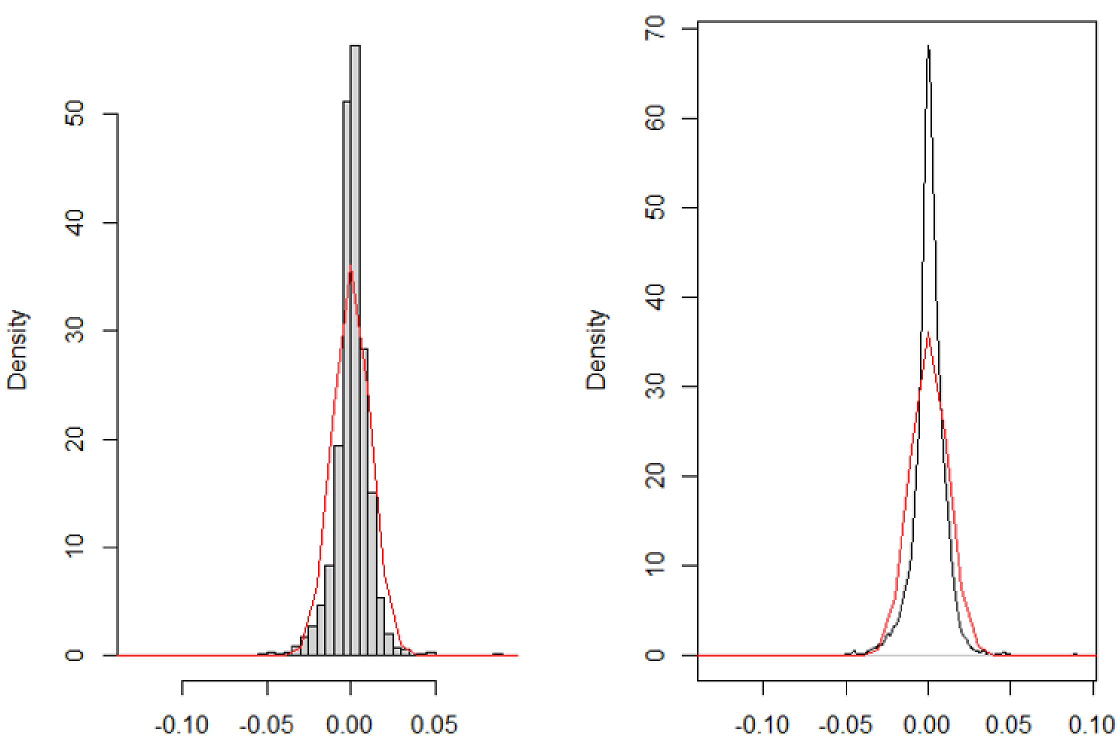

Figure 5.11 Histogram and Density Plot of S & P 500. Histogram on The Left and Density Plot on The Right

Table 5.2 Normality Test for S & P 500

Test	Statistic	pvalue
Jarque-Bera	33670	< 0.001
Kolmogorov-Smirnov	0.5084	< 0.001
Anderson-Darling	2379.4	< 0.001

The presence of an ARCH effect in the data series allows for the modelling of conditional volatility. The Ljung-Box and ARCH-LM test are performed at the 5% significant level to determine the ARCH effect in the series. The results are given in Table 5.3. The null hypothesis is the rate of returns does not have ARCH effect, while the alternative is the opposite (Forsberg and Bollerslev, 2002). Both tests show that the daily log returns are not homoscedastic but rather heteroscedastic since all p-values is approximately zero which is far less than the 5% significant level. This implies that there is an ARCH effect in the series and hence conditional variance can be computed.

Table 5.3 ARCH Effect Test on S & P 500

Test	Chi-Squared	pvalue
ARCH-LM	1024.8	< 0.001
Ljung-Box	1357	< 0.001

5.4.2 NASDAQ100

The NASDAQ100 dataset comprises 2403 observations spanning from December 5, 2011, to June 30, 2021. It was retrieved from FRED and represents a basket of the 100 largest and most actively traded non-financial companies listed on the NASDAQ Stock Exchange. The index includes companies from various industries such as retail, biotechnology, industrial technology, and healthcare, among others. Figure 5.12 displays a plot of the daily stock index of NASDAQ100, revealing periods of both large and small price movements. This suggests the clustering of volatility in the series. The log return series consists of 2402 observations, as one data point is lost when computing the return. Figure 5.13 presents the plot of the daily log returns, which further demonstrates proof of clustering of volatility in the series.

Figure 5.12 NASDAQ100 Daily Stock Index

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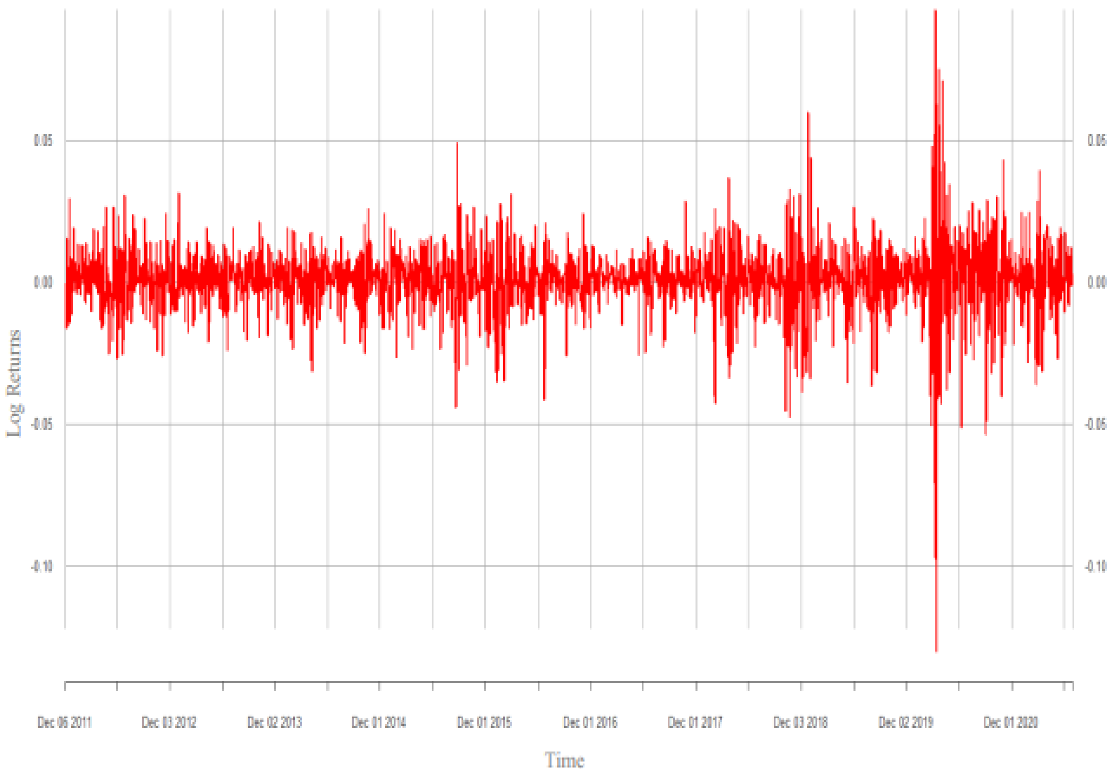

Figure 5.13 NASDAQ100 Log Returns

Table 5.4 presents summary statistics for the log returns of NASDAQ100. The results indicate that the series exhibits high kurtosis, suggesting that it deviates from a normal distribution. This finding is further supported by the normality test shown in Table 5.5 and the visual inspection of the histogram and density plot in Figure 5.14. All the tests in Table 5.5 have null hypotheses stating that the data is distributed normally, while the alternative hypotheses propose otherwise. In all cases, the significance level of 0.05 is greater than the p-values leading to the rejection of the null hypothesis. Positive skewness is also observed, indicating asymmetry in the distribution. The skewness coefficient of 0.015939 suggests that the distribution of the series has fat right tails, implying that there is asymmetry in the data.

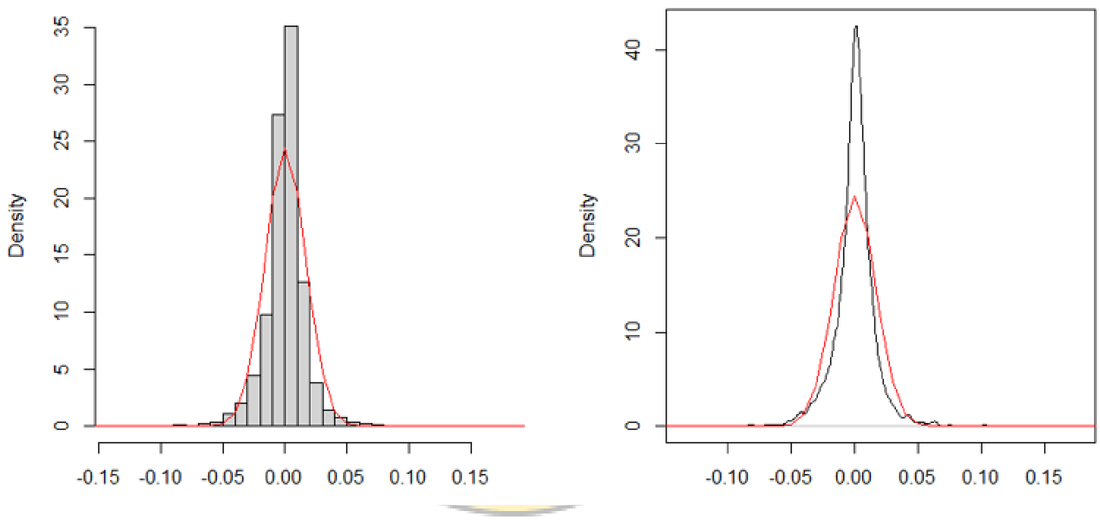

Figure 5.14 Histogram and Density Plot of NASDAQ100. Histogram on The Left and Density Plot on The Right

Table 5.4 Summary Statistics of Daily Log Returns of NASDAQ100

Statistic	NASDAQ100
Mean	0.000354
Skewness	0.015939
Maximum	0.172030
Kurtosis	8.357814
Minimum	-0.130033
Standard Deviation	0.016318

Table 5.5 Normality Test on NASDAQ100

Test	Statistic	pvalue
Jarque-Bera	15020	$<$0.001
Kolmogorov-Smirnov	0.5681	$<$0.001
Anderson-Darling	4951.3	$<$0.001

Table 5.6 gives the results for ARCH-LM and the Ljung-Box test. Both tests show that the daily log returns are not homoscedastic but rather heteroscedastic since all p-values is approximately zero which is far less than the 5% significant level. This implies that there is an ARCH effect in the series and hence conditional variance can be computed.

Table 5.6 ARCH Effect Test on NASDAQ100

Test	Chi-Squared	pvalue
ARCH-LM	1182.8	$<$0.001
Ljung-Box	908.35	$<$0.001

5.4.3 Dow Jones Industrial Average (DJIA)

The DJIA data comprises 2406 observations, covering the period from 6th September 2011 to 30th June 2021, and was retrieved from FRED. The DJIA is a stock market index that tracks the performance of 30 large publicly-owned blue chip companies listed on the New York Stock Exchange (NYSE) and the NASDAQ Exchange. Figure 5.15 shows a plot of the daily stock index of DJIA, revealing periods of both small and large price movements. This indicates clustering of volatility in the series. The log return series consists of 2405 data points. Figure 5.16 illustrates the plot of the daily log returns for the series. The plot of the log returns further confirms clustering of volatility, as the series exhibits periods of both low and high volatility. These observations suggest that the DJIA series exhibits volatility clustering, with periods of both large and small price movements.

Figure 5.15 DJIA Daily Stock Index

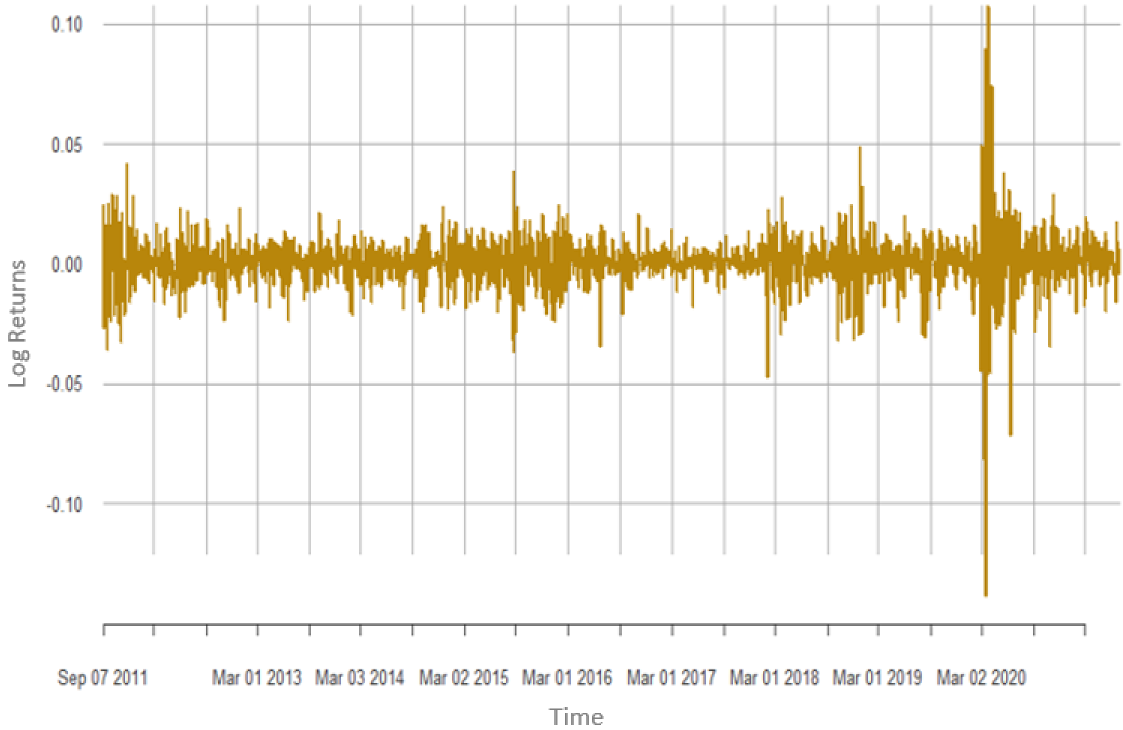

Figure 5.16 DJIA Log Returns

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Table 5.7 presents summary statistics for the log return series of DJIA. The log return series has high kurtosis, suggesting that it deviates from a normal distribution. This finding is confirmed by the tests for normality presented in Table 5.8, as well as the visual examination of the density and histogram plot shown in Figure 5.17. All the tests in Table 5.8 follow the null hypothesis stating that the data is normally distributed, while the alternative hypothesis states otherwise. The significance level of 0.05 is grater than the p-values obtained from these tests leading to the rejection of the null hypothesis. This provides further evidence that the return series of DJIA is not normally distributed. Additionally, the positive skewness coefficient of -1.018078 indicates evidence of asymmetry in the distribution of the series.

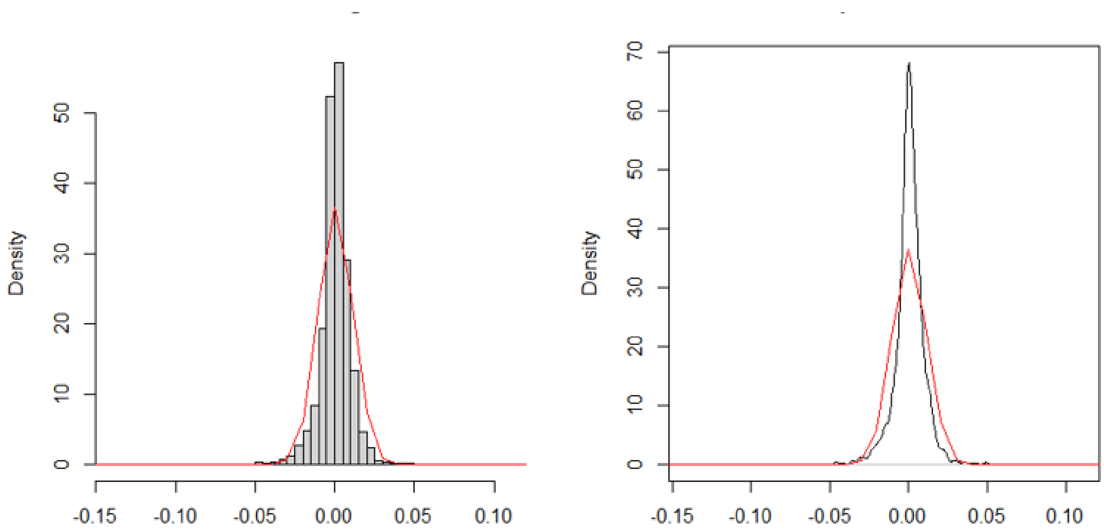

Figure 5.17 Histogram and Density Plot of DJIA. Histogram on The Left and Density Plot on The Right

Table 5.7 Summary Statistics of Daily Log Returns of DJIA

Statistic	DJIA
Mean	0.000432
Skewness	-1.018078
Maximum	0.107643
Kurtosis	27.51668
Minimum	-0.138418
Standard Deviation	0.010684

Table 5.8 Normality Test on DJIA

Test	Statistic	pvalue
Jarque-Bera	76483	$<$0.001
Kolmogorov-Smirnov	0.50795	$<$0.001
Anderson-Darling	2377.5	$<$0.001

Table 5.9 provides the Ljung-Box and the ARCH-LM test. Both tests show that the daily log returns are not homoscedastic but rather heteroscedastic since all p-values is approximately zero which is far less than the 5% significant level. This implies that there is an ARCH effect in the series and hence conditional variance can be computed.

Table 5.9 ARCH Effect Test on DJIA

Test	Chi-Squared	pvalue
ARCH-LM	1011.2	<0.001
Ljung-Box	1216.3	<0.001

5.4.4 Nikkei 225 Stock Average (NIKKEI225)

The NIKKEI225 data consists of 2075 observations spanning from January 4th, 2013, to June 30th, 2021. It was retrieved from FRED. NIKKEI225 is a price-weighted index that represents the performance of Japan's top 225 blue-chip companies traded on the Tokyo Stock Exchange. Figure 5.18 presents a plot of the daily stock index of NIKKEI225, showing periods of both large and small price movements. Figure 5.19 displays the plot of the daily log returns for the series, revealing evidence of volatility clustering.

Figure 5.18 NIKKEI225 Daily Stock Index

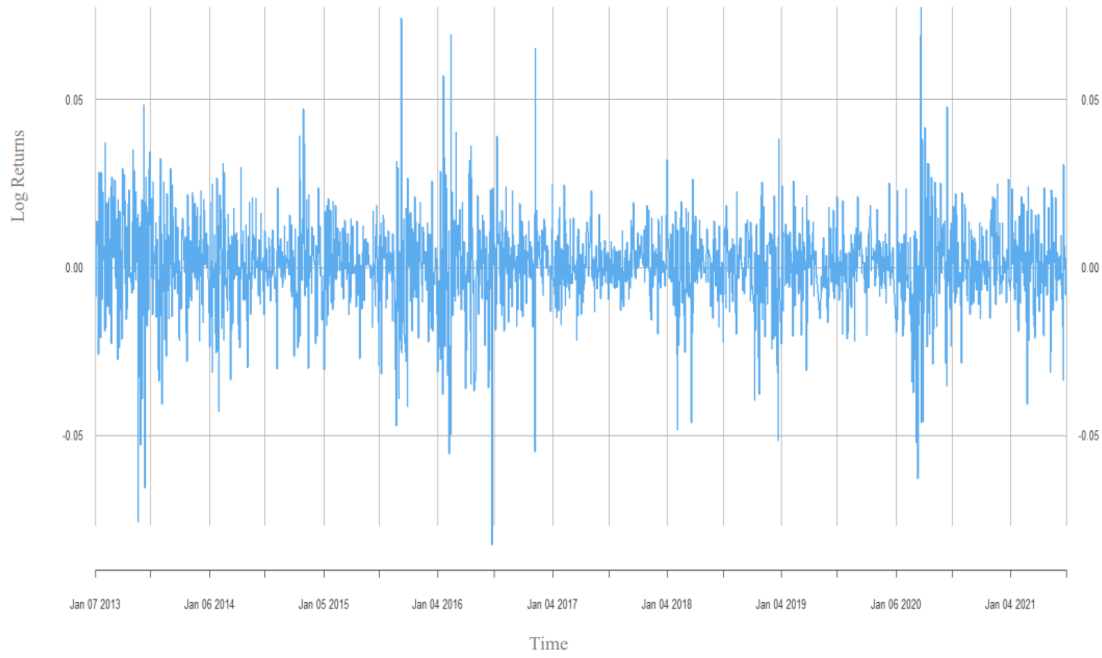

Figure 5.19 NIKKEI225 Log Returns

Table 5.10 provides summary statistics for the return series of NIKKEI225. The log return series exhibits high kurtosis, suggesting that it deviates from a normal distribution. This observation is further supported by the normality tests as shown in Table 5.11 and the visual examination of the density and histogram plot of the return displayed in Figure 5.20. In all the tests provided in Table 5.11, the significance level of 0.05 is greater the p-values, leading to the rejection of the null hypothesis. Additionally, the standard deviation value reported in Table 5.10 indicates a high level of dispersion from the average daily log returns of NIKKEI225. Furthermore, the positive skewness coefficient suggests asymmetry in the distribution. These preliminary results highlight the non-normality, high dispersion, and asymmetry in the return series of NIKKEI225, which are important considerations for modelling and forecasting the volatility of this index.

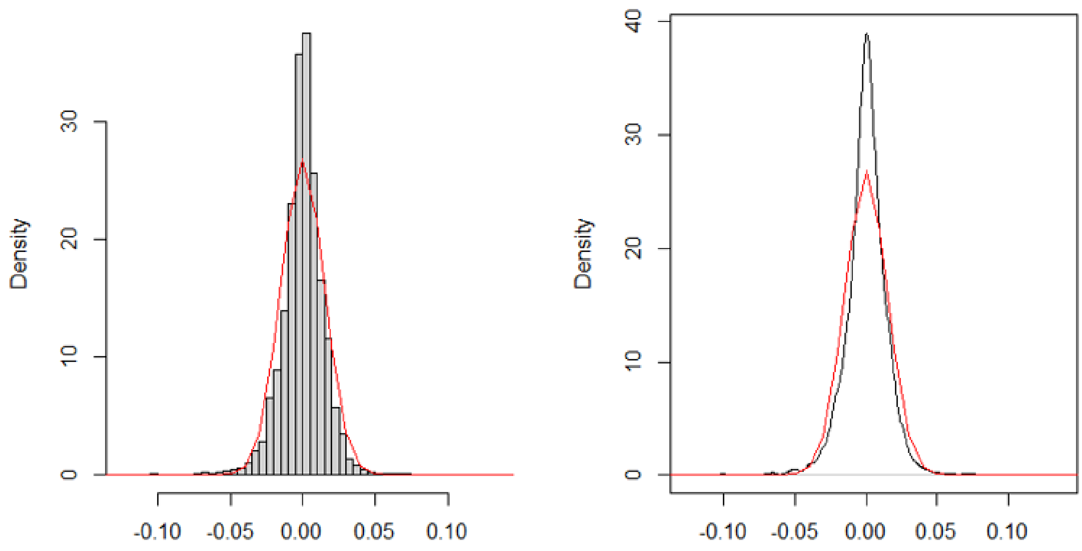

Figure 5.20 Histogram and Density Plot of NIKKEI225.Histogram on The Left and Density Plot on The Right

Table 5.10 Summary Statistics of Daily Log Returns of NIKKEI225

Statistic	NIKKEI225
Mean	0.000148
Skewness	-0.370339
Minimum	-0.12111
Maximum	0.132346
Standard Deviation	0.014872
Kurtosis	6.465696

Table 5.11 Normality Test on NIKKEI225

Test	Statistic	pvalue
Jarque-Bera	8867.5	$<$0.001
Kolmogorov-Smirnov	0.51882	$<$0.001
Anderson-Darling	4858.3	$<$0.001

Table 5.12 provides the Ljung-Box and ARCH-LM test. Both tests show that the daily log returns are not homoscedastic but rather heteroscedastic since all p-values is approximately zero which is far less than the 5% significant level. This implies that there is an ARCH effect in the series and hence conditional variance can be computed.

Table 5.12 ARCH Effect Test on NIKKEI225

Test	Chi-Squared	pvalue
ARCH-LM	1148	$<$0.001
Ljung-Box	1060.6	$<$0.001

5.4.5 Ghana Stock Exchange Composite Index (GSE-CI)

The GSE-CI dataset consists of 2403 observations, covering the period from January 3, 2012, to October 10, 2021. The data was obtained from the Ghana Stock Exchange website. The GSE-CI, is calculated as the volume-weighted average of the closing prices of all listed stocks on the Ghana Stock Exchange. It includes ordinary shares of all listed companies, except those listed on other markets. The index is market capitalization weighted, meaning that each constituent is assigned a weight based on its market capitalization. The base date for the GSE-CI is December 31, 2010, with a base index value of 1000. Figure 5.21 provides a plot of the daily stock index of GSE-CI which shows periods of both small and large price movements. This suggests clustering of volatility in the series. Figure 5.22 displays the plot of the log returns, which provides further evidence of volatility clustering. These findings highlight the dynamic nature of the GSE-CI, with periods of significant price movements and volatility clustering.

Figure 5.21 GSE-CI

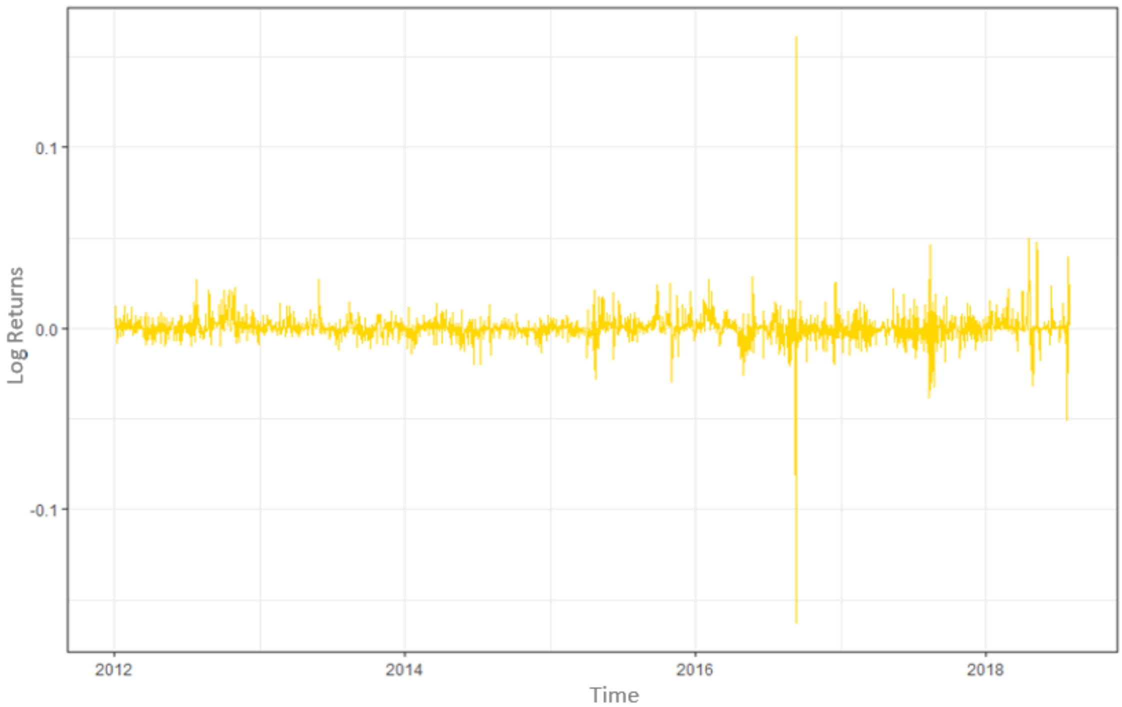

Figure 5.22 GSE-CI Log Returns

Table 5.13 provides basic statistics for the log return series of GSE-CI. The series exhibits high kurtosis, suggesting that it deviates from a normal distribution. This finding is further supported by the normality tests presented in Table 5.14 and the visual examination of the density and histogram plot shown in Figure 5.23. Moreover, the standard deviation value presented in Table 5.13 indicates a high level of dispersion from the average daily log returns of GSE-CI. Additionally, the positive skewness coefficient of 0.177693 suggests asymmetry in the distribution of the series.

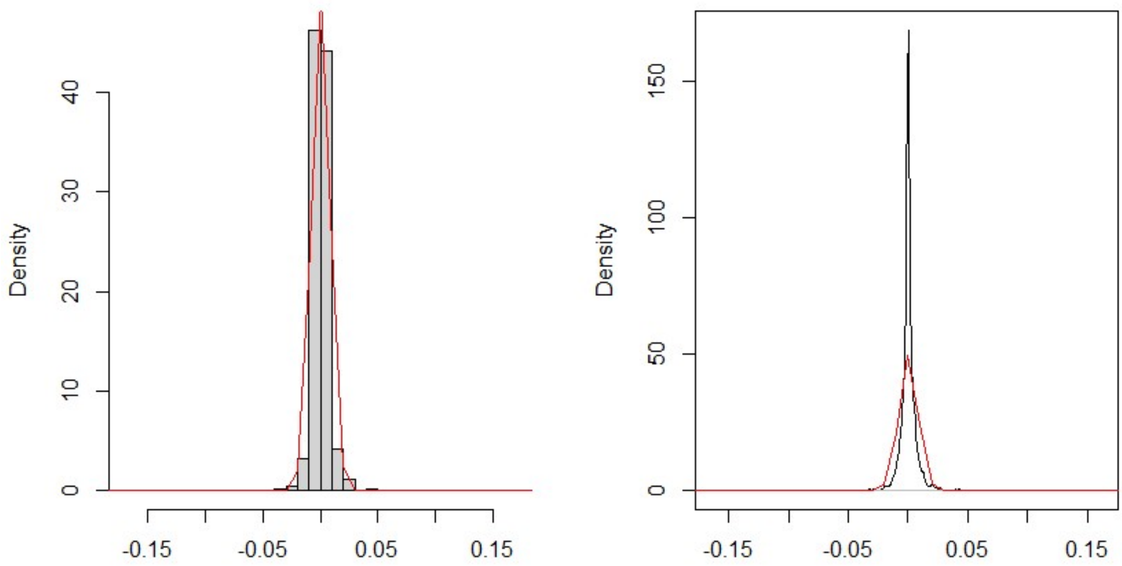

Figure 5.23 Histogram and Density Plot of GSE-CI. Histogram on The Left and Density Plot on The Right

Table 5.13 Summary Statistics of Daily Log Returns of GSE-CI

Statistic	GSE-CI
Mean	0.000450
Standard Deviation	0.008015
Maximum	0.161049
Skewness	0.177693
Minimum	-0.162585
Kurtosis	141.387

Table 5.14 Normality Test on GSE-CI

Test	Statistic	pvalue
Jarque-Bera	2003281	$<$0.001
Kolmogorov-Smirnov	0.48636	$<$0.001
Anderson-Darling	147.15	$<$0.001

Table 5.15 provides the test statistics of the ARCH-LM and the Ljung-Box test. Both tests show that the daily log returns are not homoscedastic but rather heteroscedastic since all p-values is approximately zero which is far less than the 5% significant level. This implies that there is an ARCH effect in the series and hence conditional variance can be computed.

Table 5.15 ARCH Effect Test on GSE-CI

Test	Chi-Squared	p-value
ARCH-LM	955.43	$<$0.001
Ljung-Box	581.12	$<$0.001

5.5 Performance Comparison of the New BSGARCH(1,1) Model with Conventional GARCH-Type Models on Real Financial Data

In this section, the performance of the new BSGARCH(1,1) model introduced in Chapter 4 was evaluated on real data and compared with the conventional EGARCH(1,1), APARCH(1,1), GARCH (1,1) and GJR-GARCH(1,1) models with different error distributions. The comparison with GARCH (1,1) is crucial as it is a extensively used standard model for financial volatility, which is notoriously difficult to outperform (Audrino and Bühlmann, 2009). Moreover, comparing with GJR-GARCH, EGARCH and APARCH models is also significant because these models capture the asymmetric effects in financial time series, which the GARCH (1,1) model fails to do. Therefore, this section demonstrates the practical application of the proposed model to real-world financial data.

5.5.1 S & P 500

The initial step in the analysis was to plot the optimal conditional variance estimates obtained using the proposed model against the log returns, which is shown in Figure 5.24. The estimated conditional variance functions display highly nonlinear and asymmetric patterns with respect to preceding lagged log returns of the series. The series also exhibits a leverage effect, particularly for high values of past lagged log returns. Furthermore, for the same magnitude of positive past shocks, negative past shocks tend to raise the conditional variance more than positive past shocks.

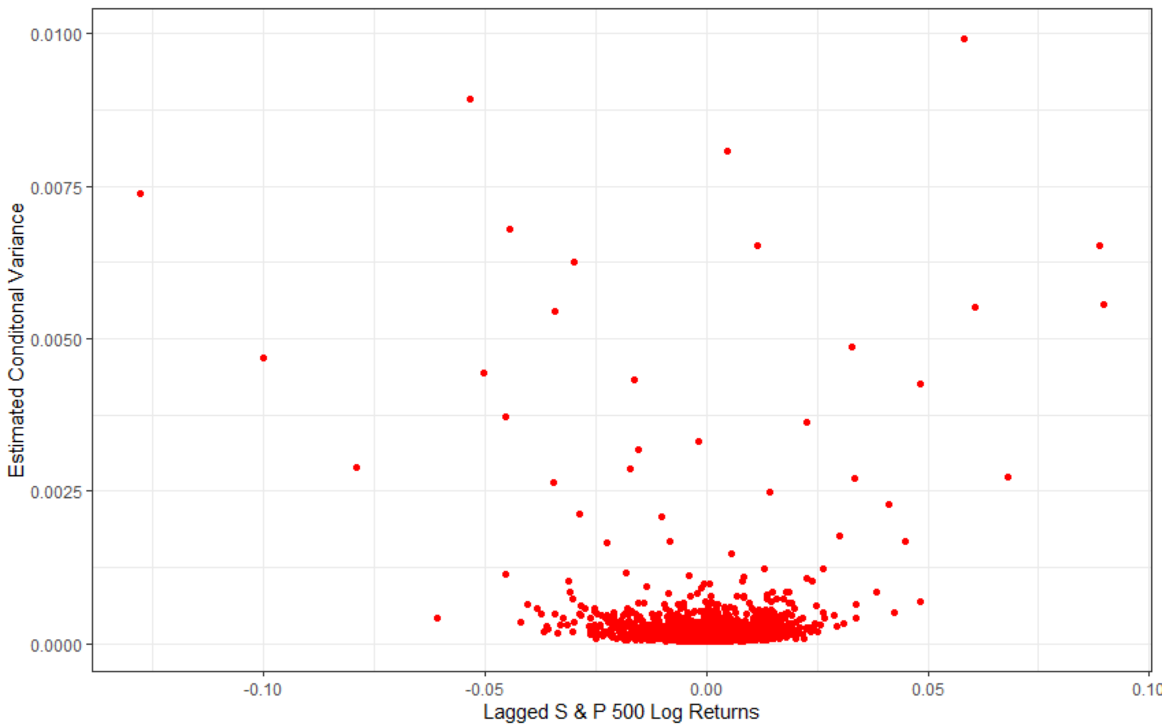

Figure 5.24 Estimated Conditional Variance against the lagged Log Returns of S&P500

Residual Analysis

The new BSGARCH (1,1) model was implemented on the datasets, and its validity was assessed using the the ACF plot of the standardized residuals and McLeod- Li test for autocorrelation. The ACF plot is shown in Figure 5.25. The results of the analysis indicated that the new BSGARCH (1,1) method provided forecasts that effectively incorporated all information available. Specifically, the expected value of the standardized residuals was found to be close to zero, indicating that the model's predictions were unbiased on average. Additionally, the ACF plot revealed no significant correlation in the series of standardized residuals.

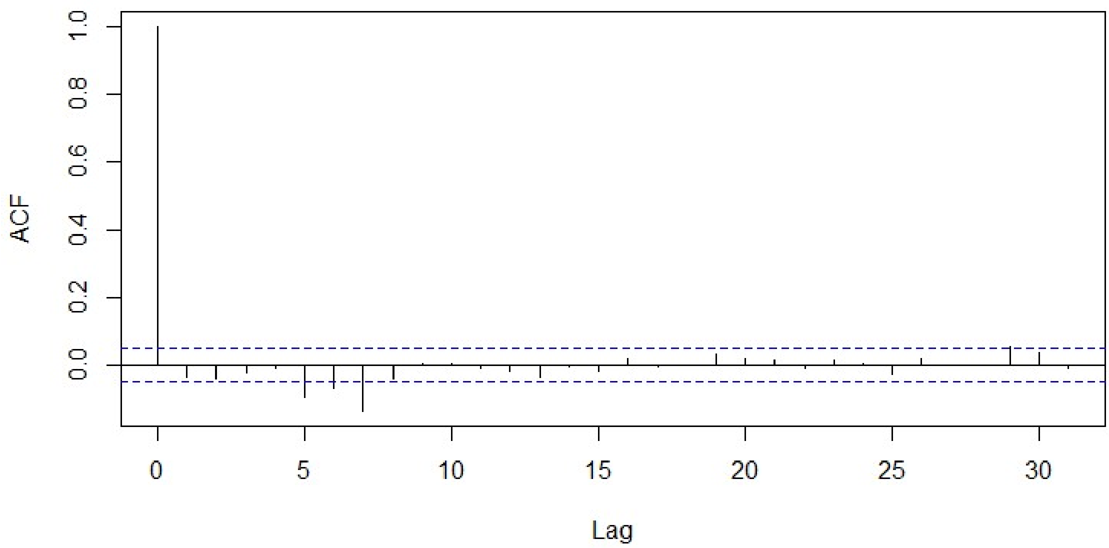

Figure 5.25 ACF Plot of Standardized Residuals (S&P 500)

To further validate the absence of autocorrelations in the standardized residuals, the McLeod-Li test was implemented. The test was carried out with different lag values, namely 10, 15, 20, 25, and 30. The null hypothesis (H_0) of the McLeod-Li test holds that the model's residuals are independent and identically distributed (i.i.d), while the alternative hypothesis H_1 suggests otherwise.

According to the results presented in Table 5.16, the H_0 of the McLeod-Li test was not rejected for the S&P 500 data at various lag values including 10, 15, 20, 25, and 30. This conclusion is supported by observing that the 5% significance level was less than the corresponding p-values. Consequently, there is insufficient proof to recommend that the residuals of the BSGARCH (1,1) model are not i.i.d. These findings indicate that the new BSGARCH (1,1) model is well-suited for the S&P 500 data. The calculated coefficients and predicted values obtained from this model can be considered unbiased and efficient.

Table 5.16 The McLeod - Li Test for ARCH effect in Standardised Residuals

Number of lags	P-Value	Test Statistic
10	0.328	11.387
15	0.173	19.975
20	0.298	22.809
25	0.561	23.282
30	0.701	25.494

Peformance Evaluation on S & P 500

The predictive performance of various models, including APARCH (1,1), GARCH (1,1), GJR-GARCH (1,1), and EGARCH (1,1) with different error distributions, was evaluated alongside the proposed BSGARCH (1,1) for S&P 500 data. The evaluation focused on the models' ability to forecast volatility. As depicted in Figure 5.26, the new BSGARCH (1,1) model consistently outperformed all the classical models examined in the study. This superiority was evident through its lower values of QLIKE, RMSE, TIC, and MAPE values. The results provide compelling evidence that the new BSGARCH (1,1) model represents a significant improvement over the classical models when it comes to forecasting volatility in financial time series, especially for huge sample size. The superior performance of the BSGARCH (1,1) model suggests its effectiveness in capturing the underlying dynamics and patterns of the S & P 500 volatility, leading to more accurate and reliable volatility forecasts.

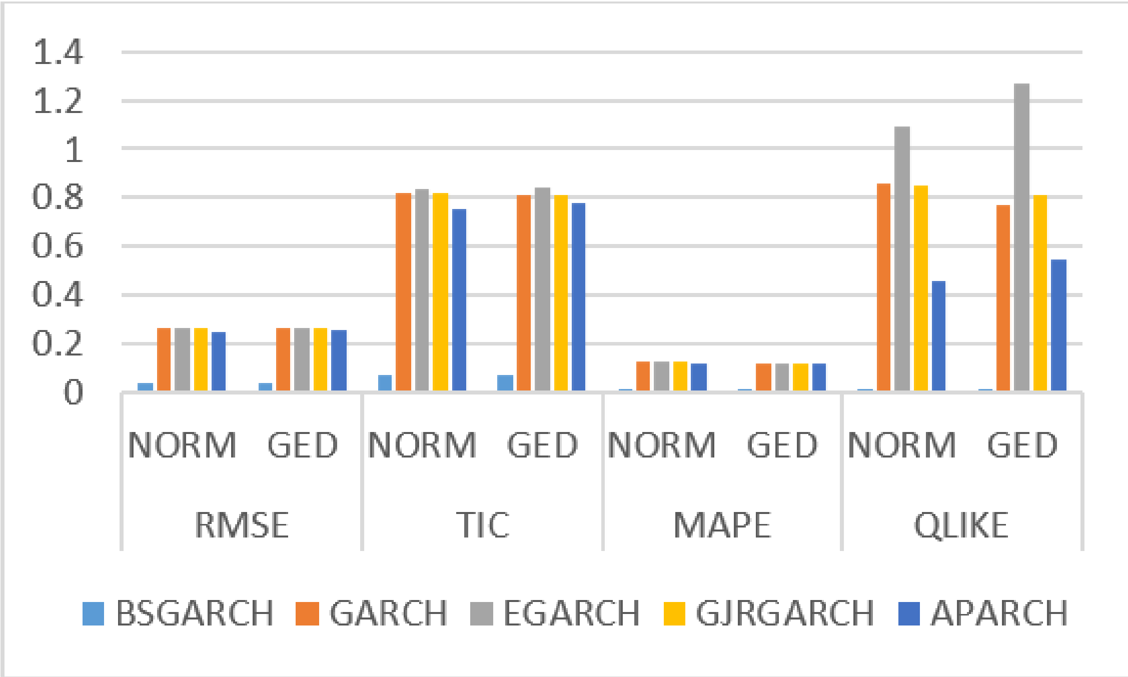

Figure 5.26 Performance Evaluation on S&P 500

The average Superior Predictive Ability (aSPA) tests, as established by Quaedvlieg (2021), were conducted to verify whether the improvements in forecasting financial time series volatility using the BSGARCH (1,1) model were statistically significant. The outcome of the tests are provided in Table 5.17. Positive statistic values are always favorable to the suggested BSGARCH(1,1) model. Table 5.17 confirms that the BSGARCH(1,1) model has higher predictive power in for conditional variance compared to the conventional GARCH-type models.

Table 5.17 Test Values for Average Superior Predictive Ability

Models	Error Distribution	Test Value	P-Value
GARCH vs BSGARCH	GED	646.22	< 0.001
	NORM	678.79	< 0.001
EGARCH vs BSGARCH	GED	721.72	< 0.001
	NORM	706.79	< 0.001
GJR-GARCH vs BSGARCH	GED	647.70	< 0.001
	NORM	661.22	< 0.001
APARCH vs BSGARCH	GED	392.86	< 0.001
	NORM	312.95	< 0.001

5.5.2 NASDAQ100

Figure 5.27 displays the optimal conditional variance estimates obtained using the proposed model, plotted against the log returns for NASDAQ100. The estimated conditional variance functions exhibit high nonlinearity and asymmetry with respect to preceding lagged log returns of the series. The series exhibits a clear leverage effect, especially when considering large values of past preceding log returns. This means that negative past returns have a greater impact on increasing the conditional variance compared to positive past returns of the same magnitude.

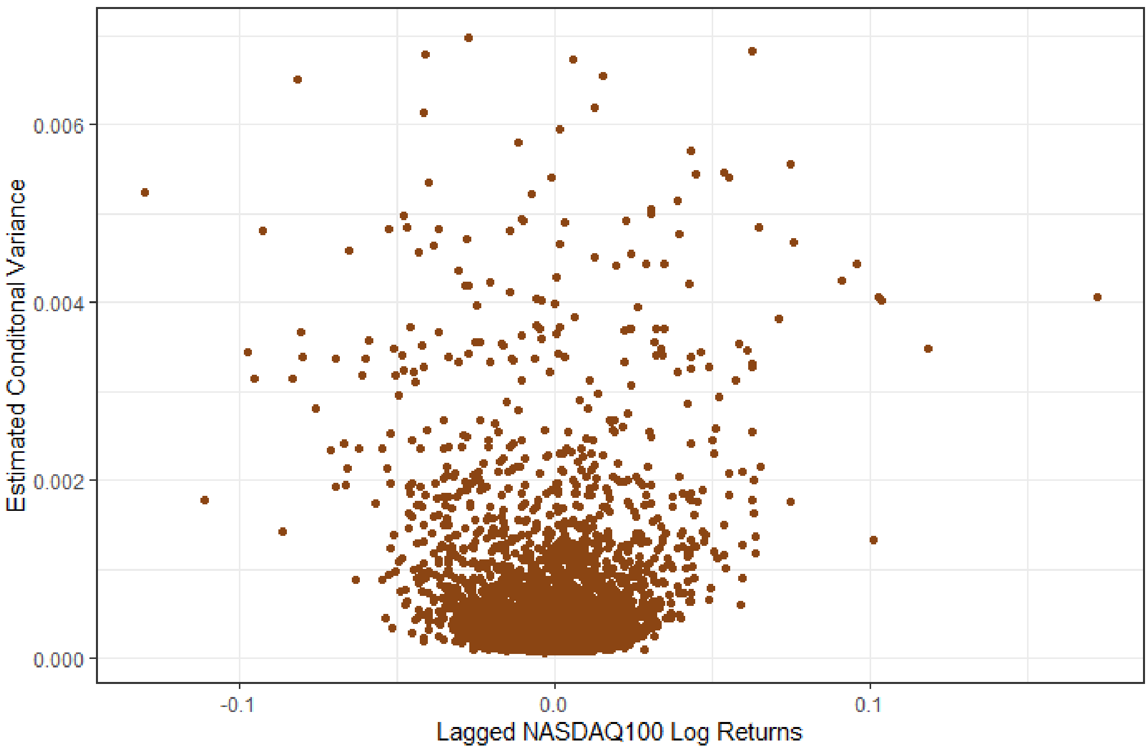

Figure 5.27 Conditional Variance Estimate of the NASDAQ100 Log Returns

Residual Analysis on NASDAQ 100

The standardized residuals' ACF plot, obtained from applying the BSGARCH (1,1) model to the NASDAQ100 dataset, is depicted in Figure 5.28. The analysis reveals that the BSGARCH (1,1) method effectively captures and incorporates all the available information in the data. The mean of the standardized residuals is found to be approximately zero, indicating that the model produces unbiased estimates.

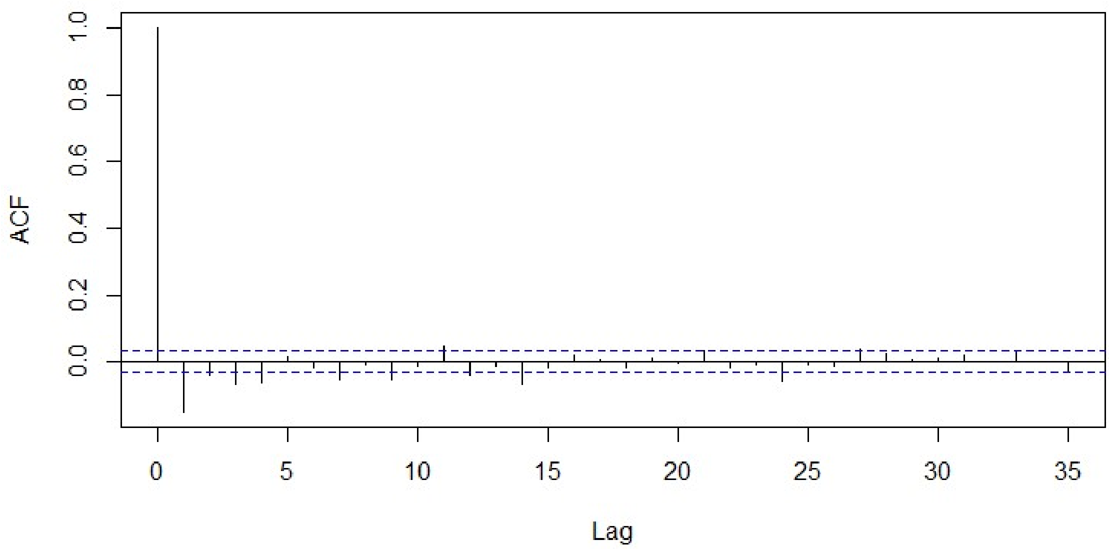

Figure 5.28 ACF Plot of Standardized Residuals (NASDAQ100)

The McLeod-Li test was conducted on the standardized residuals of the BSGARCH(1,1) model applied to the NASDAQ100 data. The test was performed for lags 10, 15, 20, 25, and 30, and the outcome are presented in Table 5.18. The p-values for all lags were greater than the 5% significance level, indicating that there is insufficient proof to recommend that the residuals of the BSGARCH (1,1) model are not i.i.d Therefore, the new BSGARCH(1,1) is deemed adequate for for predicting the NASDAQ100 data, and the calculated coefficients and forecast values are efficient and unbiased.

Table 5.18 The McLeod - Li Test for ARCH effect in Standardised Residuals For NASDAQ100

Number of lags	Test Statistic	P-Value
10	3.981	0.948
15	15.021	0.450
20	16.440	0.689
25	19.325	0.781
30	21.347	0.877

Performance Evaluation on NASDAQ 100

Figure 5.29 displays the results of the performance evaluation of the models considered in this thesis implemented on the NASDAQ100 data. The new BSGARCH(1,1) generally outperforms all the other methods, indicating that it improves upon the accuracy of the parametric models. In financial time series analysis, it is often difficult to calculate accurately the "true" conditional variances of the data. Instead, the realized conditional variances are used, which are estimated from historical data, to measure the performance of competing approaches. These realized variances are inherently noisy due to the presence of measurement error, which can lead to small differences between competing approaches (Bekierman and Manner, 2018).

However, in the case of the new BSGARCH(1,1) model, the improvement observed in this study was not just small, but rather substantial. This indicate that the BSGARCH(1,1) model is able to determine important aspects of financial time series dynamics that are not well-captured by the classical GARCH(1,1)-type, models and that this improvement is significant.

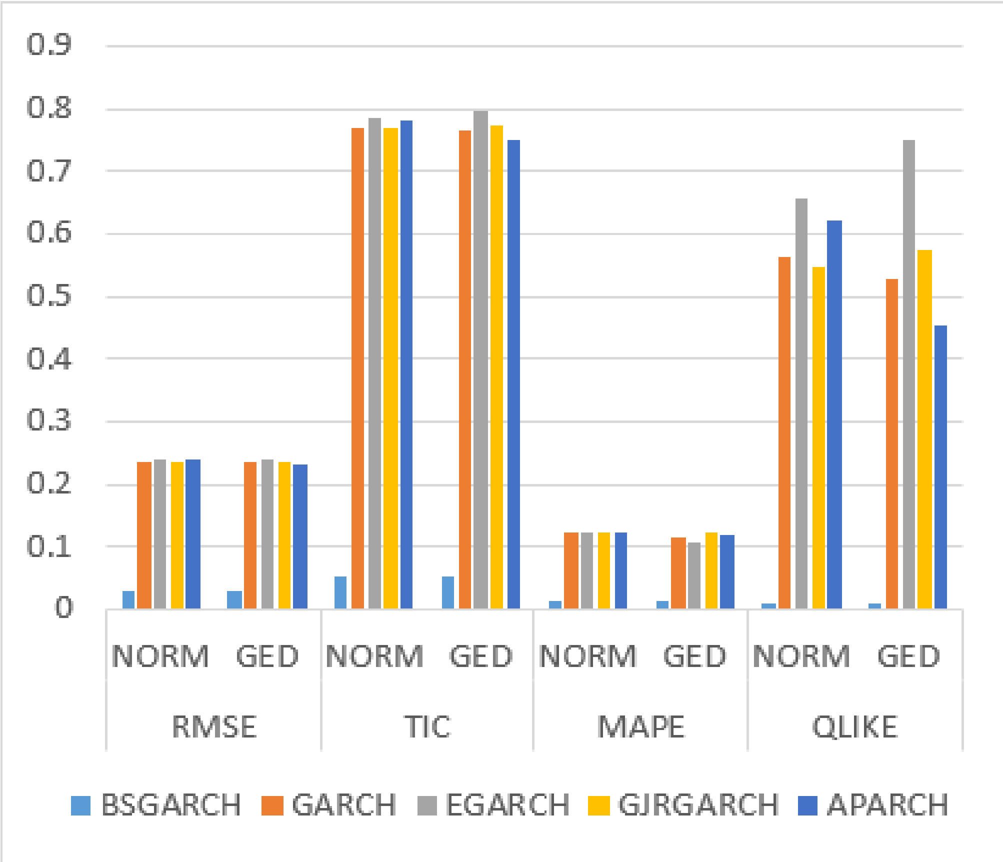

Figure 5.29 Peformance Evaluation on NASDAQ100

The aSPA test was also applied to the NASDAQ100 dataset to verify the significance of the improvements observed with the proposed BSGARCH(1,1) model. The results are presented in Table 5.19, where positive values of the statistic indicate that the BSGARCH(1,1) model outperforms the competitors.

Table 5.19 aSPA Test for NASDAQ100

Models	Error Distribution	Test Value	P-Value
GARCH vs BSGARCH	GED	175.63	< 0.001
	NORM	203.97	< 0.001
EGARCH vs BSGARCH	GED	276.30	< 0.001
	NORM	263.94	< 0.001
GJR-GARCH vs BSGARCH	GED	208.64	< 0.001
	NORM	197.29	< 0.001
APARCH vs BSGARCH	GED	152.52	< 0.001
	NORM	225.05	< 0.001

5.5.3 DJIA

The plot in Figure 5.30 depicts the optimal estimates of conditional variance obtained using the proposed model and how they relate to the log returns. The estimated function for conditional variance shows a significant degree of nonlinearity and asymmetry with respect to past lagged log returns of the series. Notably, the series exhibits a leverage effect, especially for higher values of past lagged log returns.

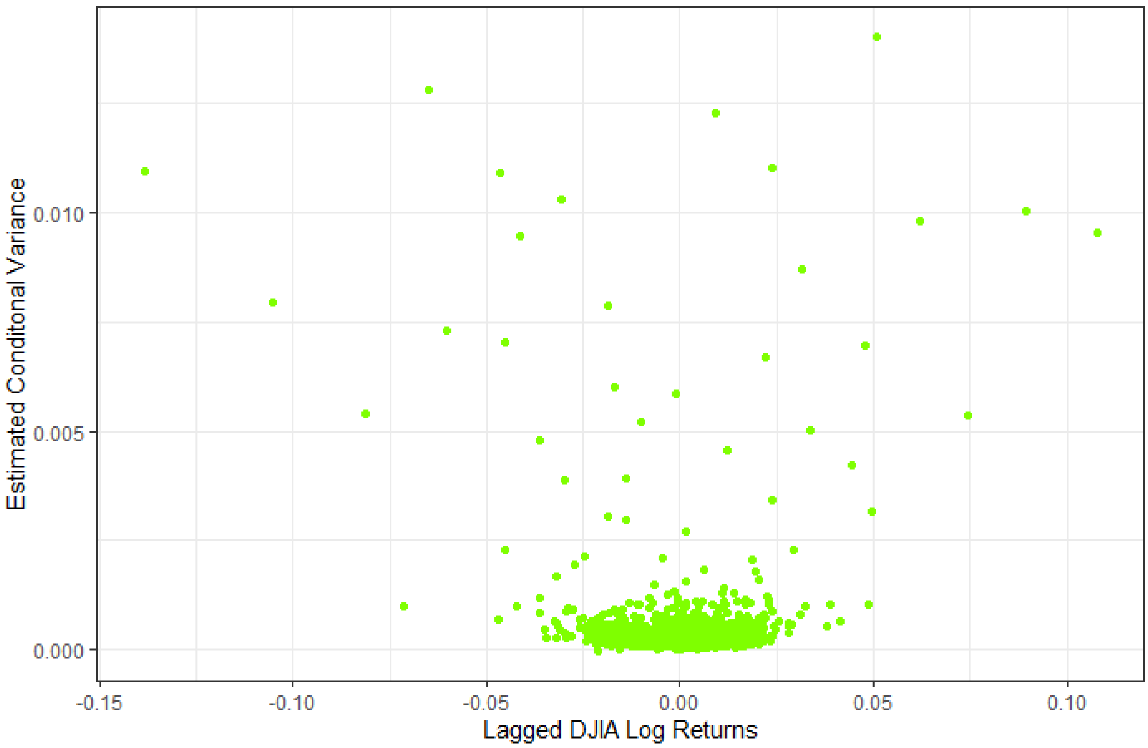

Figure 5.30 Conditional Variance Estimate of the DJIA Log Returns

Residual Analysis

The ACF plot of the standardized residuals obtained from the BSGARCH (1,1) model implemented on the DJIA is presented in Figure 5.31. The outcome indicated that the new BSGARCH (1,1) generated forecasts that included all information available. The expected value of the standardized residuals was approximately zero. There was no significant correlation in the series of standardized residuals.

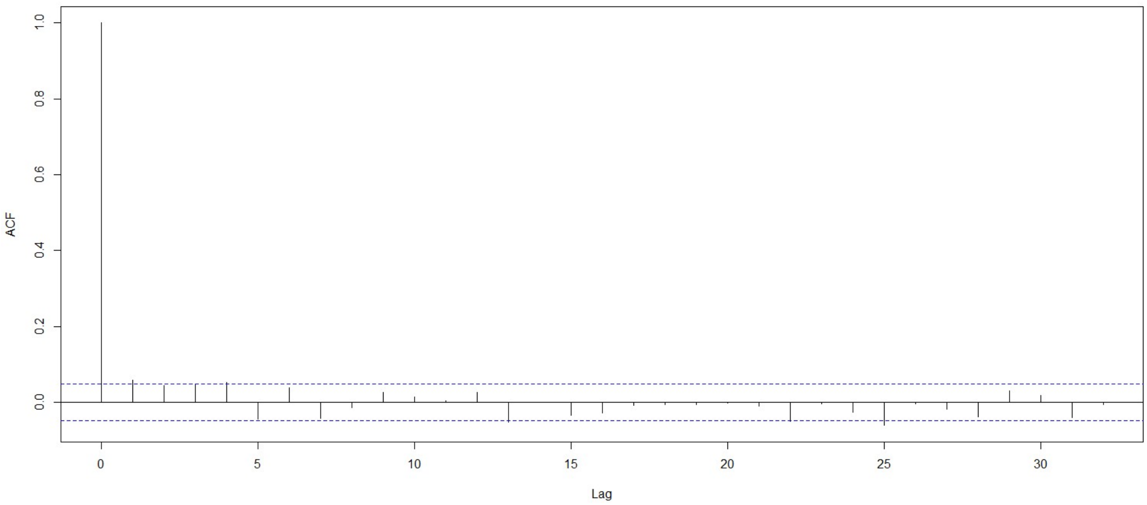

Figure 5.31 ACF Plot of Standardized Residuals (DJIA)

The McLeod-Li test was conducted on the standardized residuals of the BSGARCH(1,1) model applied to the DJIA data. The test was performed for lags 10, 15, 20, 25, and 30, and the results are presented in Table 5.20. The p-values for all lags were greater than the 5% significance level, indicating that there is insufficient evidence to reject the null hypothesis. Therefore, the BSGARCH(1,1) model is deemed adequate to fit the DJIA data, and the calculated coefficients and forecast values are efficient and unbiased.

Table 5.20 The McLeod - Li Test for ARCH effect in Standardised Residuals For DJIA

Number of lags	P-Value	Test Statistic
10	0.315	11.569
15	0.197	19.369
20	0.326	22.262
25	0.589	22.810
30	0.721	25.089

Performance Evaluation on DJIA

Figure 5.32 presents a summary of the performance results for the different volatility estimation models considered in this thesis. The outcome demonstrate that the

BSGARCH(1,1) method consistently performs better than all the competing approaches in terms of accuracy. The BSGARCH(1,1) model significantly improves upon the accuracy of the parametric models, indicating its effectiveness in capturing and predicting volatility in the DJIA dataset. This finding highlights the superior performance and predictive power of the BSGARCH(1,1) model in comparison to the conventional approaches.

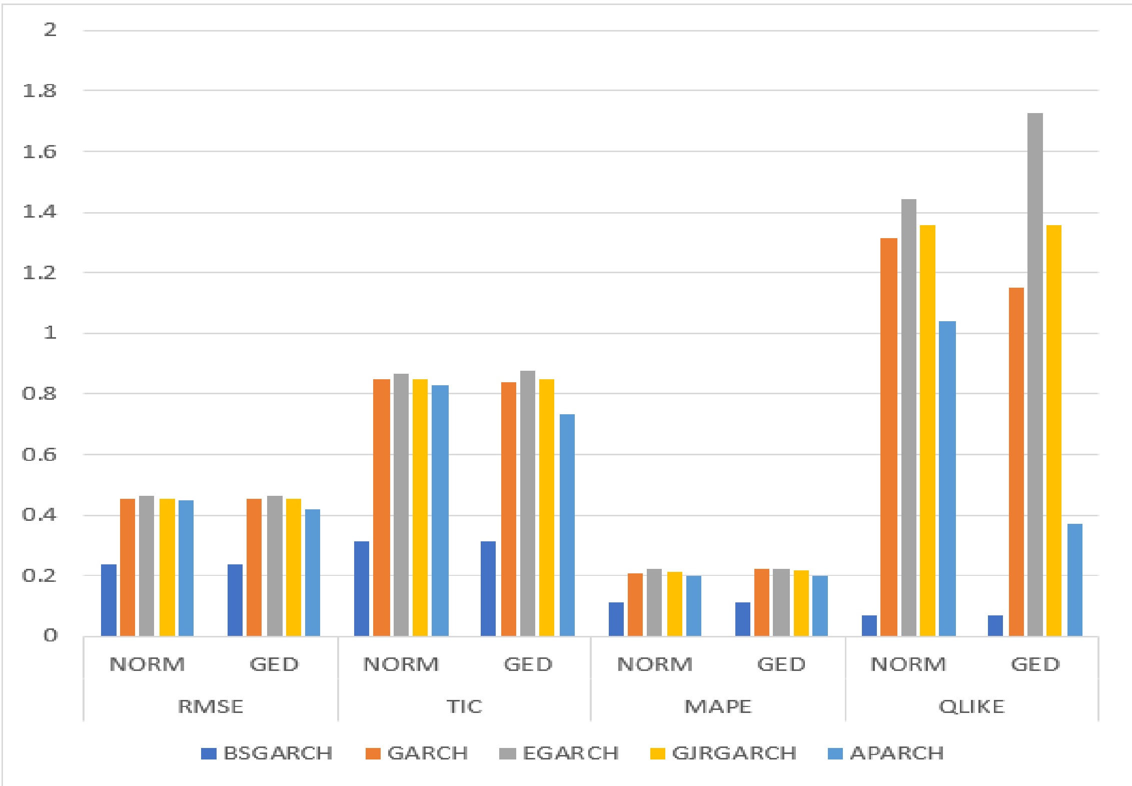

Figure 5.32 Peformance Evaluation on DJIA

The significance of the improvements made by the new BSGARCH(1,1) model was evaluated using aSPA tests. The outcome are presented in Table 5.21, where positive values of the statistic means that the new BSGARCH(1,1) model performs better than the competitors. The table confirms that the new BSGARCH(1,1) model has significantly higher predictive power in terms of conditional variance prediction than the other models considered.

Table 5.21 aSPA for DJIA

Models	Error Distribution	Test Value	P-Value
GARCH vs BSGARCH	GED	5238.18	< 0.001
	NORM	6911.40	< 0.001
EGARCH vs BSGARCH	GED	6770.40	< 0.001
	NORM	5534.06	< 0.001
GJR-GARCH vs BSGARCH	GED	3106.90	< 0.001
	NORM	3859.04	< 0.001
APARCH vs BSGARCH	GED	4784.54	< 0.001
	NORM	3859.04	< 0.001

5.5.4 NIKKEI225

The optimal conditional variance estimates obtained using the new proposed model is plotted against the log returns and presented in Figure 5.33. The estimated conditional variance functions is highly nonlinear and asymmetric in past lagged log returns of the series. Leverage effect is noticeable in the series especially for high values of past lagged log returns.

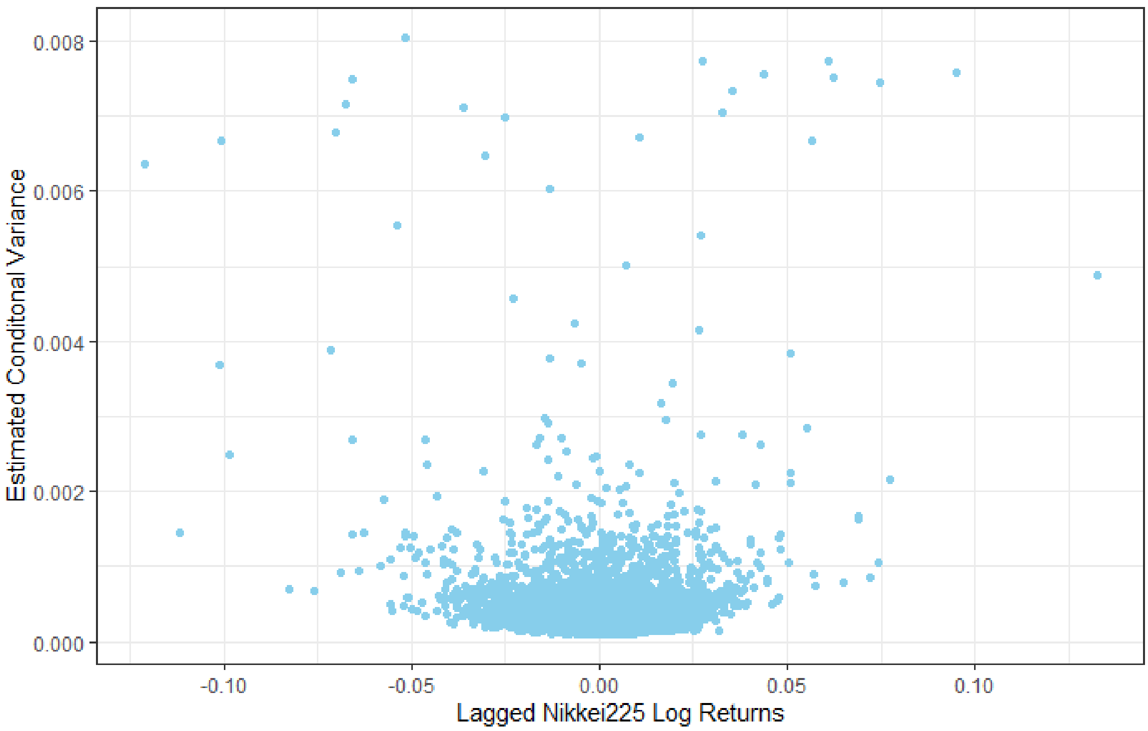

Figure 5.33 Conditional Variance Estimate of the NIKKEI225 Log Returns

Residual Analysis

The ACF plot of the standardized residuals obtained from the BSGARCH (1,1) model implemented on the NIKKEI225 is presented in Figure 5.34. The results indicated that the proposed BSGARCH (1,1) generated forecasts that included all information available. The expected value of the standardized residuals was approximately zero. There was no significant correlation in the series of standardized residuals.

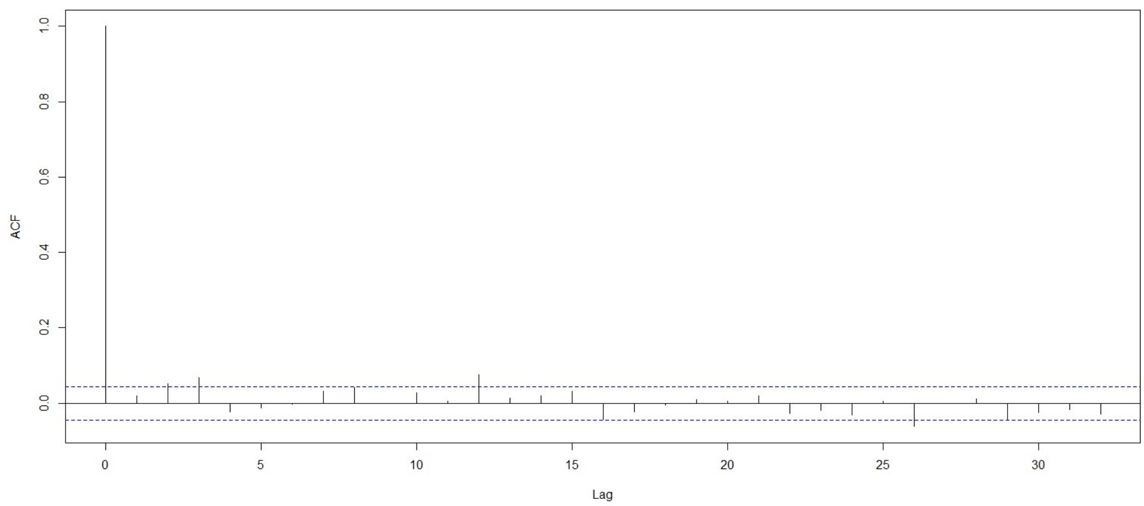

Figure 5.34 ACF of Standardized Residuals(NIKKEI225)

The McLeod-Li test was conducted on the standardized residuals of the BSGARCH(1,1) model implemented on NIKKEI225 data. The test was performed for lags 10, 15, 20, 25, and 30, and the outcome are provided in Table 5.22. The p-values for all lags were greater than the 5% significance level, indicating that there is insufficient proof to reject the null hypothesis. Therefore, the BSGARCH(1,1) model is deemed adequate to fit the NIKKEI225 data, and the calculated coefficients and forecast values are efficient and unbiased.

Table 5.22 The McLeod - Li Test for ARCH effect in Standardised Residuals For NIKKEI225

Number of lags	Test Statistic	P-Value
10	15.801	0.105
15	20.554	0.152
20	22.935	0.292
25	26.218	0.396
30	30.186	0.456

Performance Evaluation on NIKKEI225

In Figure 5.35, the performance results of different volatility estimation models are presented. Notably, the BSGARCH(1,1) method consistently performs better than all other competing approaches. This indicates that the BSGARCH(1,1) model provides improved volatility estimation compared to the traditional parametric models.

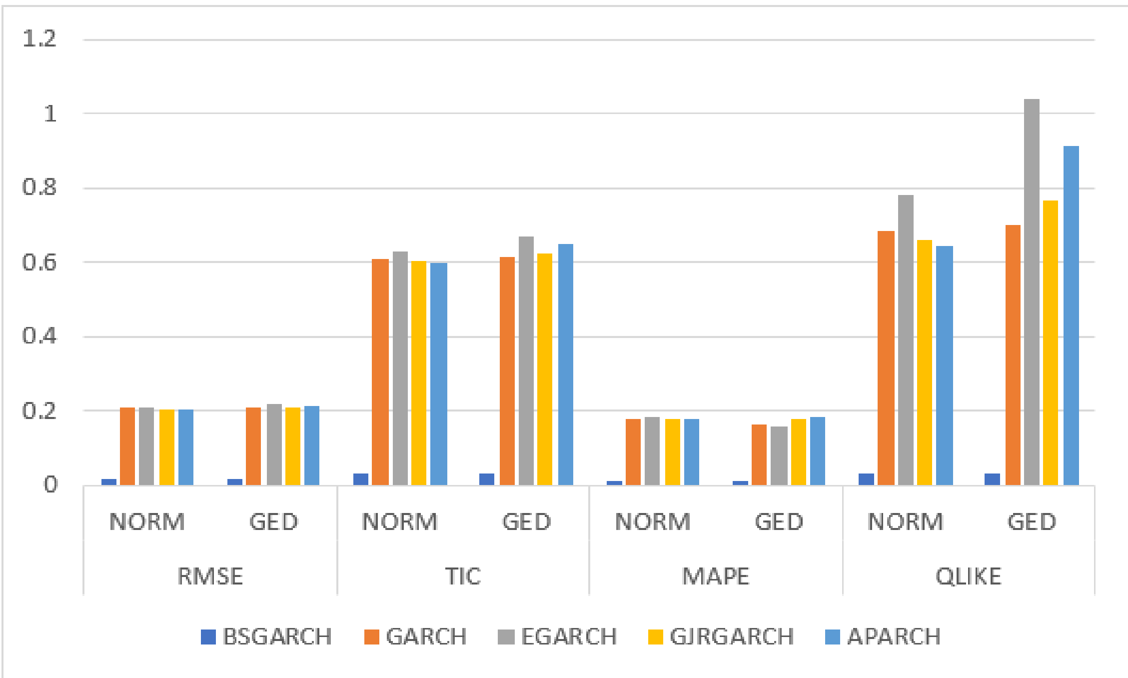

Figure 5.35 Peformance Evaluation on NIKKEI225

To assess the significance of the improvements observed, the aSPA tests, was conducted. The outcome are provided in Table 5.23. Positive values of the statistic consistently favor the proposed BSGARCH(1,1) model. The outcome in Table 5.23 confirm the superior predictive power of the BSGARCH(1,1) model over its competitors.

Table 5.23 aSPA Test For NIKKEI225

Models	Error Distribution	Test Value	P-Value
GARCH vs BSGARCH	GED	71.72	< 0.001
	NORM	72.20	< 0.001
EGARCH vs BSGARCH	GED	84.72	< 0.001
	NORM	79.91	< 0.001
GJR-GARCH vs BSGARCH	GED	76.79	< 0.001
	NORM	76.26	< 0.001
APARCH vs BSGARCH	GED	82.53	< 0.001
	NORM	76.26	< 0.001

5.6 GSE-CI

The optimal conditional variance estimates obtained using the new proposed model is plotted against the log returns and presented in Figure 5.36. The estimated conditional variance function exhibits a highly nonlinear and asymmetric relationship with past lagged log returns of the series. This suggests the presence of a leverage effect, particularly for high values of past lagged log returns.

Figure 5.36 Conditional Variance Estimate of the GSE-CI Log Returns

Residual Analysis

The ACF plot of the standardized residuals derived from applying the BSGARCH (1,1) model to the GSE-CI data is shown in Figure 5.37. The results demonstrate that the BSGARCH (1,1) method effectively captured and included all available information in the data. The expected value of the standardized residuals was observed to be approximately zero, indicating that the model produced unbiased estimates.

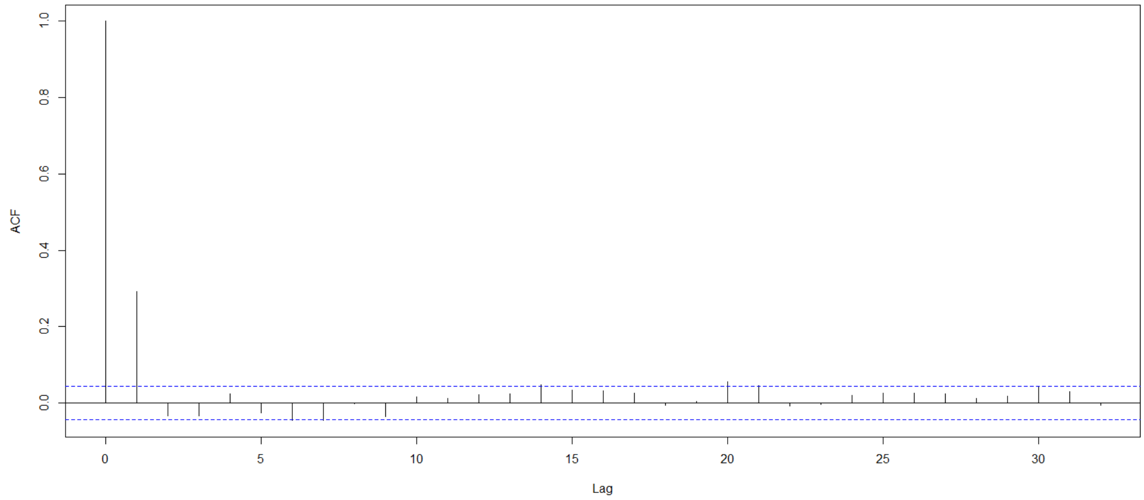

Figure 5.37 ACF of Standardized Residuals (GSE-CI)

The standardized residuals of the BSGARCH(1,1) model applied to the GSE-CI data were subjected to the McLeod-Li test for autocorrelation at lags 10, 15, 20, 25, and 30. The outcome of the test are presented in Table 5.24. The p-values corresponding to each lag were found to be larger than the 5% significance level, indicating that there is insufficient proof to reject the null hypothesis.This suggests that the new BSGARCH(1,1) is suitable for the GSE-CI data, and the calculated coefficients and forecast values are efficient and unbiased.

Table 5.24 The McLeod - Li Test for ARCH effect in Standardised Residuals For GSE-CI

Number of lags	Test Statistic	P-Value
10	5.288	0.871
15	12.543	0.638
20	16.390	0.692
25	33.833	0.112
30	37.297	0.169

Performance Evaluation on GSE-CI

The performance of the various volatility models, was assessed on the GSE-CI dataset. The evaluation was based on four metrics: QLIKE, MAPE, RMSE and TIC. The outcome are summarized in Figure 5.38. The performance analysis reveals that the BSGARCH (1,1) model consistently performs better than all other models across all metrics. It achieves the lowest values for RMSE and MAPE, indicating superior accuracy in estimating conditional variance. Additionally, the BSGARCH (1,1) model exhibits the lowest TIC value, suggesting better forecasting performance compared to the competing models. Although the QLIKE values for the BSGARCH (1,1) model are comparable to the GJR-GARCH (1,1) and APARCH (1,1) models, they are significantly lower than those of the GARCH (1,1) and EGARCH (1,1) models. Overall, these results highlight the improved predictive power of the BSGARCH (1,1) model in capturing the volatility dynamics of the GSE-CI dataset.

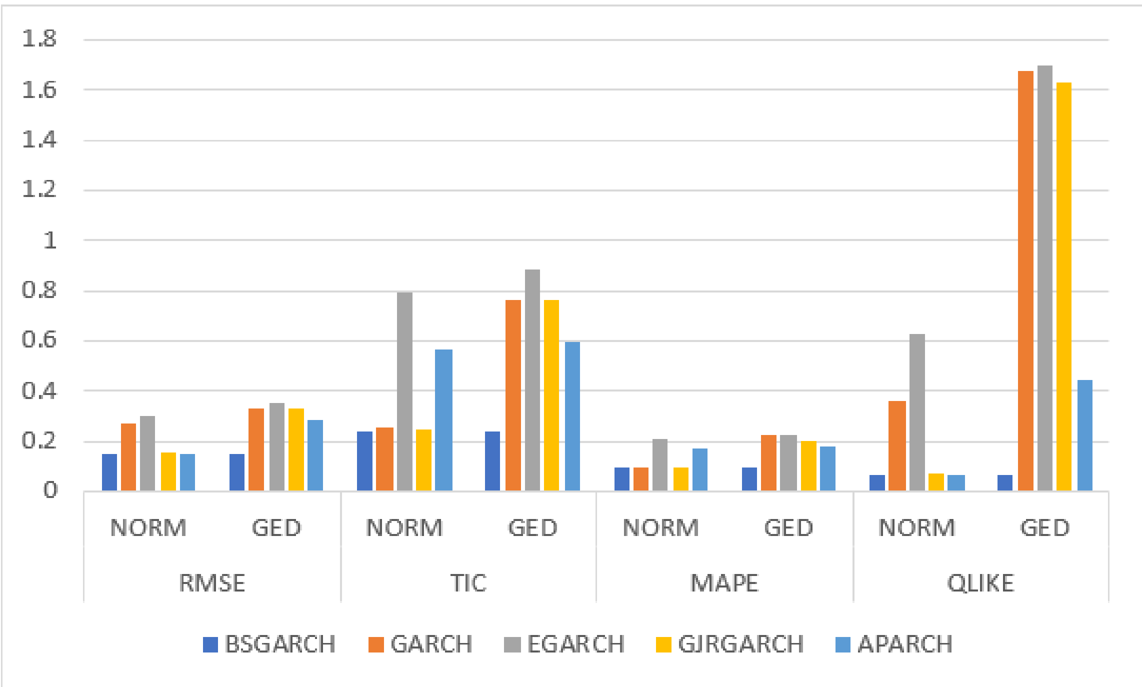

Figure 5.38 Peformance Evaluation on GSE-CI

To assess the significance of the improvements observed, the aSPA tests, was conducted. The outcome are provided in Table 5.25. Positive values of the statistic consistently favor the new BSGARCH(1,1) model. The outcome in Table 5.25 confirm the superior predictive power of the BSGARCH(1,1) model over its competitors.

Table 5.25 aSPA Test for GSE-CI

Models	Error Distribution	Test Value	P-Value
BSGARCH vs GARCH	GED	412.49	< 0.001
	NORM	629.66	< 0.001
BSGARCH vs EGARCH	GED	626.17	< 0.001
	NORM	628.22	< 0.001
BSGARCH vs GJR-GARCH	GED	634.81	< 0.001
	NORM	627.43	< 0.001
BSGARCH vs APARCH	GED	623.91	< 0.001
	NORM	627.43	< 0.001

Comparison of the SplineGARCH Model with BSGARCH (1,1) Model

A comparative analysis was conducted between the proposed BSGARCH (1,1) model and the Spline-GARCH model introduced by Engle and Rangel (2008). Utilizing data from (Engle and Rangel, 2008), the performance of these models was assessed using RMSE, MAPE, TIC, and QLIKE as evaluation metrics. The main objective was to evaluate their predictive capabilities using out-of-sample data. The results, presented in Figure 5.39, indicated that the Spline-GARCH model slightly outperformed the proposed BSGARCH (1,1) model.

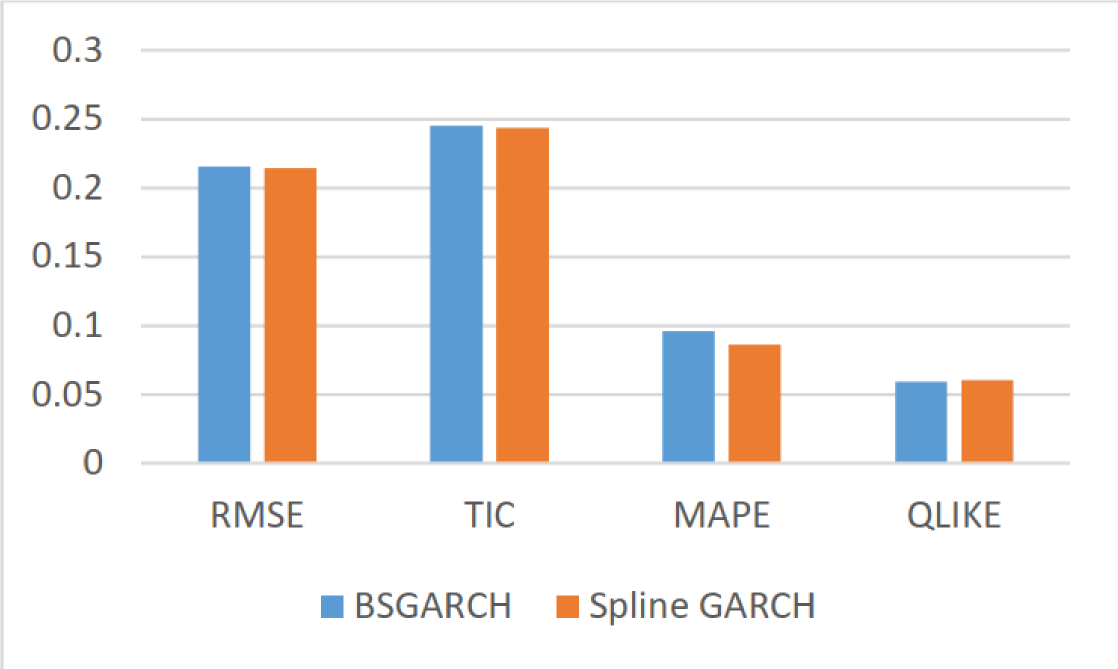

Figure 5.39 BSGARCH (1,1) vs SplineGARCH (1,1) Model

To statistically test the difference in performance, a two-tailed T-test was conducted (see Table 5.26). The null hypothesis (H_0) stated that there is no significant difference between the performance of the two models, while the alternative hypothesis (H_1) stated that there is a significant difference. Based on the T-test results, the p-value was calculated as 0.9134, which is greater than the significance level of 0.05. Therefore, we fail to reject the null hypothesis, indicating that there is no significant difference in performance between the two models. These findings suggest that the new BSGARCH (1,1) model can be considered as a valid alternative to the Spline-GARCH model, offering comparable predictive accuracy and easier implementation.

Table 5.26 Comparison of Performances: Two Tailed Test Results

Test	Statistic	P-value
t-test	0.1134	0.9134

CHAPTER 6

CONCLUSIONS, CONTRIBUTION AND RECOMMENDATIONS

6.1 Conclusions

The research presented a novel approach, the BSGARCH(1,1) model, for estimating the conditional variance of financial time series. The model exhibited superior performance compared to popular models such asAPARCH(1,1), GARCH(1,1), GJR-GARCH(1,1) and EGARCH(1,1) across multiple datasets and various performance metrics. The consistent outperformance of the new BSGARCH(1,1) model in terms of RMSE, TIC, MAPE, and QLIKE metrics demonstrates its effectiveness in capturing the volatility patterns inherent in financial data.

Moreover, the statistically significant improvements in conditional variance prediction demonstrated by the classical average Superior Predictive Ability (aSPA) test further support the superiority of the BSGARCH(1,1) model. These findings indicate that the proposed model can enhance the accuracy and predictive power of conditional variance evaluation.

The research contributes to the existing body of knowledge by introducing a new model that performs better than the traditional GARCH-type models. By incorporating the BSpline framework, the BSGARCH(1,1) model captures the nonlinear and asymmetrical dynamics of financial time series, as well as the leverage effect observed in the data. The empirical results validate the potential of the BSGARCH(1,1) model as a valuable tool for volatility modelling and forecasting, highlighting its superiority over widely used alternatives.

These findings have practical implications for market participants, risk analysts, and financial decision-makers. The accurate estimation of conditional variance is crucial for various financial applications, including option pricing, portfolio optimization, and risk management strategies. The BSGARCH(1,1) model can provide more reliable volatility forecasts, enabling market participants to better manage their exposure to market risks and make informed decisions.

In conclusion, the research demonstrates the effectiveness and superiority of the proposed BSGARCH(1,1) model in estimating the conditional variance of financial time series. The model's improved accuracy, predictive power, and ability to capture the dynamics of financial markets make it a valuable tool for researchers and practitioners in the field of financial econometrics.

6.2 Recommendations

The study finally recommend the following:

i. From the outcome obtained from the study, it is recommended that financial analysts and researchers consider using the proposed BSGARCH (1,1) model in their financial time series modelling. This is because the model outperformed other popular models such as GJRGARCH, GARCH, APARCH and EGARCH in terms of accuracy and predictive power. Using the BSGARCH (1,1) model as an alternative could lead to more reliable and accurate financial forecasts.

ii. The findings suggest that using more sophisticated models such as the BSGARCH (1,1) model can lead to more accurate predictions of the conditional variance of financial returns. Therefore, it is recommended to explore more sophisticated models in financial modelling.

iii. The results suggest that financial returns exhibit a high degree of nonlinearity and asymmetry with respect to past lagged log returns of the series. This implies that simple linear models may not be appropriate for modelling the financial returns. Therefore, it is recommended to consider nonlinear models in financial modelling.

iv. Future research could investigate the applicability of the model to other financial datasets, as well as explore ways to further improve its performance.

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APPENDIX A VALUES OF PERFORMANCE EVALUATION ON SIMULATED DATA

Table A.1 Performance Evaluation on Simulated Data (Scenario 1, Scenario 2 and Scenario 3)

	ED	BSGARCH	GARCH	EGARCH	GJRGARCH	APARCH
Scenario 1						
RMSE	NORM	4.9332	5.5791	5.5809	5.5778	5.5773
	GED	5.7862	6.5377	6.5386	6.5365	6.5367
TIC	NORM	0.9391	0.9630	0.9649	0.9628	0.9632
	GED	0.9453	0.9629	0.9629	0.9633	0.9634
MAPE	NORM	1.2391	1.2571	1.2592	1.2566	1.2576
	GED	1.2525	1.2714	1.2721	1.2710	1.2707
QLIKE	NORM	1.7449	4.8728	5.1087	4.7125	5.2820
	GED	1.9904	5.3272	5.9296	5.1521	5.4620
Scenario 2						
RMSE	NORM	0.9035	0.9165	0.9169	0.9164	0.9159
	GED	0.9035	0.9164	0.9169	0.9160	0.9160
TIC	NORM	0.9044	0.9493	0.9507	0.9493	0.9471
	GED	0.9044	0.9493	0.9507	0.9480	0.9476
MAPE	NORM	0.1580	0.1623	0.1624	0.1623	0.1621
	GED	0.1580	0.1623	0.1624	0.1622	0.1621
QLIKE	NORM	0.0977	0.2698	0.2826	0.2672	0.2543
	GED	0.0977	0.2683	0.2822	0.2554	0.2572
Scenario 3						
RMSE	NORM	2.2200	2.2200	2.2253	2.2224	2.2200
	GED	2.2200	2.2200	2.2256	2.2217	2.2200
TIC	NORM	0.9157	0.9121	0.9198	0.9175	0.9149
	GED	0.9157	0.9119	0.9201	0.9170	0.9149
MAPE	NORM	0.9372	0.9353	0.9395	0.9382	0.9355
	GED	0.9372	0.9352	0.9396	0.9379	0.9368
QLIKE	NORM	0.2162	0.2192	0.2394	0.2260	0.2165
	GED	0.2162	0.2184	0.2409	0.2230	0.2297

Table A.2 Performance Evaluation on Simulated Data (Scenario 4, Scenario 5 and Scenario 6)

	ED	BSGARCH	GARCH	EGARCH	GJRGARCH	APARCH
Scenario 4			0			
RMSE	NORM	0.1474	0.1643	0.1657	0.1653	0.1642
	GED	0.1474	0.1643	0.1657	0.1644	0.1641
TIC	NORM	0.1307	0.1461	0.1485	0.1478	0.1454
	GED	0.1307	0.1462	0.1485	0.1462	0.1432
MAPE	NORM	0.1427	0.1591	0.1604	0.1601	0.1587
	GED	0.1427	0.1591	0.1605	0.1592	0.1588
QLIKE	NORM	0.0249	0.0315	0.0454	0.0404	0.0279
	GED	0.0249	0.0315	0.0458	0.0317	0.0288
Scenario 5						
RMSE	NORM	0.5259	0.5622	0.5667	0.5664	0.5613
	GED	0.5259	0.5671	0.5667	0.5670	0.5657
TIC	NORM	0.7468	0.7970	0.7971	0.7961	0.7818
	GED	0.7468	0.7982	0.7970	0.7977	0.7942
MAPE	NORM	0.7277	0.7765	0.7829	0.7824	0.7753
	GED	0.7337	0.7834	0.7829	0.7832	0.7814
QLIKE	NORM	0.4198	0.5406	0.6466	0.4830	0.7316
	GED	0.4198	0.6522	0.6198	0.6912	0.8232
Scenario 6						
RMSE	NORM	0.0161	0.2049	0.0226	0.0348	0.4000
	GED	0.0161	0.2054	0.2038	0.3418	0.4084
TIC	NORM	0.1162	0.1961	0.1417	0.1802	0.3312
	GED	0.1162	0.1961	0.3592	0.1961	0.3318
MAPE	NORM	0.0375	0.2124	0.0779	0.1202	0.3454
	GED	0.0375	0.2128	0.1412	0.3541	0.3526
QLIKE	NORM	0.0045	0.0090	0.0041	0.0041	0.0095
	GED	0.0045	0.0090	0.0217	0.0090	0.0095

Table A.3 Performance Evaluation on Simulated Data (Corresponding to Figure 5.7 and Figure 5.8)

	ED	BSGARCH	GARCH	EGARCH	GJRGARCH	APARCH
Figure 5.7			0			
RMSE	NORM	0.7409	0.7420	0.7431	0.7428	0.7420
	GED	0.7409	0.7420	0.7430	0.7420	0.7420
TIC	NORM	0.4894	0.4908	0.4922	0.4918	0.4908
	GED	0.4894	0.4908	0.4922	0.4909	0.4908
MAPE	NORM	0.1413	0.1415	0.1417	0.1417	0.1415
	GED	0.1413	0.1415	0.1417	0.1415	0.1415
QLIKE	NORM	0.1557	0.2102	0.2906	0.2646	0.2108
	GED	0.1557	0.2110	0.2896	0.2117	0.2095
Figure 5.8						
RMSE	NORM	0.6232	1.0388	1.0403	1.0399	1.0388
	GED	0.6232	1.0388	1.0402	1.0388	1.0388
TIC	NORM	0.4393	0.9817	0.9844	0.9837	0.9817
	GED	0.4393	0.9817	0.9844	0.9818	0.9817
MAPE	NORM	0.0365	0.1415	0.1417	0.1417	0.1415
	GED	0.0365	0.1415	0.1417	0.1415	0.1415
QLIKE	NORM	0.0488	0.2102	0.2906	0.2646	0.2108
	GED	0.0488	0.2110	0.2896	0.2117	0.2095

APPENDIX B VALUES OF PERFORMANCE EVALUATION ON REAL TIME SERIES

Table B.1 Performance Evaluation for S&P500, NASDAQ100 and DJIA

	ED	BSGARCH	GARCH	EGARCH	GJRGARCH	APARCH
S&P500						
RMSE	NORM	0.0393	0.2631	0.2664	0.2629	0.2523
	GED	0.0393	0.2615	0.2682	0.2621	0.2555
TIC	NORM	0.0720	0.8165	0.8354	0.8154	0.7576
	GED	0.0720	0.8076	0.8461	0.8112	0.7751
MAPE	NORM	0.0187	0.1249	0.1264	0.1248	0.1197
	GED	0.0187	0.1237	0.1237	0.1244	0.1213
QLIKE	NORM	0.0161	0.8601	1.0932	0.8487	0.4586
	GED	0.0161	0.7737	1.2658	0.8070	0.5444
NASDAQ						
RMSE	NORM	0.0302	0.2366	0.2391	0.2361	0.2382
	GED	0.0302	0.2355	0.2412	0.2369	0.2326
TIC	NORM	0.0526	0.7708	0.7856	0.7683	0.7804
	GED	0.0526	0.7647	0.7981	0.7728	0.7484
MAPE	NORM	0.0154	0.1209	0.1222	0.1207	0.1218
	GED	0.0154	0.1141	0.1071	0.1211	0.1189
QLIKE	NORM	0.0084	0.5622	0.6541	0.5480	0.6196
	GED	0.0084	0.5291	0.74855	0.5737	0.4536
DJIA						
RMSE	NORM	0.2381	0.4554	0.4615	0.4561	0.4504
	GED	0.2381	0.4526	0.4645	0.4561	0.4197
TIC	NORM	0.3151	0.8460	0.8668	0.8481	0.8287
	GED	0.3151	0.8363	0.8776	0.8481	0.7310
MAPE	NORM	0.1130	0.2101	0.2206	0.2153	0.1992
	GED	0.1130	0.2243	0.2252	0.2165	0.1992
QLIKE	NORM	0.0712	1.3152	1.4419	1.3554	1.0416
	GED	0.0712	1.1520	1.7268	1.3551	0.3740

Table B.2 Performance Evaluation for NIKKEI225 and GSE-CI

	ED	BSGARCH	GARCH	EGARCH	GJRGARCH	APARCH
NIKKEI						
RMSE	NORM	0.0159	0.2074	0.2114	0.2060	0.2052
	GED	0.0159	0.2081	0.2196	0.2109	0.2160
TIC	NORM	0.0299	0.6087	0.6278	0.6024	0.5986
	GED	0.0299	0.6120	0.6687	0.6257	0.6505
MAPE	NORM	0.0137	0.1785	0.1819	0.1773	0.1766
	GED	0.0137	0.1639	0.1597	0.1815	0.1859
QLIKE	NORM	0.0333	0.6862	0.7791	0.6588	0.6427
	GED	0.0333	0.7012	1.012	0.7679	0.9113
GSE-CI						
RMSE	NORM	0.1467	0.2704	0.2953	0.1521	0.1463
	GED	0.1467	0.3270	0.3538	0.3263	0.2810
TIC	NORM	0.2412	0.2512	0.7964	0.2469	0.5639
	GED	0.2412	0.7641	0.8820	0.7613	0.5931
MAPE	NORM	0.0917	0.0947	0.2091	0.0934	0.1701
	GED	0.0917	0.2228	0.2230	0.2039	0.1756
QLIKE	NORM	0.0668	0.3596	0.6220	0.0681	0.0666
	GED	0.0668	1.6750	1.6980	1.631	0.4470

APPENDIX C A 10-DAY VOLATILITY FORECAST USING THE BSGARCH(1,1) MODEL

Table C.1 A 10-Day Volatility Forecast Using BSGARCH(1,1)

Day Forecast	S&P500	NASDAQ100	DJIA	NIKKEI225	GSE-CI
T+1	0.053602	0.061514	0.051121	0.055735	0.044303
T+2	0.053604	0.061534	0.051166	0.055781	0.044276
T+3	0.053632	0.061525	0.051211	0.055850	0.044265
T+4	0.053614	0.061500	0.051181	0.055614	0.044276
T+5	0.053620	0.061516	0.051184	0.055589	0.044297
T+6	0.053793	0.061496	0.051183	0.055638	0.044305
T+7	0.053773	0.061484	0.051192	0.055515	0.044310
T+8	0.053742	0.061462	0.051195	0.055476	0.044309
T+9	0.053701	0.061451	0.051209	0.055473	0.044280
T+10	0.053671	0.061449	0.051221	0.055472	0 044253

APPENDIX D LIST OF ACRONYMS AND ABBREVIATIONS

Acronym	**Meaning**
APARCH	Asymmetric Power Autoregressive Conditional Heteroskedasticity
ARCH	Autoregressive Conditional Heteroskedasticity
BSGARCH	Basis Spline Generalized Autoregressive Conditional Heteroskedasticity
DJIA	Dow Jones Industrial Average
ED	Error Distribution
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedasticity
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GED	Generalized Error Distribution
GJR-GARCH	Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity
GSE-CI	Ghana Stock Exchange Composite Index
MAPE	Mean Absolute Percentage Error
NASDAQ	National Association of Securities Dealers Automated Quotations
NORM	Normal Distribution
QLIKE	Quasi-Likelihood Loss Function
RMSE	Root Mean Squared Error
S & P 500	Standard & Poor's 500
TIC	Theil's Inequality Coefficient

APPENDIX E PUBLICATIONS ARISING FROM RESEARCH

PAPER 1

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Hindawi

Research Article

Hybrid Model for Stock Market Volatility

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Empirical evidence suggests that the traditional GARCH-type models are unable to accurately estimate the volatility of financial markets. To improve on the accuracy of the traditional GARCH-type models, a hybrid model (BSGARCH (1, 1)) that combines the flexibility of B-splines with the GARCH (1, 1) model has been proposed in the study. The lagged residuals from the GARCH (1, 1) model are fitted with a B-spline estimator and added to the results produced from the GARCH (1, 1) model. The proposed BSGARCH (1, 1) model was applied to simulated data and two real financial time series data (NASDAQ 100 and S&P 500). The outcome was then compared to the outcomes of the GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1), and APARCH (1, 1) with different error distributions (ED) using the mean absolute percentage error (MAPE), the root mean square error (RMSE), Theil's inequality coefficient (TIC) and QLIKE. It was concluded that the proposed BSGARCH (1, 1) model outperforms the traditional GARCH-type models that were considered in the study based on the performance metrics, and thus, it can be used for estimating volatility of stock markets.

1. Introduction

Extensive empirical and theoretical research has been conducted on modelling and forecasting stock market volatility over the past three decades [1–3]. This line of inquiry is motivated by a number of factors. Arguably, volatility is one of the most significant concepts in the whole of finance. Volatility is frequently used as a rough indicator of the total risk of financial assets. The estimation or forecast of a volatility parameter is used in many value-at-risk models

estimation has played a significant role in the fields of statistics, economics, and finance since the seminal work of Engle [8, 9]. Several different techniques attempt to address the issue of estimating the volatility of a financial asset, but the Generalised Autoregressive Conditional Heteroskedastic (GARCH) models have often been used in estimating the volatility of financial time series of stock returns [10, 11]. Especially, the GARCH (1, 1) has been proven to be one of the best predictive models in estimating the volatility of the stock market.

PAPER 2

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Modelling the volatility of the Ghana stock market: A comparative study

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Abstract
The Ghana stock market is considered attractive to both local and international investors, as it is a developing market with potential for growth. The volatility of stock returns is one of the crucial features of Ghana's stock market that should be carefully taken into account by any investor or policymaker. As a result, the GARCH, TGARCH, and EGARCH models were used in this study to analyze the volatility of the Ghanaian stock market. The models were assessed using Akaike Information Criterion (AIC), RMSE and MAPE. The TGARCH (1,1) with generalized error distribution was the model that suited the data the best based on the AIC, RMSE, and MAPE values.

Keywords: Ghana Stock Exchange (GSE), Volatility, GARCH Models, Log Returns

1. Introduction

Wealth creation is the aim of every investor as the value of financial assets changes over time. The stock market is a desirable place to invest. A stock market may be defined as the regulatory environment that permits the trading of shares of several businesses or organizations (Gregoriou, 2009) [1]. Long regarded as significant economic development stimulators are stock exchanges. They give people and organizations that want to invest their savings or extra money by buying securities access to a regulated market for trading securities. Since the Ghana Stock Exchange is a developing market, both local and foreign investors are thought to be interested in taking advantage of the chance to make money on stock market. The volatility of stock returns is one of the crucial factors that must be carefully taken into account by any investor or policymaker (Dufitinema, 2021 Henriksen, 2011) [2, 3]. Volatility refers to a statistical calculation that measures the level of dispersion or variability in the returns of a particular security or market index (Brooks, 2008) [4]. In the last three decades

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